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ANALYSIS OF RELIABILITY, AVAILABILITY AND MAINTAINABILITY FOR $E_r/E_r/1/N$ PHASE-TYPE QUEUEING MODEL

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Abstract. Analysis of Reliability, Availability and Maintainability of the finite capacity Erlangian phase type queueing system are modelled in this paper. The interarrival times and the service times follow Erlang distribution along with different environmental states such as Working and Working-Breakdown states. These states were considered along with the failure and recovery rates that follows exponential distribution. The differential-difference equations describing the system in the transient state are obtained from the state-transition diagram. The general equations that are formed are solved by using numerical method for the special case of $N = 4$. The results of RAM along with some special metrics are shown numerically and graphically. The performance measures of RAM were also studied for different parametric values by using Sensitivity Analysis.

1. Introduction

Queueing theory is a branch of Operational Research that assists the users to make business decisions in order to build efficient and cost-effective workflow systems. The phase-concept is a widely used method for analysing queueing systems. The principle of this phase-concept was developed by A.K. Erlang, the view of breaking down the service times into equally distributed intervals which are called phases. The members of the phase-type distributions are the Erlangian, the Hyperexponential distributions, the Hypoexponential and the Coxian distributions. Bux and Herzog [2] proposed a numerical method for the analysis of the inter-arrival and service time distributions for the phase-type method. Neuts [8], presented a detailed survey of the markov chain with renewal processes of the $M/G/1$ queueing model by using the matrix-analytic methodology.

Mostly, in practice, the arrivals and their service times cases depend on the system state. In order to understand the system evolution, the transient performance measures are very important which is much more difficult to analyse than the steady state. Takacs [10] in his book derived the transient probabilities of the $M/M/1/K$, referred to as $P_{ij}(t)$. Gray et al. [4] analysed a multiple vacation queueing model where the service station will be in the following operation states, namely, vacation, service, or breakdown states. Hanumantha Rao et.al. [6] proposed a two-phase unreliable $M/E_k/1$ queuing system with server start up, delayed

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repair and N-Policy for the state dependent arrival rates. Gerald and Wheatley [5], Ong and Taaffe [9] solved the ordinary differential equations (ODEs) numerically by using Chapman-Kolmogorov equations and by applying Runge-Kutta method.

A lot of techniques have been adapted to improve the reliability, availability and maintainability of the systems, but recently some researchers have exhibited that the queueing models are used to describe various activities in the system. Antoniol et al. [1] proposed a queueing model approach to plan and to control the project staffing in the multiphase maintenance process. Reliability concepts are very much linked to queueing systems Kan Cheng [7] discussed, $M/G/1$ queueing system with a repairable service station where the lifespan of the service station is exponentially distributed where the repair time has a general distribution. Recently, using the supplementary variable method, the repairable $M/G/1$ queueing model was discussed by Dinghua shi [3] where the service station is assumed the be an Erlangian distribution of the $M^X/G/1$ repairable queueing model in the occurrence of the batch arrival.

The Reliability, Availability and Maintainability analysis of the $E_r/E_r/1/N$ Phase-type queueing model was taken into consideration for study. The differential-difference for the transient state was formed for the general case as well as special case with two different environmental states such as the Working state and Working-Breakdown states from the state-transition diagram. The results are obtained numerically and graphically for this model. Furthermore, the "What-if analysis otherwise known as the Sensitivity analysis is also carried out to find the changes in the system for different parametric values.

2. Assumptions and Notations

The assumptions that are used in this model are described below:

- ➢ The machines to be repaired arrive independently according to Erlangian distribution with parameter $r\lambda$, where r represents the number of phases in arrival process
- ➢ The service process of the machines that are to be repaired follows Erlang distribution with parameter $r\mu$, where r represents the number of phases in service process
- ➢ Two different environmental states such as Working state and Working-Breakdown state are assumed in the service process along with multi-phases
- ➢ In the phase-type arrival and service process, when the queueing system is in the Working-state the failure occurs in the system and the process is moved to the Working-Breakdown state where the Queuing system is continued with lower rate. The Failure rate is exponentially distributed
- ➢ Whenever the queueing system is in the Working-breakdown state, recovery process take place which is also exponentially distributed. Once the recovery process is completed the queueing system performs its activity at a normal rate

The notations used in this model are

$N(t)$: Total number of machines in the system at any time t
E_r : Erlang distribution with r identical phases
$S(t)$: The environmental state at any instant of time t which is given by:

$$S(t) = \begin{cases} 0, & \text{if the server is in the working environment state for Phase 1 \& 2} \\ 1, & \text{if the server is in the working breakdown state for Phase 1 \& 2} \end{cases}$$

λ : Arrival rate
μ_1 : Service rate for working state
μ_2 : Service rate for working-breakdown state ($\mu_1 > \mu_2$)
α : Failure rate in the service process
β : Recovery state in the service process

The probabilities of the transient-state that are used in this model are:
$P_{0,0,0}(t)$ - The Probability that there are no machines in the system
$P_{n,i,j}(t)$ - The Probability that there are n machines in the i^{th} phase of the system with j^{th} state

3. Description of the System

The Reliability, Availability and Maintainability for Erlang Phase-type queueing model are analysed in this paper. The arrival and service time process follows Erlang distribution with r phases is defined by the continuous time interval of $t \geq 0$ and with finite state space. The transient state probabilities for the Erlangian Queueing system with r phases are studied with two different environmental states such as the Working and Working-Breakdown state. When there is at least one machine in the system the failure takes place in working state of the queueing system and the process is moved to the Working-Breakdown state where the process performs its activity at a low rate. In the working-breakdown state of the Queueing system, recovery process also takes place and once it is recovered the process is moved to the Working state where the Queueing system performs at the normal rate. The State-transition diagram for the RAM analysis of the $E_r/E_r/1/N$ Queueing model is given below:

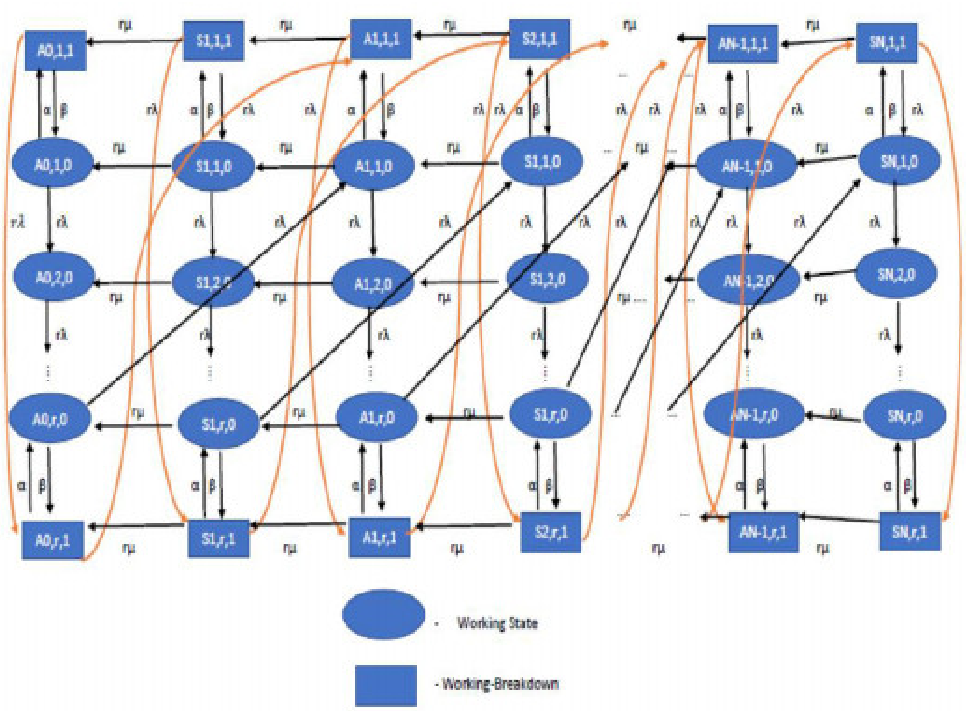

FIGURE 1. State-transition diagram of $E_r/E_r/1/N$

The differential-difference equations for the transient state of the Erlangian Phase-type queueing system for different environmental states are given below:
Working State

$$\frac{dP_{A0,1,0}}{dt} = -(2\lambda+\alpha)P_{A0,1,0}(t) + 2\mu P_{S1,1,0} + \beta P_{A0,1,1}, \qquad n = 0 \tag{3.1}$$

$$\frac{dP_{A0,r,0}}{dt} = -(2\lambda+\alpha)P_{A0,r,0}(t) + 2\lambda P_{A0,r-1,0} + 2\mu P_{S1,r,0} + \beta P_{A0,r,1}, \qquad n = 0 \tag{3.2}$$

$$\frac{dP_{Sn,1,0}}{dt} = -(2\mu+2\lambda+\alpha)P_{Sn,1,0}(t) + 2\mu P_{An,1,0} + \beta P_{Sn,1,1}, \qquad n \geq 1 \tag{3.3}$$

$$\frac{dP_{Sn,r,0}}{dt} = -(2\mu+2\lambda+\alpha)P_{Sn,r,0}(t) + 2\mu P_{An,r,0} + 2\lambda P_{Sn,r-1,0} + \beta P_{Sn,r,1}, \qquad n \geq 1 \tag{3.4}$$

$$\frac{dP_{An,1,0}}{dt} = -(2\mu+2\lambda+\alpha)P_{An,1,0}(t) + 2\mu P_{Sn+1,1,0} + 2\lambda P_{An-1,r,0} + \beta P_{An,1,1}, \qquad n \geq 1 \tag{3.5}$$

$$\frac{dP_{An,r,0}}{dt} = -(2\mu+2\lambda+\alpha)P_{An,r,0}(t) + 2\mu P_{Sn+1,1,0} + 2\lambda P_{An,r-1,0} + \beta P_{An,r,1}, \quad n \geq 1 \quad (3.6)$$

$$\frac{dP_{SN,1,0}}{dt} = -(2\mu+2\lambda+\alpha)P_{Sn,1,0}(t) + 2\lambda P_{SN-1,r,0} + \beta P_{SN,1,1}, \quad n = N \quad (3.7)$$

$$\frac{dP_{SN,r,0}}{dt} = -(2\mu+\alpha)P_{SN,r,0}(t) + 2\lambda P_{SN,r-1,0} + \beta P_{SN,r,1}, \qquad n = N \quad (3.8)$$

$$\frac{dP_{AN,1,0}}{dt} = -(2\mu+2\lambda+\alpha)P_{AN,1,0}(t) + 2\mu P_{SN,1,0} + 2\lambda P_{AN-1,r,0} + \beta P_{AN,1,1}, \quad n = N \quad (3.9)$$

$$\frac{dP_{AN,r,0}}{dt} = -(2\mu+\alpha)P_{AN,r,0}(t) + 2\lambda P_{AN,r-1,0} + \beta P_{AN,r,1}, \qquad n = N \quad (3.10)$$

Working-Breakdown State

$$\frac{dP_{A0,1,1}}{dt} = -(2\lambda+\beta)P_{A0,1,1}(t) + 2\mu P_{S1,1,1} + \alpha P_{A0,1,0}, \qquad n = 0 \quad (3.11)$$

$$\frac{dP_{A0,r,1}}{dt} = -(2\lambda+\beta)P_{A0,r,1}(t) + 2\lambda P_{A0,r-1,1} + 2\mu P_{S1,r,1} + \alpha P_{A0,r,0}, \quad n = 0 \quad (3.12)$$

$$\frac{dP_{Sn,1,1}}{dt} = -(2\mu+2\lambda+\beta)P_{Sn,1,1}(t) + 2\mu P_{An,1,1} + \alpha P_{Sn,1,0}, \qquad n \geq 1 \quad (3.13)$$

$$\frac{dP_{Sn,r,1}}{dt} = -(2\mu+2\lambda+\beta)P_{Sn,r,1}(t) + 2\mu P_{An,r,1} + 2\lambda P_{Sn,r-1,1} + \alpha P_{Sn,r,0}, \quad n \geq 1 \quad (3.14)$$

$$\frac{dP_{An,1,1}}{dt} = -(2\mu+2\lambda+\beta)P_{An,1,1}(t) + 2\mu P_{Sn+1,1,1} + 2\lambda P_{An-1,r,1} + \alpha P_{An,1,0}, \quad n \geq 1 \quad (3.15)$$

$$\frac{dP_{An,r,1}}{dt} = -(2\mu+2\lambda+\beta)P_{An,r,1}(t) + 2\mu P_{Sn+1,1,1} + 2\lambda P_{An,r-1,1} + \alpha P_{An,r,0}, \quad n \geq 1 \quad (3.16)$$

$$\frac{dP_{SN,1,1}}{dt} = -(2\mu+2\lambda+\beta)P_{Sn,1,1}(t) + 2\lambda P_{SN-1,r,1} + \alpha P_{SN,1,0}, \quad n = N \quad (3.17)$$

$$\frac{dP_{SN,r,1}}{dt} = -(2\mu+\beta)P_{SN,r,1}(t) + 2\lambda P_{SN,r-1,1} + \alpha P_{SN,r,0}, \qquad n = N \quad (3.18)$$

$$\frac{dP_{AN,1,1}}{dt} = -(2\mu + 2\lambda + \beta)P_{AN,1,1}(t) + 2\mu P_{SN,1,1} + 2\lambda P_{AN-1,r,0} + \alpha P_{AN,1,0}, \quad n = N \tag{3.19}$$

$$\frac{dP_{AN,r,1}}{dt} = -(2\mu + \beta)P_{AN,r,1}(t) + 2\lambda P_{AN,r-1,1} + \alpha P_{AN,r,0}, \qquad n = N \tag{3.20}$$

without loss of generality the initial state conditions are given by
$P_{0,1,0}(0) = 1,\ P_{N,i,j}(0) = 0,\ \forall\ n = 1, 2, \ldots, N;\ i = 1, 2, \ldots, r;\ j = 0, 1$

The total system probabilities for the transient state of N machines are evaluated to understand the distribution of the machines in the system. This is done by:

$$P_n(t) = P_{w,n}(t) + P_{wb,n}(t) \tag{3.21}$$

Where, $P_{w,n}(t)$ and $P_{wb,n}(t)$ are the environmental states for the working normal and working-breakdown state.

The Reliability of the system for time t is calculated as follows:

$$R(t) = \sum_{n=0}^{N}\sum_{i=1}^{r}\sum_{j=0}^{1} P_{n,i,j} \tag{3.22}$$

The Availability of the system for time t is calculated by considering up all the working states as follows:

$$A(t) = \sum_{n=0}^{N}\sum_{i=1}^{r}\sum_{j=0}^{1} P_{n,i,j} \tag{3.23}$$

The Maintainability of the system for time t is calculated by considering working-breakdown state which is calculated as follows:

$$M(t) = \sum_{n=0}^{N}\sum_{i=1}^{r}\sum_{j=0} P_{n,i,j} \tag{3.24}$$

Other Metrics formula such as $MTBF$ (Mean time between failures) and $MTTR$ (Mean Time till Recovery) were also calculated along with RAM are as follows:

$$MTBF = \frac{1}{\alpha}$$
$$MTTR = \frac{1}{\beta}$$

4. Special Case

As a special case of $N = 4$ the differential difference equations for the Erlangian Phase-type queueing model with Working and Working-Breakdown states are given below:

Working State

$$\frac{dP_{A0,1,0}(t)}{dt} = -(2\lambda+\alpha)P_{A0,1,0}(t) + 2\mu_1 P_{S1,1,0}(t) + \beta P_{A0,1,1}(t) \tag{4.1}$$

$$\frac{dP_{A0,2,0}}{dt} = -(2\lambda+\alpha)P_{A0,2,0}(t) + 2\mu_1 P_{S1,2,0}(t) + \beta P_{A0,2,1}(t) \tag{4.2}$$

$$\frac{dP_{S1,1,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{S1,1,0}(t) + 2\mu_1 P_{A1,1,0}(t) + \beta P_{S1,1,1}(t) \tag{4.3}$$

$$\frac{dP_{S1,2,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{S1,2,0}(t) + 2\mu_1 P_{A1,2,0}(t) + 2\lambda P_{S1,1,0}(t) + \beta P_{S1,2,1}(t) \tag{4.4}$$

$$\frac{dP_{A1,1,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{A1,1,0}(t) + 2\mu_1 P_{S2,1,0}(t) + 2\lambda P_{A0,2,0}(t) + \beta P_{A1,1,1}(t) \tag{4.5}$$

$$\frac{dP_{A1,2,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{A1,2,0}(t) + 2\mu_1 P_{S2,2,0}(t) + 2\lambda P_{A1,1,0}(t) + \beta P_{A1,2,1}(t) \tag{4.6}$$

$$\frac{dP_{S2,1,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{S2,1,0}(t) + 2\mu_1 P_{A2,1,0}(t) + \beta P_{S2,1,1}(t) \tag{4.7}$$

$$\frac{dP_{S2,2,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{S2,2,0}(t) + 2\mu_1 P_{A2,2,0}(t) + 2\lambda P_{S2,1,0}(t) + \beta P_{S2,2,1}(t) \tag{4.8}$$

$$\frac{dP_{A2,1,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{A2,1,0}(t) + 2\mu_1 P_{S3,1,0}(t) + 2\lambda P_{A1,2,0}(t) + \beta P_{A2,1,1}(t) \tag{4.9}$$

$$\frac{dP_{A2,2,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{A2,2,0}(t) + 2\mu_1 P_{S3,2,0}(t) + 2\lambda P_{A2,1,0}(t) + \beta P_{A2,2,1}(t) \tag{4.10}$$

$$\frac{dP_{S3,1,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{S3,1,0}(t) + 2\mu_1 P_{A3,1,0}(t) + \beta P_{S3,1,1}(t) \tag{4.11}$$

$$\frac{dP_{S3,2,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{S3,2,0}(t) + 2\mu_1 P_{A3,2,0}(t) + 2\lambda P_{S3,1,0}(t) + \beta P_{S3,2,1}(t) \tag{4.12}$$

$$\frac{dP_{A3,1,0}}{dt} = -(2\mu_1+2\lambda+\alpha)P_{A3,1,0}(t) + 2\mu_1 P_{S4,1,0}(t) + 2\lambda P_{A2,2,0}(t) + \beta P_{A3,1,1}(t) \tag{4.13}$$

$$\frac{dP_{A3,2,0}}{dt} = -(2\mu_1 + \alpha)P_{A3,2,0}(t) + 2\lambda P_{A3,1,0}(t) + \beta P_{A3,2,1}(t) \tag{4.14}$$

$$\frac{dP_{S4,1,0}}{dt} = -(2\mu_1 + 2\lambda + \alpha)P_{S4,1,0}(t) + 2\mu_1 P_{S3,1,0}(t) + 2\lambda P_{S3,1,0}(t) + \beta P_{S4,1,1}(t) \tag{4.15}$$

$$\frac{dP_{S4,2,0}}{dt} = -(2\mu_1 + \alpha)P_{S4,2,0}(t) + 2\lambda P_{S4,1,0}(t) + \beta P_{S4,2,1}(t) \tag{4.16}$$

Working-Breakdown State

$$\frac{dP_{A0,1,1}(t)}{dt} = -(2\lambda + \beta)P_{A0,1,1}(t) + 2\mu_2 P_{S1,1,1}(t) + \alpha P_{A0,1,0}(t) \tag{4.17}$$

$$\frac{dP_{A0,2,1}}{dt} = -(2\lambda + \beta)P_{A0,2,1}(t) + 2\mu_2 P_{S1,2,1}(t) + 2\lambda P_{A0,1,1}(t) + \alpha P_{A0,2,0}(t) \tag{4.18}$$

$$\frac{dP_{S1,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S1,1,1}(t) + 2\mu_2 P_{A1,1,1}(t) + \alpha P_{S1,1,0}(t) \tag{4.19}$$

$$\frac{dP_{S1,2,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S1,2,1}(t) + 2\mu_2 P_{A1,2,1}(t) + 2\lambda P_{S1,1,1}(t) + \alpha P_{S1,2,0}(t) \tag{4.20}$$

$$\frac{dP_{A1,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{A1,1,1}(t) + 2\mu_2 P_{S2,1,1}(t) + 2\lambda P_{A0,1,1}(t) + \alpha P_{A1,1,0}(t) \tag{4.21}$$

$$\frac{dP_{A1,2,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{A1,2,1}(t) + 2\mu_2 P_{S2,2,1}(t) + 2\lambda P_{A1,1,1}(t) + \alpha P_{A1,2,0}(t) \tag{4.22}$$

$$\frac{dP_{S2,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S2,1,1}(t) + 2\lambda P_{S1,2,1}(t) + 2\mu_2 P_{A2,1,1}(t) + \alpha P_{S2,1,0}(t) \tag{4.23}$$

$$\frac{dP_{S2,2,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S2,2,1}(t) + 2\mu_2 P_{A2,2,1}(t) + 2\lambda P_{S2,1,1}(t) + \alpha P_{S2,2,0}(t) \tag{4.24}$$

$$\frac{dP_{A2,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{A2,1,1}(t) + 2\mu_2 P_{S3,1,1}(t) + 2\lambda P_{A1,1,1}(t) + \alpha P_{A2,1,0}(t) \tag{4.25}$$

$$\frac{dP_{A2,2,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{A2,2,1}(t) + 2\mu_2 P_{S3,2,1}(t) + 2\lambda P_{A2,1,1}(t) + \alpha P_{A2,2,0}(t) \quad (4.26)$$

$$\frac{dP_{S3,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S3,1,1}(t) + 2\mu_2 P_{A3,1,1}(t) + 2\lambda P_{S2,2,1}(t) + \alpha P_{S3,1,0}(t) \quad (4.27)$$

$$\frac{dP_{S3,2,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S3,2,1}(t) + 2\mu_2 P_{A3,2,1}(t) + 2\lambda P_{S3,1,1}(t) + \alpha P_{S3,2,0}(t) \quad (4.28)$$

$$\frac{dP_{A3,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{A3,1,1}(t) + 2\mu_2 P_{S4,1,1}(t) + 2\lambda P_{A2,2,1}(t) + \alpha P_{A3,1,0}(t) \quad (4.29)$$

$$\frac{dP_{A3,2,1}}{dt} = -(2\mu_2 + \beta)P_{A3,2,1}(t) + 2\lambda P_{A3,1,1}(t) + \alpha P_{A3,2,0}(t) \quad (4.30)$$

$$\frac{dP_{S4,1,1}}{dt} = -(2\mu_2 + 2\lambda + \beta)P_{S4,1,1}(t) + 2\lambda P_{S3,2,1}(t) + \alpha P_{S4,1,0}(t) \quad (4.31)$$

$$\frac{dP_{S4,2,1}}{dt} = -(2\mu_2 + \beta)P_{S4,2,1}(t) + 2\lambda P_{S4,1,1}(t) + \alpha P_{S4,2,0}(t) \quad (4.32)$$

4.1. Numerical Illustration. The RAM analysis of the transient behaviour of $E_r/E_r/1/N$ Queueing model for the finite capacity of $N = 4$ is analysed by using the differential-difference equations from (4.1) - (4.32). These equations are solved for the transient probabilities by using Fourth-order Runge-Kutta numerical method for $t = 0$ to $t = 200$ and parametric values such as $\lambda = 0.0070$, $\mu_1 = 0.0090$, $\mu_2 = 0.0085$, $\alpha = 0.004$, $\beta = 0.003$.

Figure 2 shows the various probability distribution $P_n(t)$, for the Erlangian Phase-type queueing system. These probability trend curves are displayed to understand the distribution trend of the system probabilities for the specified time intervals.

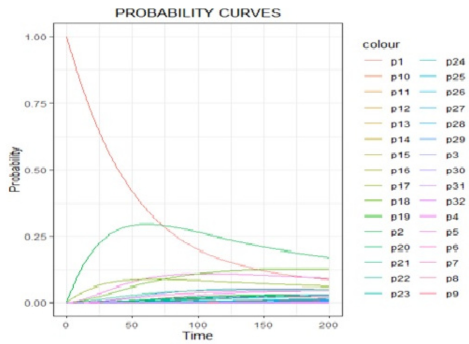

FIGURE 2. Probability Trend Curves

Figure 3, represents the Reliability of the $E_r/E_r/1/N$ Queueing model. It is observed from the figure that as the time increases the Reliability of the system decreases. It was also found out that after 200 hours the $R(t)$ of the system will be approximately 57%.

Figure 4, shows the Availability of the $E_r/E_r/1/N$ Queueing model. It is seen from the figure as the time range increases, the Availability of the system decreases. After 200 hours the $A(t)$ of the system was found to be 44%.

Figure 5, depicts the Maintainability of the $E_r/E_r/1/N$ Queueing model. From the figure it was seen that as time increases, Maintainability of the system also increases. It is observed that after 200 hours $M(t)$ is found out to be 42%.

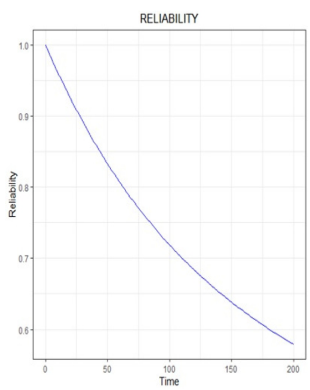

FIGURE 3

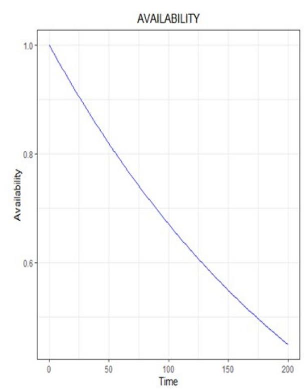

FIGURE 4

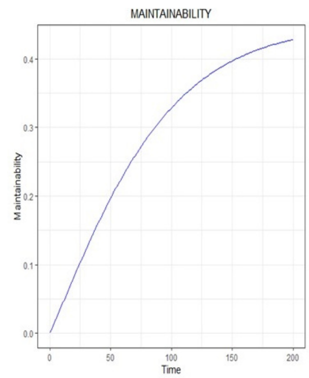

FIGURE 5

Along with RAM important metric values such as $MTBF$ and $MTTR$ for the $E_r/E_r/1/N$ Queueing model is found to be 250 hours/failure and 333 hours/recovery respectively.

5. Sensitivity Analysis

Sensitivity Analysis is used in Reliability, Availability and Maintainability of the Erlangian Phase-type queueing model for different parametric values. Figures 6, 7, 8 represents the graphs for different Failure rate values (0.004, 0.005, 0.006) with regard to Reliability, Availability and Maintainability. From the graphs it can be seen that as the failure rate value increases Reliability and Availability decreases and the Maintainability increases.

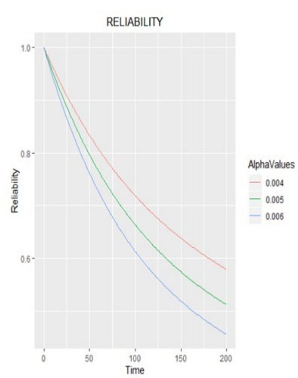

FIGURE 6

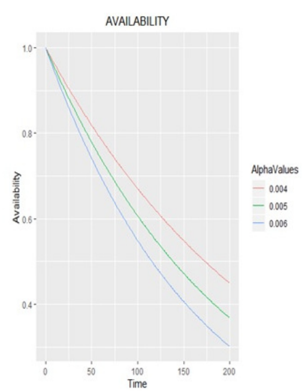

FIGURE 7

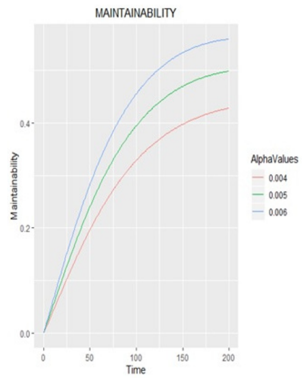

FIGURE 8

Figures 9 and 10 illustrates the graphs for different Recovery rate values (0.001, 0.002, 0.003) of Reliability and Maintainability functions by keeping the other parameters constant. From the figures it is observed that as the Recovery rate values increases Reliability decreases, whereas Maintainability increases.

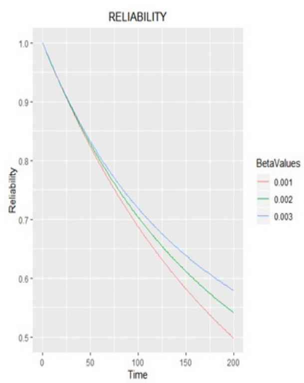

FIGURE 9

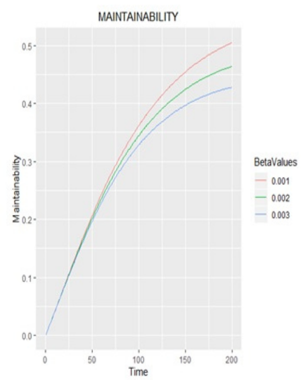

FIGURE 10

6. Conclusion

The Reliability, Availability and Maintainability analysis of the $E_r/E_r/1/N$ Phase-type queueing system are analysed in this model. The state-transition diagram for the $E_r/E_r/1/N$ with different Environmental states such as the Working state and Working-Breakdown state are formed and the Differential-Difference equations are obtained. A special case for the finite capacity of $N = 4$ and $r = 2$ is considered and are solved by using Numerical method for a time of 200 hours. The probability trend curves, Reliability, Availability and Maintainability curves were obtained. The curves show that as the time increases the Reliability and Availability of the Erlangian Phase-type Queueing system decreases and after 200 hours the values are found to be 57% and 44%, whereas the Maintainability increases with 42% after 200 hours. Sensitivity analysis were also used to analyse the Reliability, Availability and Maintainability for different values of Failure and Recovery rates. It was observed that as the Failure rate value increases the Reliability and Availability increases whereas the Maintainability decreases and vice versa results were observed with regard to Recovery rate.

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