Stochastic Modeling and Applications
Vol.25 No.1 (January-June, 2021)

Received: 25th January 2021 Revised: 22nd March 2021 Selected: 30th March 2021

MARKOV MODEL OF COVID-19 DISEASE PROGRESSION

TIRUPATHI RAO PADI, KANIMOZHI.V *, AND SAKKEEL P.T

Abstract. This study is on Markov Modeling of the disease progression of COVID-19. Transition probability matrices are obtained for different states of disease progression. Probability distributions for the successive occurrences of same state with a run length of two and three days are derived from the deduced transition probability matrices. Model behavior is analysed by deriving different mathematical relations of statistical measures using pearson coefficient's. Numerical illustrations were given for sensitivity analysis. This study will be helpful to the health care takers for getting the indicators of disease intensity.

1. Introduction

A novel coronavirus, officially called COVID-19 by world health organization (WHO) caused an outbreak of a typical pneumonia. The first case was detected in the city of Wuhan, China which is the capital of Hubei province on december 2019 and then rapidly spread out in the entire world [5] [9]. The WHO has declared COVID-19 as a pandemic. The COVID-19 is spread through the droplets of infected individuals by coughing, sneezing, or close interaction with infected individuals. The symptoms of COVID-19 appear within 2 to 14 days after exposure and include fever, cough, a runny nose, and difficulty in breathing [2].

In India, first case was reported in kerala on 27^{th} January, 2020. A 20 year old female was admitted in Thrissur general hospital with the symptom of sore throat and dry cough [6]. As of March 8, 2021 around 11.2 million cases were found infective with the virus and 158 thousands of deaths occur in India [7]. The government of India implemented preventive measures such as lockdown, isolation, and quarantining the infected individuals, promoting social distancing and wearing face mask, individual Public sanitation etc.

Mathematical modeling of infectious diseases has an important role in epidemiology, it gives a better understanding of disease progression. In this paper, we study the dynamic behavior of the COVID-19 and its characteristics based on the markov processes to predict the probable number of positive cases of infected patients in the future. The basic assumption of the markov process is that the future condition entirely depends on the current condition, not on the previous one.

Key words and phrases. Markov Model, Transition Probability Matrices, Disease Progression, COVID-19.

*corresponding author .

Sha He et al (2020) developed a discrete-time stochastic model with the help of binomial distribution in order to study the transmission of covid-19 (corona virus-2019) disease [10]. Asymptomatic covid-19 patients changed as a super-spreader unknowingly. If we identify that type of infected patients well in advance then will avoid further spread to the susceptible group of people so, Shreekanth et al (2020) used hidden markov model in their study to know better appraisal of the spread of the disease on Indian context [11]. Non-linear markov chain model with three states namely infectious, recovered, dead was used to evaluate daily covid-19 cases by Muammer et al (2020) [8]. Respiratory related disease like middle east respiratory syndrome (MERS) and severe acute respiratory syndrome (SARS), influenza's symptoms and corona virus symptoms are similar, it was a hurdle to the health care people to provide proper treatment to the patients. So, the government lags in tracing the corona virus infected patients, in order to avoid such issues Larsen et al (2020) used markov process to differentiate covid-19 from other diseases [4]. Sultan Hussain et al (2020) developed an SIR (susceptible-infectious-recovered) model in order to understand the control and spread of covid-19 infectious disease and also the developed model's standard was investigated [12].

This study has given focus on development of constructing a markov model based on transition probabilities. We develop probability distributions for different states in a span of 1 day, 2 day and 3 day sequences. The study also derived the statistical characteristic based on the probability distributions. All pearson coefficient's mathematical formulae were derived from the developed probability distributions. In order to understand the model behaviour in a common man's perception, numerical data through from the sources of wikipedia is collected and analysed. Indicators on intensity of states like *Decrease State*, *Remain Same State* and *Increase State* are estimated from the data. Interpretation and inferences are arrived from the numerical illustration.

2. Stochastic Model

This model intends to derive probability mass functions of the discrete distribution of number of states. Let the states of transitions be of three categories, namely State – 1: *Decrease*; State – 2: *Remain Same*; State – 3: *Increase*.
p_{ij} - probability of transition from i^{th} state to j^{th} state.

$$p_{ij} : pr\left\{x_n = j/x_{n-1} = i\right\}; i, j = 1, 2, 3$$
$$p_{ij} \geq 0 \;\forall\; i, j = 1, 2, 3 \text{ and } \sum_{j=1}^{3} p_{ij} = 1; \forall\; i = 1, 2, 3$$

Let there be 'i' and 'j' states in which 'i' is the state of previous trial and 'j' be the state of current trial $i, j = 1, 2, 3$ where, $1, 2, 3$ represents the identified positive cases in the states of *Decrease*, *Remain Same*, *Increase* respectively.

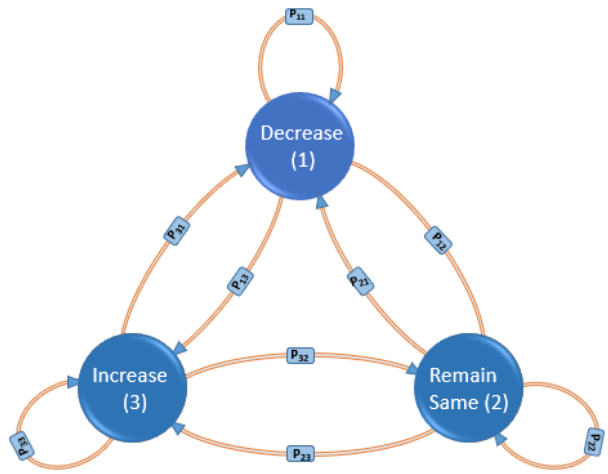

FIGURE 1. Schematic diagram for three state Markov model of COVID-19 spread

3. Probability mass function for one day length of sequence

$P[X(S)] = n$, Let S be the random variable which refers to the number of times the specific state of study is occurred. S can be of three different states namely *Decrease*, *Remain Same* and *Increase* (i.e) S = D (or) R (or) I. n will be the number of times the events tend to happen in that particular state, 'n' can be 0,1. Where 0 will be non-happening of the event, 1 will be the event of occurring once of the specific state under study.

3.1. Probability mass function of "*Decrease State*".

$$P[X(D)] = \begin{cases} \sum_{j=2}^{3} p(j); & X(D) = 0 \\ p(1); & X(D) = 1 \end{cases} \tag{3.1}$$

Characteristic function, $$\Phi_{[X(D)]}(t) = \sum_{j=2}^{3} p(j) + e^{it} p(1) \tag{3.2}$$

3.2. Probability mass function of "*Remain Same State*".

$$P[X(R)] = \begin{cases} \sum_{\substack{j=1 \\ j\neq 2}}^{3} p(j); & X(R) = 0 \\ p(2); & X(R) = 1 \end{cases} \tag{3.3}$$

Characteristic function, $$\Phi_{[X(R)]}(t) = \sum_{\substack{j=1 \\ j\neq 2}}^{3} p(j) + e^{it} p(2) \tag{3.4}$$

3.3. Probability mass function of "*Increase State*".

$$P[X(I)] = \begin{cases} \sum_{j=1}^{2} p(j); & X(I) = 0 \\ p(3); & X(I) = 1 \end{cases} \tag{3.5}$$

$$\text{Characteristic function, } \Phi_{[X(I)]}(t) = \sum_{j=1}^{2} p(j) + e^{it} p(3) \tag{3.6}$$

3.4. Statistical characteristics. In this section, some statistical characteristics are derived for the probability distribution given in the equations (3.1),(3.2) and (3.3) respectively. In this p(i) will be the probability for happening of the states where the values of 'i' will be 1 which indicates probability of "*Decrease state*", 2 which indicates probability of "*Remain Same state*" and 3 indicates probability of "*Increase state*".

$$\text{Mean, } E[X(S)] = p(i); \forall\ i = 1, 2, 3 \tag{3.7}$$

$$\text{Variance, } V[X(S)] = p(i)[1 - p(i)]; \forall\ i = 1, 2, 3 \tag{3.8}$$

Third and fourth central moments

$$\mu_3[X(S)] = p(i)(1 - p(i))(1 - 2p(i)); \forall\ i = 1, 2, 3 \tag{3.9}$$

$$\mu_4[X(S)] = p(i)(1 - p(i))[1 - 3p(i)(1 - p(i))]; \forall\ i = 1, 2, 3 \tag{3.10}$$

Shaping of measure

$$\beta_1[X(S)] = \{p(i)(1 - p(i))(1 - 2p(i))\}^2 \{[p(i)(1 - p(i))]^3\}^{-1}; \forall\ i = 1, 2, 3 \tag{3.11}$$

Kurtosis measure

$$\beta_2[X(S)] = \{p(i)(1 - p(i))(1 - 3p(i))(1 - p(i))\}\{[p(i)(1 - p(i))]^2\}^{-1}; \forall\ i = 1, 2, 3 \tag{3.12}$$

$$\text{Coefficient of variation} = [\{p(i)[1 - p(i)]\}^{1/2} p(i)^{-1}] * 100;\ \forall\ i = 1, 2, 3 \tag{3.13}$$

4. Probability mass function for two day length of sequence

On the similar lines of section '3' the successive occurrence of the study states in a length of two days are considered in this section. The event of none (or) once (or) twice in a run of two occurrences are modeled for deriving the probability distributions. The number of occurrences 'n' is having the possibilities $n = 0, 1, 2$.

4.1. Probability mass function of "*Decrease State*".

$$P[X(D)] = \begin{cases} \sum_{j=2}^{3} [\sum_{i=2}^{3} p(i) p_{ij}^{(2)}]; & X(D) = 0 \\ \sum_{i=2}^{3} p(i).p_{i1}^{(2)} + p(1) \sum_{j=2}^{3} p_{1j}^{(2)}; & X(D) = 1 \\ p(1).p_{11}^{(2)}; & X(D) = 2 \end{cases} \tag{4.1}$$

4.2. Probability mass function of "*Remain Same State*".

$$P[X(R)] = \begin{cases} \sum_{\substack{j=1 \\ j\neq 2}}^{3} \sum_{\substack{i=1 \\ i\neq 2}}^{3} p(i)p_{ij}^{(2)}; & X(R) = 0 \\ \sum_{\substack{i=1 \\ i\neq 2}}^{3} p(i)p_{i2}^{(2)} + p(2) \sum_{\substack{j=1 \\ j\neq 2}}^{3} p_{2j}^{(2)}; & X(R) = 1 \\ p(2)p_{22}^{(2)}; & X(R) = 2 \end{cases} \tag{4.2}$$

4.3. Probability mass function of "*Increase State*".

$$P[X(I)] = \begin{cases} \sum_{j=1}^{2} [\sum_{i=1}^{2} p(i)p_{ij}^{(2)}]; & X(I) = 0 \\ \sum_{i=1}^{2} p(i)p_{i3}^{(2)} + p(3) \sum_{j=1}^{2} p_{3j}^{(2)}; & X(I) = 1 \\ p(3)p_{33}^{(2)}; & X(I) = 2 \end{cases} \tag{4.3}$$

4.4. Statistical characteristics. Some statistical characteristics are explored for the probability distributions shown in the above equations (4.1),(4.2),(4.3). Let us consider, α is the probability of happening of the sate once (i.e) $P[X(S)] = 1$, β is the probability of happening of the state twice (i.e) $P[X(S)] = 2$ and γ is the probability of non-happening of the state (i.e) $P[X(S)] = 0$.

$$\text{Mean}, E[X(S)] = \alpha + 2\beta \tag{4.4}$$

$$\text{Variance}, V[X(S)] = \alpha\ (1-\alpha) + 4\beta\ (1-\alpha-\beta) \tag{4.5}$$

Third and fourth central moments

$$\mu_3[X(S)] = 2\alpha^3 + 16\beta^3 - 3\alpha^2\ (1-4\beta) - 24\beta^2\ (1-\alpha) + \alpha\ (1-18\beta) + 8\beta \tag{4.6}$$

$$\begin{aligned}\mu_4[X(S)] =& \alpha + 4\beta(4-10\alpha) - 4\alpha^2(1-12\beta) - 8\beta^2(8-15\alpha+9\alpha^2) \\ &+ 6\alpha^3(1-4\beta) + 96\beta^3(1-\alpha) - 3\alpha^4 - 48\beta^4\end{aligned} \tag{4.7}$$

Shaping of measure

$$\begin{aligned}\beta_1[X(S)] =& \{2\alpha^3 + 16\beta^3 - 3\alpha^2\ (1-4\beta) - 24\beta^2\ (1-\alpha) + \alpha\ (1-18\beta) + 8\beta\}^2 \\ &\{[\alpha\ (1-\alpha) + 4\beta\ (1-\alpha-\beta)]^3\}^{-1}\end{aligned} \tag{4.8}$$

Kurtosis measure

$$\begin{aligned}\beta_2[X(S)] =& \alpha + 4\beta(4-10\alpha) - 4\alpha^2(1-12\beta) - 8\beta^2(8-15\alpha+9\alpha^2) \\ &+ 6\alpha^3(1-4\beta) + 96\beta^3(1-\alpha) - 3\alpha^4 - 48\beta^4 \\ &\left\{[\alpha\ (1-\alpha) + 4\beta\ (1-\alpha-\beta)]^2\right\}^{-1}\end{aligned} \tag{4.9}$$

$$\text{Coefficient of variation} = [\{\alpha\ (1-\alpha) + 4\beta\ (1-\alpha-\beta)\}^{1/2}\{\alpha+2\beta\}^{-1}] * 100 \tag{4.10}$$

$$\text{Characteristic function}, \Phi_{[X(S)]}(t) = \gamma + e^{it}\alpha + e^{2it}\beta \tag{4.11}$$

5. Probability mass function for three day length of sequence

As mentioned in section '3' the successive occurrence of the study states in a length of three days are considered in the section. The event of none (or) once (or) twice (or) thrice in a run of three occurrences are modeled for deriving the probability distributions. The number of occurrences 'n' is having the possibilities $n = 0, 1, 2, 3$.

5.1. Probability mass function of "*Decrease State*".

$$P[X(D)] = \begin{cases} \sum_{i=2}^{3}\sum_{j=2}^{3}\sum_{k=2}^{3} p(i,j)p_{jk}^{(3)}]; & X(D)=0 \\ \sum_{k=2}^{3}[\sum_{i=1}^{2}\sum_{\substack{j=1\\ i\neq j}}^{2} p(i,j)p_{jk}^{(3)} + \sum_{\substack{i=1\\ i\neq 2}}^{3}\sum_{\substack{j=1\\ j\neq 2\\ i\neq j}}^{3} p(i,j)p_{jk}^{(3)}] + \sum_{i=2}^{3}\sum_{j=2}^{3} p(i,j)p_{j1}^{(3)}; & X(D)=1 \\ \sum_{k=2}^{3} p(1,1)p_{1k}^{(3)} + \sum_{i\neq j=1}^{2} p(i,j)p_{j1}^{(3)} + \sum_{\substack{i\neq j=1\\ i\neq 2\\ j\neq 2}}^{3} p(i,j)p_{j1}^{(3)}; & X(D)=2 \\ p(1,1)p_{11}^{(3)}; & X(D)=3 \end{cases} \tag{5.1}$$

5.2. Probability mass function of "*Remain Same State*".

$$P[X(R)] = \begin{cases} \sum_{\substack{i=1\\ i\neq 2}}^{3}\sum_{\substack{j=1\\ j\neq 2}}^{3}\sum_{\substack{k=1\\ k\neq 2}}^{3} p(i,j)p_{jk}^{(3)}]; & X(R)=0 \\ \sum_{\substack{k=1\\ k\neq 2}}^{3}[\sum_{i=1}^{2}\sum_{\substack{j=1\\ i\neq j}}^{2} p(i,j)p_{jk}^{(3)} + \sum_{i=2}^{3}\sum_{\substack{j=2\\ i\neq j}}^{3} p(i,j)p_{jk}^{(3}] + \sum_{\substack{i=1\\ i\neq 2}}^{3}\sum_{\substack{j=1\\ j\neq 2\\ i\neq j}}^{3} p(i,j)p_{j2}^{(3)}; & X(R)=1 \\ \sum_{\substack{k=1\\ k\neq 2}}^{3} p(2,2)p_{2k}^{(3)} + \sum_{i\neq j=1}^{2} p(i,j)p_{j2}^{(3)} + \sum_{i\neq j=2}^{3} p(i,j)p_{j2}^{3}; & X(R)=2 \\ p(2,2)p_{22}^{(3)}; & X(R)=3 \end{cases} \tag{5.2}$$

5.3. Probability mass function of "*Increase State*".

$$P[X(I)] = \begin{cases} \sum_{\substack{i=1\\ i\neq 2}}^{3}\sum_{\substack{j=1\\ j\neq 2}}^{3}\sum_{\substack{k=1\\ k\neq 2}}^{3} p(i,j)p_{jk}^{(3)}]; & X(I)=0 \\ \sum_{k=1}^{3}[\sum_{\substack{i=1\\ i\neq 2}}^{3}\sum_{\substack{j=1\\ j\neq 2\\ i\neq j}}^{3} p(i,j)p_{jk}^{(3)} + \sum_{i=2}^{3}\sum_{\substack{j=2\\ i\neq j}}^{3} p(i,j)p_{jk}^{(3)}] + \sum_{i=1}^{2}\sum_{\substack{j=1\\ i\neq j}}^{2} p(i,j)p_{j3}^{(3)}; & X(I)=1 \\ \sum_{k=1}^{2} p(3,3)p_{3k}^{(3)} + \sum_{\substack{i\neq j=1\\ i\neq 2\\ j\neq 2}}^{3} p(i,j)p_{j3}^{(3)} + \sum_{i\neq j=2}^{3} p(i,j)p_{j3}^{3}; & X(I)=2 \\ p(3,3)p_{33}^{(3)}; & X(I)=3 \end{cases} \tag{5.3}$$

5.4. Statistical characteristics. In this section, statistical characteristics are obtained for the probability distribution given in the equations (5.1),(5.2),(5.3). Let us consider, a is the probability of happening of the state once, (i.e) $P[X(S)] = 1$, b is the probability of happening of the state twice (i.e) $P[X(S)] = 2$, c is the probability of happening of the state thrice (i.e) $P[X(S)] = 3$ and d is the probability of non-happening of the state (i.e) $P[X(S)] = 0$

$$\text{Mean}, E[X(S)] = a + 2b + 3c \tag{5.4}$$

$$\text{Variance}, V[X(S)] = a[1 - a - 6c] + 4b[1 - b - a] + 3c[3 - 3c - 4b] \tag{5.5}$$

Third and fourth central moments

$$\begin{aligned}\mu_3[X(S)] =& 2a^3 + 16b^3 + 54c^3 - 3a^2[1 - 4b - 6c] - 24b^2[1 - a - 3c] \\ &- 9c^2[9 - 6a - 12b] + a[1 - 18b] + b[8 - 90c] + c[27 - 36a] + 72abc \end{aligned} \tag{5.6}$$

$$\begin{aligned}\mu_4[X(S)] =& a[1 - 120c] + 4b[4 - 10a] + 3c[27 - 104b] + a^2[48b - 4 + 90c - 72b^2 \\ &- 162c^2 - 216bc] + b^2[120a - 64 + 504c - 648c^2 - 432ac] + c^2 \\ &[378a - 324 + 864b - 648ab] + 6a^3[1 - 4b - 6c] + 96b^3[1 - a - 3c] \\ &+ 18c^3[27 - 18a - 36b] - 3a^4 - 48b^4 - 243c^4 + 432abc \end{aligned} \tag{5.7}$$

Shaping of measure

$$\begin{aligned}\beta_1[X(S)] = &\{2a^3 + 16b^3 + 54c^3 - 3a^2[1 - 4b - 6c] - 24b^2[1 - a - 3c] - 9c^2 \\ &[9 - 6a - 12b] + a[1 - 18b] + b[8 - 90c] + c[27 - 36a] + 72abc\}^2 \\ &\{[a(1 - a - 6c) + 4b(1 - b - a) + 3c(3 - 3c - 4b)]^3\}^{-1} \end{aligned} \tag{5.8}$$

Kurtosis measure

$$\begin{aligned}\beta_2[X(S)] =& a[1 - 120c] + 4b[4 - 10a] + 3c[27 - 104b] + a^2[48b - 4 + 90c \\ &- 72b^2 - 162c^2 - 216bc] + b^2[120a - 64 + 504c - 648c^2 - 432ac] \\ &+ c^2[378a - 324 + 864b - 648ab] + 6a^3[1 - 4b - 6c] + 96b^3[1 - a - 3c] \\ &+ 18c^3[27 - 18a - 36b] - 3a^4 - 48b^4 - 243c^4 + 432abc \\ &\{\{a[1 - a - 6c] + 4b[1 - b - a] + 3c[3 - 3c - 4b]\}^2\}^{-1} \end{aligned} \tag{5.9}$$

$$\begin{aligned}\text{Coefficient of variation} =& [\{a[1 - a - 6c] + 4b[1 - b - a] + 3c[3 - 3c - 4b]\}^{1/2} \\ &\{a + 2b + 3c\}^{-1}] * 100 \end{aligned} \tag{5.10}$$

Characteristic function, (5.11)

$$\Phi_{[X(S)]}(t) = d + e^{it}a + e^{2it}b + e^{3it}c$$

6. Results and discussion

6.1. Description of methodology. This model behaviour is studied with the help of corona data set which was collected from internet sources [3]. The data is obtained as total number of positive cases from 30^{th} January to 25^{th} August 2020. The number of incidences are arrived by subtracting the previous day total number of cases from the current day total number of cases. The number of incidences per day are categorised as three states of transitions namely *Decrease*, *Remain Same* and *Increase*.
The Transition of "*Decrease*" is arrived if the current day number of cases are less than the previous day. The state "*Remain Same*" is arrived if there is no change in the number of cases between the current and previous days. The state of "*Increase*" is arrived when the number of cases on the current day are more than the previous day. The transition frequency table and transition probability matrices are obtained from the arrived states of transition while considering the definition.

$$p_{ij} = pr\,\{\text{the state of the current day} = j/\text{the state of the previous day} = i\}$$

The numerical values of probabilities are obtained from the developed equations (i.e) (3.1,3.2 and 3.3), (4.1,4.2 and 4.3) and (5.1,5.2 and 5.3)

6.2. Model behaviour for the state of "*Decrease*". In this section, behaviour of the *Decrease* state is analysed and its result is given.

TABLE 1. Probability distribution for one day length of sequence

X(D)	0	1
P[X(D)]	0.5764	0.4236

From the table 1, it is observed that non happening of the state is having the chance 0.5764 and happening of the state have chance 0.4236. Hence, it may be inferred that non happening of *Decrease State* has more likely than the happening of *Decrease State*.

TABLE 2. Probability distribution for two day length of sequence

X(D)	0	1	2
P[X(D)]	0.3367	0.4814	0.1820

From table 2, it is observed that non occurrence of the state is having chance 0.3367 and happening of state once is 0.4814 and occurrence of state twice in a run have chance 0.1820. Hence, we may interpret the result that occurrence of *Decrease State* once has more likely when compared to other.

TABLE 3. Probability distribution for three day length of sequence

X(D)	0	1	2	3
P[X(D)]	0.1952	0.4194	0.3081	0.0772

From table 3, it is observed that non-happening of *Decrease State* have the chance of 0.1952 and chance of happening of state once in three day's run is 0.4194, chance of happening of *Decrease State* twice is 0.3081, chance of happening of *Decrease State* thrice is 0.0772. Hence, we may infer the result that happening of *Decrease State* once has more likely when compared to other states in a study of three days successive happenings. Graphical representation of the table is given for better understanding.

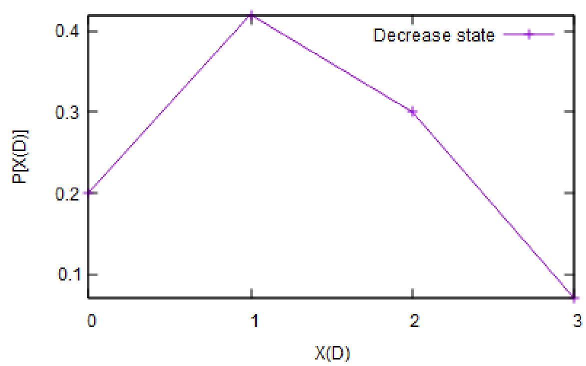

FIGURE 2. Probability of *Decrease State* for 3 day's

TABLE 4. Statistical Results

Statistical measures	1 day	2 day's	3 day's
Mean	0.4236	0.8453	1.2672
Variance	0.2442	0.4947	0.7407
3^{rd} central moment	0.0373	0.0786	0.1174
4^{th} central moment	0.0653	0.4957	1.2903
Beta 1	0.0956	0.0510	0.0339
Beta 2	1.0956	2.0253	2.3518
Coefficient of variation	116.64	83.20	67.92

It is observed from the table 4 that the average occurrence of "*Decrease State*" is 1 out of three days run study, less than one day out of 2 day's study. Variance of 3 day's study is 0.74 and for 2 day's it is 0.49. Therefore, it may be inferred that occurrence of *Decrease State* has more variability in 3 day's study when compared to 2 day's study.
$\mu_3 = 0.12$ and $\beta_1 = 0.03$, which reveals that there is a positive skewness in the distribution of *Decrease State* in a study of 3 day's run. Which indicates that the average number of occurrences is more than the model number of occurrences of *Decrease State*. Measure of β_2 indicates the increased peakedness for the increased length of study regarding the state of "*Decrease*"

6.3. Model behaviour for the state of "*Remain Same*". Behaviour of *Remain Same* state is studied in this section and the result is given below.

TABLE 5. Probability distribution for one day length of sequence

X(R)	0	1
P[X(R)]	0.9811	0.0189

From table 5, it is observed that the non happening of state is having the chance 0.98 and state of *Remain Same* have chance 0.01. Hence, it may be inferred that non happening of *Remain Same* State has more likely when compared to the happening of *Remain Same* State.

TABLE 6. Probability distribution for two day length of sequences

X(R)	0	1	2
P[X(R)]	0.9659	0.0318	0.0024

From table 6, it is observed that non happening of *Remain Same* state is having the chance 0.97 and occurrence of state once have chance 0.03 and occurrence of state twice have chance 0.0024. Hence, it is observed that non happening of *Remain Same* state has more likely when compared to the others.

TABLE 7. Probability distribution for three day length of sequences

X(R)	0	1	2	3
P[X(R)]	0.9496	0.0470	0.0033	0.00012

From table 7, it is observed that non-happening of *Remain Same* state have chance 0.95, happening of the state once is 0.047, occurrence of the state twice have chance 0.0033 and occurrence of *Remain Same* state thrice have chance 0.00012. Hence, it may be concluded that non-occurrence of *Remain Same* state has more likely than the others. And also graph is given below for clear understanding.

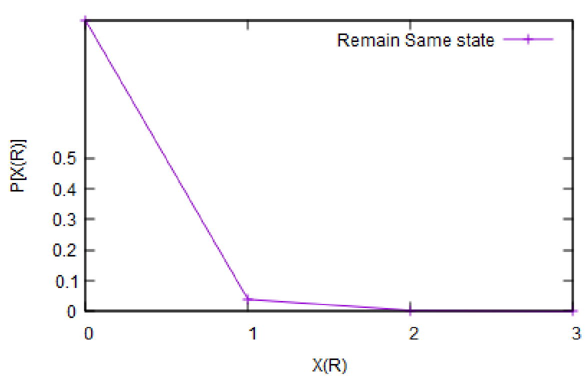

FIGURE 3. Probability of *Remain Same* state for 3 day's

Table 8. Statistical Results

Statistical measures	1 day	2 day's	3 day's
Mean	0.0189	0.0365	0.0540
Variance	0.0185	0.0399	0.0585
3^{rd} central moment	0.0178	0.0462	0.0674
4^{th} central moment	0.0175	0.0624	0.0950
Beta 1	50.0563	33.6789	22.6671
Beta 2	51.0563	39.2406	27.7210
Coefficient of variation	721.37	547.08	447.81

It is observed from table 8 that the average occurrence of *Remain Same* state is less than one day out of 1 day, 2 day's and 3 day's study. Variance of 2 day's study is 0.03 and for 3 day's it is 0.05. Hence, it may be inferred that occurrence of *Remain Same* has more variability in 3 day's study when compared to 2 day's study.
$\mu_3 = 0.06$ and $\beta_1 = 22.66$, which reveals that there is a positive skewness in the distribution of *Remain Same* states in a study of 3 day's run. Which indicates the average number of occurrence of *Remain Same* is more than the model number of occurence of *Remain Same* state. Measure of β_2 specifies the decreased peakedness for the increased length of study for "*Remain Same*" state.

6.4. Model behaviour for the state of "*Increase*". Behaviour of *Increase* state is analysed and its result is given below.

Table 9. Probability distribution for one day length of sequence

X(I)	0	1
P[X(I)]	0.4425	0.5575

From table 9, it is observed that non happening of *Increase State* have chance 0.4425 and happening of the state have chance 0.5575. Hence, it may be concluded that occurrence of *Increase State* has more likely than non occurrence of *Increase State*.

Table 10. Probability distribution for two day length of sequence

X(I)	0	1	2
P[X(I)]	0.1950	0.4919	0.3131

From table 10, it is noticed that non happening of *Increase State* have chance 0.19, occurrence of the state once have chance 0.49 and chance of *Increase State* twice have 0.31. Hence, it may be inferred that happening of *Increase State* once has more likely when compared to the others.

Table 11. Probability distribution for three day length of sequence

X(I)	0	1	2	3
P[X(I)]	0.0857	0.3254	0.4133	0.1755

From table 11, it is observed that non occurrence of *Increase State* have chance 0.08, happening of *Increase State* once have chance 0.32 occurrence of the state twice have chance 0.41 and happening of *Increase State* thrice have chance 0.17. Hence, it may be inferred that occurrence of *Increase State* twice has more likely when compared to other. Graph is given for the above table.

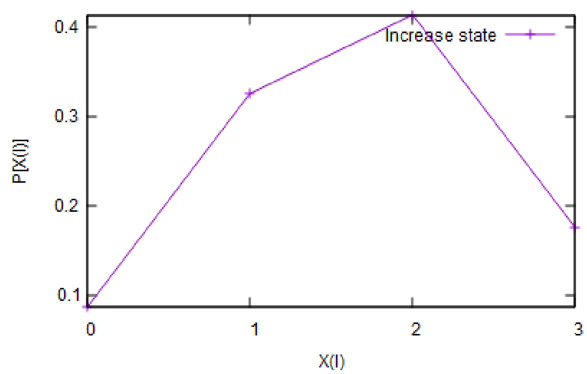

FIGURE 4. Probability of *Increase State* for 3 day's

TABLE 12. Statistical Results

Statistical measures	1 day	2 day's	3 day's
Mean	0.5575	1.1182	1.6788
Variance	0.2467	0.4941	0.7404
3^{rd} central moment	-0.0283	-0.0587	-0.0885
4^{th} central moment	0.0641	0.4942	1.2889
Beta 1	0.0537	0.0285	0.0193
Beta 2	1.0537	2.0240	2.3506
Coefficient of variation	89.08	62.87	51.26

It is observed from table 12, that the average occurrence of *Increase State* is more than 1 out of 3 day's study, 1 out of 2 day's study. Variance of 3 day's study is 0.74 and for 2 day's it is 0.49. Therefore it may be inferred that occurrence of *Increase State* has more variable in 3 day's study when compared to two day's study.
$\mu_3 = -0.0885$ which indicates that there is a negative skewness in the distribution of *Increase State* in a study of 3 day's run. Which indicates the average number of occurrence of *Increase State* is more than the model number of occurrence of *Increase State*. Value of β_2 shows high growth of peakedness when the length of study is increased regarding the "*Increase State*".

7. Summary and conclusion

The study focused on development of Markov model based on the transition states. Discrete Markov processes have been considered in deriving the transition frequency table so as transition probability matrices. Separate probability distribution's are derived to know the behaviour of the states namely, *Decrease*, *Remain Same* and *Increase*.

Construction of the probability mass functions helped us in deriving the mathematical formulae of various statistical characteristics.

This study is a mix of classical and empirical notions because it consists of, part one deals with development of mathematical theory for COVID progression based on mathematical biology of the disease. Assumptions and postulates are defined and considered with all its relevance of mathematics and spreading biology of the disease. Part two deals with data collection from wikipedia sources regarding the prevalence of newly identified COVID positive cases. Transition probability matrices are obtained and respective probability distribution's are also explored from the derived formula to the collected data. Observations and interpretations were made based on the numerical illustrations. The vital part of study is on empirical lines for understanding the data dynamics. User interfaces can be developed for exploring different indicators for different regions is the possible contribution of this work. Healthcare Management may make use of this study for their decision making protocols.

References

1. Brahim Boukanjime, Tomás Caraballo, Mohamed El Fatini, Mohamed El Khalifi.: Dynamics of a stochastic coronavirus (COVID-19) epidemic model with Markovian switching, *Chaos,Solitons and Fractals* (2020) Volume-141, 110361.
2. *Clinical management of COVID-19: living guidance – coronavirus disease (COVID-19)*. Geneva: World Health Organization, 2020.
3. COVID-19 pandemic data/India medical cases by state and union territory, https://en.wikipedia.org/wiki/Template:COVID-19_pandemic_data/India_medical_cases _by_state_and_union_territory.
4. Larsen JR,Martin MR,Martin JD, Kuhn P and Hicks JB.: Modeling the onset of symptoms of COVID-19, *Frontiers in public health* (2020) Volume-8, 473.
5. Liangrong Peng, Wuyue Yang, Dongyan Zhang, Changjing Zhuge, Liu Hong.: Epidemic analysis of COVID-19 in China by dynamical modeling, *arXiv preprint arXiv:2002.06563* (2020).
6. M.A. Andrews Binu Areekal,K.R. Rajesh,Jijith Krishnan, R. Suryakala, Biju Krishnan ,C.P. Muraly & P.V. Santhosh.: First confirmed case of COVID-19 infection in India: A case report, *The Indian Journal of Medical Research* (2020) Volume-151, no 5, 490.
7. Ministry of Health and Family Welfare, mohfw.gov.in
8. Muammer Catak and Necati Duran.: Nonlinear Markov Chain Modelling of the Novel Coronavirus (COVID-19) pandemic, *medRxiv* (2020).
9. R. Prasad, Bindu Shajan Perappadan, Jyoti Shelar and Jacob Koshy.: *The Pandemic Notebook: A handy guide from The Hindu on understanding the coronavirus pandemic and staying protected against COVID-19*, The Hindu, India, 2020.
10. Sha He, Sanyi Tang and Libin Rong.: A discrete stochastic model of the COVID-19 outbreak: Forecast and control, *Mathematical Biosciences and Engineering* (2020) Volume-17, no 4,2792-2804.
11. Shreekanth M.Prabhu and Natarajan Subramaniam.: Surveillance of Covid-19 pandemic using Hidden Markov Model, *ArXiv, abs/2008.07609* (2020).
12. Sultan Hussain,Anwar Zebl,Akhter Rasheed and Tareq Saeed.: Stochastic mathematical model for the spread and control of Corona virus, *Advances in Difference Equations* (2020) Volume 2020, no 1, 1–11.

Department of Statistics, Pondicherry University, Puducherry-605 014, India
Email address: drtrpadi@gmail.com

Department of Statistics, Pondicherry University, Puducherry-605 014, India
Email address: vinkani7@gmail.com

Department of Statistics, Pondicherry University, Puducherry-605 014, India
Email address: sakkeelptnkt@gmail.com