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STATISTICAL INFERENCE ON STRESS-STRENGTH RELIABILITY OF MULTICOMPONENT SYSTEM UNDER WEIBULL-PARETO TYPE DISTRIBUTIONS.

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Abstract. The problem of estimation of s-out-of-k system reliability in stress-strength setup is studied. The reliability of the system is derived assuming strength and stress variables to follow new Weibull-Pareto distribution. Maximum likelihood and Bayesian approaches are used to estimate system reliability. Approximate Bayes estimators are obtained using Lindley's approximation technique. The mean squared errors of the estimators of reliability are computed using simulation and the estimators are compared using mean squares error criteria. The estimation procedures are illustrated using real data analysis.
Keywords: new Weibull-Pareto distribution, stress-strength reliability, maximum likelihood estimator, Bayes estimator.

Mathematics Subject Classification number: 62F10

1. Introduction

The stress-strength model has Vast practical applications in the field of Engineering and other disciplines of science. The estimation of stress-strength reliability for s out of k system has received much attention by researchers. Rao and Kantam (2010) addressed the estimation of reliability of such a system when the underlying distribution of the stress and strength variables are log-logistic distribution, followed by Rao (2014) for Rayleigh distribution. The estimation of multicomponent stress-strength reliability for parallel and series systems is studied by Pandit and Kantu (2013) when strength and stress variables follow exponential distribution. Kizilaslan and Nadar (2015,2016) studied the problem of estimating stress-strength reliabilityof an s out of k system when the underlying distributions are Weibull and bivariate Kumarswamy.
In this paper, s-out-of-k system in stress-strength environment which has k independent and identical strength components and a common stress is studied. These kinds of situations may occur in real life. For example, in an electrical power station containing eight generating units which produce the electricity only if at least six units are working; the demand for the electricity of a district is fulfilled only if s-out-of-k wind rose are operating at all times and in a communication system for a navy can be successful only if six transmitters out of ten are operational to cover a district (refer Nadar, M.and Kizilaslan, F (2015)). The application of s out of k system can be seen in many real life situations, particularly in industry and military(refer Kuo and Zuo (2003)).

This paper considers stress strength reliability of s out of k system when the underlying distribution is new Weibull-Pareto. Tahir et. al (2014) introduced new Weibull-Pareto distribution and it is further studied by Nasiru and Luguterah (2015).probability density function and Cumulative distribution function of new Weibull-Pareto distribution respectively are given below:

$$f(x) = \frac{\beta\delta}{\theta}(\frac{x}{\theta})^{\beta-1}\ e^{-\delta(\frac{x}{\theta})^{\beta}}\ ;\ x > 0,\ \delta,\ \beta\ \ \theta > 0$$

and

$$F(x) = 1 - e^{-\delta(\frac{x}{\theta})^{\beta}}\ ;\ x > 0,\ \delta,\ \beta\ \ \theta > 0$$

This distribution has constant failure rate when $\beta = 1$ and increasing (decreasing) failure rates when $\beta > 1(\beta < 1)$.
Rest of the paper is organized as follows. In section 2, maximum likelihood estimation of Rs,k is considered. Approximate Bayes estimators of Rs,k is obtained by using Lindley's approximation method in section 3. In section 4, simulation study is conducted by estimating MSEs to compare the estimators of reliability and real data analysis is given in section 5. Section 6 contains the summary and conclusions.

2. Maximum likelihood estimation of System reliability

First, the system reliability of s out of k system is presented. Let the random variables $Y, X_1, X_2, ..., X_k$ be independent, $G(y)$ be the cummulative distribution function of Y and $F(x)$ be the common cummulative distribution function of $X_1, X_2, ..., X_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya and Johnson (1974) is

$$\begin{aligned} R_{s,k} &= P(at\ least\ s\ of\ the\ (X_1, X_2, ..., X_k)\ exceed\ Y\ ) \\ &= \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} (1 - F(y))^i (F(y))^{k-i} dG(y) \end{aligned} \tag{1}$$

The system with k identical components functions if s $(1 \le s \le k)$ or more components simultaneously operate. Now, the system is subjected to a stress Y which is a random variable with distribution function $G(.)$. The strengths of the components, that is the minimum stresses to cause failure, are indendently and identically distributed random variables with distribution function $F(.)$. Then the reliability of the system is given in equation (1).
In our case, multicomponent system reliability when $X_1, X_2, ..., X_k$ follow new Weibull Pareto distribution with parameters $(\delta_1, \theta, \beta)$ and Y follow new Weibull

Pareto distribution with parameters $(\delta_2, \theta, \beta)$

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_0^\infty \frac{\delta_2 \beta}{\theta} [1 - (1 - e^- \delta_1 (\frac{y}{\theta})^\beta)]^i$$

$$[1 - e^- \delta_1 (\frac{y}{\theta})^\beta]^{k-i} (\frac{y}{\theta})^{\beta-1} e^- \delta_2 (\frac{y}{\theta})^\beta dy$$

$$= \sum_{i=s}^{k} \sum_{j=0}^{k-i} \binom{k}{i} \binom{k-i}{j} (-1)^j \frac{\delta_2}{[\delta_1(i+j)+\delta_2]}$$

Next, we consider the estimation of system reliability using method of maximum likelihood.
Let $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_m$ be two random samples of size n, m respectively. Here, strength and stress variables from NWP distribution with parameters δ_1, δ_2, θ and β, then the likelihood function is given by

$$L_s(\delta_1, \delta_2, \theta, \beta) = \frac{\delta_1^n \delta_2^m \beta^{n+m}}{\theta^{n+m}} \left[\prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right) \prod_{j=1}^{m} \left(\frac{y_j}{\theta}\right) \right]^{\beta-1} e^{\delta_1 \sum_{i=1}^{n} (\frac{x_i}{\theta})^\beta} e^{\delta_2 \sum_{j=1}^{m} (\frac{y_j}{\theta})^\beta}$$

Then, the log-likelihood function of $\delta_1, \delta_2, \theta$ and β is

$$logL_s(\delta_1, \delta_2, \theta, \beta) = nlog\delta_1 + mlog\delta_2 + (n+m)log\beta - (n+m)log\theta$$

$$+ (\beta - 1) \sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right) + (\beta - 1) \sum_{j=1}^{m} log\left(\frac{y_j}{\theta}\right)$$

$$- \delta_1 \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^\beta - \delta_2 \sum_{j=1}^{m} \left(\frac{y_j}{\theta}\right)^\beta$$

The likelihood equations are

$$\frac{\partial logL_s}{\partial \delta_1} = 0 \Rightarrow \frac{n}{\delta_1} - \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^\beta$$

$$\frac{\partial logL_s}{\partial \delta_2} = 0 \Rightarrow \frac{m}{\delta_2} - \sum_{j=1}^{m} \left(\frac{y_j}{\theta}\right)^\beta$$

$$\frac{\partial logL_s}{\partial \theta} = 0 \Rightarrow -\frac{n+m}{\theta} - \frac{n(\beta-1)}{\theta} - \frac{m(\beta-1)}{\theta} + \delta_1 \sum_{i=1}^{n} \beta \frac{x_i}{\theta^2} \left[\frac{x_i}{\theta}^\beta - 1\right]$$

$$+ \delta_2 \sum_{j=1}^{m} \beta \frac{y_j}{\theta^2} \left[\frac{y_j}{\theta}^\beta - 1\right]$$

$$\frac{\partial logL_s}{\partial \beta} = 0 \Rightarrow -\frac{n+m}{\beta} + \sum_{i=1}^{n} log\left[\frac{x_i}{\theta}\right] + \sum_{j=1}^{m} log\left[\frac{y_j}{\theta}\right] - \delta_1 \sum_{i=1}^{n} log\frac{x_i}{\theta} \left[\frac{x_i}{\theta}\right]^\beta$$

$$- \delta_2 \sum_{j=1}^{m} log\frac{y_j}{\theta} \left[\frac{y_j}{\theta}\right]^\beta$$

The MLEs of δ_1, δ_2 are obtained as functions of β and θ.
The maximum likelihood equations are

$$\hat{\delta_1} = \frac{n}{\sum_{i=1}^{n} (\frac{x_i}{\theta})^\beta}, \quad \hat{\delta_2} = \frac{m}{\sum_{j=1}^{m} (\frac{y_j}{\theta})^\beta}$$

where $\hat{\beta}$ and $\hat{\theta}$ are the solution of the nonlinear equation of the form, $h(\beta) = \beta$.

$$h(\beta) = (n+m)\left[-\sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right) - \sum_{j=1}^{m} log\left(\frac{y_j}{\theta}\right) + \delta_1 \sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right)\left(\frac{x_i}{\theta}\right)^{\beta} + \delta_2 \sum_{j=1}^{m} log\left(\frac{y_j}{\theta}\right)\left(\frac{y_j}{\theta}\right)^{\beta}\right]^{-1}$$

and $h(\theta) = \theta$.

$$h(\theta) = (n+m)\left[\frac{n(\beta-1)}{\theta} - \frac{m(\beta-1)}{\theta} + \delta_1 \sum_{i=1}^{n} \beta \frac{x_i}{\theta^2}\left[\frac{x_i}{\theta}^{\beta} - 1\right] + \delta_2 \sum_{j=1}^{m} \beta \frac{y_j}{\theta^2}\left[\frac{y_j}{\theta}^{\beta} - 1\right]\right]^{-1}$$

Here, $\hat{\beta}$ and $\hat{\theta}$ can be obtained by using any iterative scheme like Newton-Raphson method and then using invariance principle, the MLEs of δ_1 and δ_2 are obtained from (3).
Hence, the MLE of $\hat{R_{s,k}}$ is given by,

$$\hat{R}_{s,k} = \sum_{i=s}^{k} \sum_{j=0}^{k-i} \binom{k}{i} \binom{k-i}{j} (-1)^j \frac{\hat{\delta_2}}{[\hat{\delta_1}(i+j) + \hat{\delta_2}]}$$

3. Bayes estimation of $R_{s,k}$

Here,we assume that all parameters $\delta_1, \delta_2, \theta$ and β are random variables and have independent gamma prior with parameters $(c_i, d_i), i = 1, 2, 3, 4$ respectively. The pdf of a gamma random variable X with parameters (c_i, d_i) is

$$f(x) = \frac{d_i^{c_i}}{\Gamma(d_i)} x^{c_i - 1} e^{-x d_i};\ x > 0,\ c_i,\ d_i > 0$$

where $c_i,\ d_i > 0,\ i = 1, 2, 3, 4$. Thus the joint prior δ_1 , δ_2, θ and β is

$$\begin{aligned} g(\delta_1, \delta_2, \theta, \beta) = & \frac{d_1^{c_1}}{\Gamma(c_1)} \frac{d_2^{c_2}}{\Gamma(c_2)} \frac{d_3^{c_3}}{\Gamma(c_3)} \frac{d_4^{c_4}}{\Gamma(c_4)} \delta_1^{c_1-1} \delta_2^{c_2-1} \theta^{c_3-1} \beta^{c_4-1} e^{-d_1\delta_1} \\ & e^{-d_2\delta_2} e^{-d_3\theta} e^{-d_4\beta}; \\ & \delta_1,\ \delta_2,\ \theta,\ \beta > 0,\ c_i,\ d_i > 0,\ i = 1, 2, 3, 4 \end{aligned}$$

The corresponding joint posterior distribution is given by

$$\begin{aligned} \pi(\delta_1, \delta_2, \theta, \beta | X, Y) &= \frac{g(\delta_1, \delta_2, \theta, \beta) L_s(\delta_1, \delta_2, \theta, \beta)}{\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty g(\delta_1, \delta_2, \theta, \beta) L_s(\delta_1, \delta_2, \theta, \beta) d\delta_1 d\delta_2 d\theta d\beta} \\ &= \frac{A}{\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty A d\delta_1 d\delta_2 d\theta d\beta} \end{aligned}$$

where

$$\begin{aligned} A = & \delta_1^{n+c_1-1} \delta_2^{m+c_2-1} \theta^{-(n+m)+c_3-1} \beta^{n+m+c_4-1} \prod_{i=1}^{n} x_i^{\beta-1} \prod_{j=1}^{m} y_j^{\beta-1} \\ & \exp -\delta_1 \left(\sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta} + d_1\right) \exp -\delta_2 \left(\sum_{j=1}^{m} \left(\frac{y_j}{\theta}\right)^{\beta} + d_2\right) \exp^{-} d_3\theta \exp^{-} d_4\beta \end{aligned}$$

Here, the bayes estimator of R is obtained as the posterior expectation of reliability under squared error (SE) loss function

$$\hat{R}_{s,k}^{B} = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty R_{s,k} \pi(\delta_1, \delta_2, \theta, \beta | X, Y)\ d\delta_1 d\delta_2 d\theta d\beta$$

The evaluation of posterior mean is not tractable. However, approximate posterior mean can be obtained using Lindley's approximation method.
The simplest method to approximate is Lindley's (Lindley (1980)) approximation method.

The Lindley's approximation method evaluates the ratio of the integrals as a whole and-produces a single numerical result.
The Bayes estimator under squared error loss function is

$$\hat{R}_B = u + (u_1 a_1 + u_2 a_2 + a_5 + a_6) + \frac{1}{2}[A(u_1\sigma_{11} + B(u_2\sigma_{22}) + C(u_1\sigma_{31} + u_2\sigma_{32} + D(u_1\sigma_{41} + u_2\sigma_{42})]$$

where

$$a_1 = \rho_1\sigma_{11} + \rho_3\sigma_{13} + \rho_4\sigma_{14}, \quad a_2 = \rho_2\sigma_{22} + \rho_3\sigma_{23} + \rho_4\sigma_{24}$$
$$a_6 = \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22} + u_{33}\sigma_{33} + u_{44}\sigma_{44}),$$
$$A = \sigma_{11}L_{111} + \sigma_{33}L_{331} + 2\sigma_{34}L_{341} + 2\sigma_{44}L_{441},$$
$$B = 2\sigma_{13}L_{132} + 2\sigma_{14}L_{142} + 2\sigma_{23}L_{232} + \sigma_{24}L_{242} + \sigma_{22}L_{222}\sigma_{33}L_{332} + 2\sigma_{34}L_{342} + \sigma_{44}L_{442},$$
$$C = \sigma_{11}L_{113} + 2\sigma_{12}L_{123} + 2\sigma_{13}L_{133} + 2\sigma_{14}L_{143} + 2\sigma_{23}L_{233} + 2\sigma_{24}L_{243} + \sigma_{22}L_{223} + \sigma_{33}L_{333} + 2\sigma_{34}L_{343} + \sigma_{44}L_{443},$$
$$D = \sigma_{11}L_{114} + 2\sigma_{12}L_{124} + 2\sigma_{13}L_{134} + 2\sigma_{14}L_{144} + 2\sigma_{23}L_{234} + 2\sigma_{24}L_{244} + \sigma_{22}L_{224} + \sigma_{33}L_{334} + 2\sigma_{34}L_{344} + \sigma_{44}L_{444},$$

here

$$\rho_1 = \frac{\partial\rho}{\partial\delta_1} = \frac{c_1 - 1}{\delta_1} - d_1, \quad \rho_2 = \frac{\partial\rho}{\partial\delta_2} = \frac{c_2 - 1}{\delta_2} - d_2,$$
$$\rho_3 = \frac{\partial\rho}{\partial\theta} = \frac{c_3 - 1}{\theta} - d_3, \rho_4 = \frac{\partial\rho}{\partial\beta} = \frac{c_4 - 1}{\beta} - d_4$$

The values of L_{ij} can be obtained as follows for $i, j = 1, 2$.

$$L_{11} = \frac{\partial^2 logL_s}{\partial\delta_1^2} = -\frac{n}{\delta_1^2}, \quad L_{22} = \frac{\partial^2 logL_s}{\partial\delta_2^2} = -\frac{m}{\delta_2^2}$$
$$L_{13} = L_{31} = \sum_{i=1}^{n} \beta\frac{x_i}{\theta^2}\left[\frac{x_i}{\theta}^{\beta} - 1\right] + \delta_2\sum_{j=1}^{m} \beta\frac{y_j}{\theta^2}\left[\frac{y_j}{\theta}^{\beta} - 1\right]$$
$$L_{23} = L_{32} = \sum_{j=1}^{m} \beta\frac{y_j}{\theta^2}\left[\frac{y_j}{\theta}^{\beta} - 1\right]$$
$$L_{14} = L_{41} = -\sum_{i=1}^{n} \frac{x_i}{\theta^2}^{\beta} log\left[\frac{x_i}{\theta}\right],$$
$$L_{24} = L_{42} = -\sum_{j=1}^{m} \left(\frac{y_j}{\theta^2}\right)^{\beta} log\left[\frac{y_j}{\theta}\right]$$
$$L_{33} = \frac{\partial^2 logL_s}{\partial\theta^2} = \frac{n+m}{\theta^2} - \delta_1\beta\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta-1}\left(\frac{\beta+1}{\theta^3}\right) - \delta_2\beta\sum_{j=1}^{m}\left(\frac{y_j}{\theta}\right)^{\beta-1}\left(\frac{\beta+1}{\theta^3}\right) + (n+m)\left(\frac{\beta-1}{\theta^2}\right)$$
$$L_{44} = -\frac{n+m}{\beta^2} - \delta_1\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}\left[log\left(\frac{x_i}{\theta}\right)\right]^2 - \delta_2\sum_{j=1}^{m}\left(\frac{y_j}{\theta}\right)^{\beta}\left[log\left(\frac{y_j}{\theta}\right)\right]^2$$

and the values of L_{ijk} for $i,j,k=1,2,3$

$$L_{111}=\frac{2n}{\delta_1^3}, L_{222}=\frac{2m}{\delta_2^3}$$

$$L_{331}=L_{133}=L_{313}=-\sum_{i=1}^{n}\beta x_i\left(\frac{x_i}{\theta}\right)^{\beta-1}\left(\frac{\beta+1}{\theta^3}\right)$$

$$L_{341}=L_{431}=L_{134}=-\sum_{i=1}^{n}\frac{1}{\theta}\left(\frac{x_i}{\theta}\right)^{\beta}\left(1+\beta log\left(\frac{x_i}{\theta}\right)\right)$$

$$L_{332}=L_{233}=L_{323}=-\sum_{j=1}^{m}\beta y_j\left(\frac{y_j}{\theta}\right)^{\beta-1}\left(\frac{\beta+1}{\theta^3}\right)$$

$$L_{441}=-\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}\left[log\left(\frac{x_i}{\theta}\right)\right]^2$$

$$L_{442}=-\sum_{j=1}^{m}\left(\frac{y_j}{\theta}\right)^{\beta}\left[log\left(\frac{y_j}{\theta}\right)\right]^2$$

$$L_{342}=L_{432}=L_{234}=-\sum_{j=1}^{m}\frac{1}{\theta}\left(\frac{y_j}{\theta}\right)^{\beta}\left(1+\beta log\left(\frac{y_j}{\theta}\right)\right)$$

$$L_{333}=-\frac{2(n+m)\beta}{\theta^3}+\delta_1\beta\sum_{i=1}^{n}x_i\left(\frac{x_i}{\theta}\right)^{\beta-1}\frac{(\beta+1)(\beta+2)}{\theta^4}+\delta_2\beta\sum_{j=1}^{m}y_j\left(\frac{y_j}{\theta}\right)^{\beta-1}\frac{(\beta+1)(\beta+2)}{\theta^4}$$

$$L_{343}=\frac{n+m}{\theta^2}-\delta_1\sum_{i=1}^{n}x_i^{\beta}\left[\frac{\beta}{\theta^{\beta+2}}\left(\beta log\frac{x_i}{\theta}\right)+1\right]-\delta_2\sum_{j=1}^{m}y_j^{\beta}\left[\frac{\beta}{\theta^{\beta+2}}\left(\beta log\frac{y_j}{\theta}\right)+1\right]$$

$$L_{444}=\frac{2(n+m)}{\beta^3}-\delta_1\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}\left[log\left(\frac{x_i}{\theta}\right)\right]^3-\delta_2\sum_{j=1}^{m}\left(\frac{y_j}{\theta}\right)^{\beta}\left[log\left(\frac{y_j}{\theta}\right)\right]^3$$

$$L_{443}=\delta_1\sum_{i=1}^{n}\left[\left(\frac{x_i}{\theta}\right)^{\beta}log\left(\frac{x_i}{\theta}\right)\left[\beta log\left(\frac{x_i}{\theta}\right)+2\right]\right]+\delta_2\sum_{j=1}^{m}\left[\left(\frac{y_j}{\theta}\right)^{\beta}log\left(\frac{y_j}{\theta}\right)\left[\beta log\left(\frac{y_j}{\theta}\right)+2\right]\right]$$

since, $u=u(\delta_1,\delta_2,\theta,\beta)=R_{s;k}$,

$$u_1=\frac{\partial u}{\partial\delta_1}=\sum_{i=s}^{k}\sum_{j=0}^{k-i}\binom{k}{i}\binom{k-i}{j}(-1)^{j+1}\frac{\delta_2(i+j)}{[\delta_1(i+j)+\delta_2]^2}$$

$$u_2=\frac{\partial u}{\partial\delta_2}=\sum_{i=s}^{k}\sum_{j=0}^{k-i}\binom{k}{i}\binom{k-i}{j}(-1)^{j}\frac{\delta_1(i+j)}{[\delta_1(i+j)+\delta_2]^2}$$

$$u_{11}=\frac{\partial^2 u}{\partial\delta_1^2}=\sum_{i=s}^{k}\sum_{j=0}^{k-i}\binom{k}{i}\binom{k-i}{j}(-1)^{j}\frac{2\delta_2(i+j)^2}{[\delta_1(i+j)+\delta_2]^3}$$

$$u_{12}=\frac{\partial^2 u}{\partial\delta_1\partial\delta_2}=\sum_{i=s}^{k}\sum_{j=0}^{k-i}\binom{k}{i}\binom{k-i}{j}(-1)^{j+1}\frac{[\delta_1(i+j)-\delta_2](i+j)}{[\delta_1(i+j)+\delta_2]^3}$$

$$u_{22}=\frac{\partial^2 u}{\partial\delta_2^2}=\sum_{i=s}^{k}\sum_{j=0}^{k-i}\binom{k}{i}\binom{k-i}{j}(-1)^{j+1}\frac{2\delta_1(i+j)}{[\delta_1(i+j)+\delta_2]^3}$$

4. Simulation Study

Simulation study consists of estimating multicomponent stress-strength reliability when the sample is generated from new Weibull- Pareto distribution under ML and Bayesian approaches. The comparison of the estimates are done though mean squared error criteria based on 100000 random samples of size n and m. We have evaluated empirical mean square errors for different sets of values for $(\delta_1, \delta_2, \theta, \beta)$ for an s out of k system. For the present study, the values of $(\delta_1, \delta_2, \theta, \beta)$ are $(0.2, 0.1, 1, 1), (1.4, 1.2, 0.4, 0.7), (0.5, 0.6, 0.5, 0.7)$ and $(0.2, 0.7, 1, 0.6)$.
The corresponding true values of stress-strength reliability for s-out-of-k system with $(s, k) = (1, 3)$ are $0.5428, 0.7068, 0.7970$ and 0.9627 that for $(s, k) = (2, 4)$ are $0.3904, 0.5516, 0.6565,\ 0.9104$. The Bayesian estimators under squared error loss function using gamma prior are $c_1 = 2, c_2 = 7, c_3 = 4, d_1 = 3, d_2 = 5, d_3 = 2$ (prior1) and $c_1 = 1, c_2 = 1, c_3 = 1, d_1 = 1, d_2 = 1, d_3 = 1$ (prior2).

Table 1. MLE and Bayes estimators and MSE for estimates of $R_{s,k}$

$\delta_1 = 0.2, \delta_2 = 0.1, \theta = 1, \beta = 1$, prior1						
(s, k)	$R_{s,k}$	n = m	$\hat{R^{srs}_{s,k}}$	$\hat{R^{B}_{s,k}}$	$MSE(\hat{R^{srs}_{s,k}})$	$MSE(\hat{R^{B}_{s,k}})$
(1, 3)	0.5428	10	0.5532	0.5524	0.0091	0.0077
		15	0.5528	0.5511	0.0089	0.0059
		20	0.5464	0.5447	0.0076	0.0050
		30	0.5459	0.5441	0.0071	0.0039
		35	0.5458	0.5437	0.0054	0.0023
		40	0.5446	0.5436	0.0065	0.0014
		50	0.5431	0.5425	0.0012	0.0009
(2, 4)	0.3904	10	0.4010	0.3991	0.0095	0.0082
		15	0.3996	0.3986	0.0083	0.0072
		20	0.3993	0.3981	0.0055	0.0047
		30	0.3981	0.3974	0.0043	0.0035
		35	0.3984	0.3971	0.0038	0.0023
		40	0.3952	0.3945	0.0017	0.0015
		50	0.3932	0.3912	0.0012	0.0008

TABLE 2. MLE and Bayes estimators and MSE for estimates of $R_{s,k}$

$\delta_1 = 1.4, \delta_2 = 1.2, \theta = 0.7, \beta = 0.4$, prior2						
(s, k)	$R_{s,k}$	n = m	$\hat{R_{s,k}^{srs}}$	$\hat{R_{s,k}^{B}}$	$MSE(\hat{R_{s,k}^{srs}})$	$MSE(\hat{R_{s,k}^{B}})$
(1, 3)	0.7068	10	0.7193	0.7091	0.0074	0.0065
		15	0.7092	0.7088	0.0054	0.0052
		20	0.7094	0.7084	0.0051	0.0046
		30	0.7085	0.7079	0.0042	0.0031
		35	0.7083	0.7073	0.0032	0.0018
		40	0.7077	0.7071	0.0019	0.0009
		50	0.7070	0.7061	0.0012	0.0005
(2, 4)	0.5516	10	0.5627	0.5612	0.0089	0.0084
		15	0.5610	0.5601	0.0081	0.0079
		20	0.5606	0.5583	0.0073	0.0061
		30	0.5592	0.5568	0.0062	0.0052
		35	0.5564	0.5547	0.0044	0.0041
		40	0.5540	0.5522	0.0023	0.0019
		50	0.5532	0.5510	0.0016	0.0008

TABLE 3. MLE and Bayes estimators and MSE for estimates of $R_{s,k}$

$\delta_1 = 0.5, \delta_2 = 0.6, \theta = 0.7, \beta = 0.5$, prior2						
(s, k)	$R_{s,k}$	n = m	$\hat{R_{s,k}^{srs}}$	$\hat{R_{s,k}^{B}}$	$MSE(\hat{R_{s,k}^{srs}})$	$MSE(\hat{R_{s,k}^{B}})$
(1, 3)	0.7970	10	0.7995	0.7992	0.0098	0.0087
		15	0.7991	0.7989	0.0091	0.0082
		20	0.7982	0.7975	0.0076	0.0073
		30	0.7980	0.7973	0.0065	0.0046
		35	0.7979	0.7969	0.0047	0.0029
		40	0.7974	0.7972	0.0032	0.0015
		50	0.7972	0.7968	0.0017	0.0012
(2, 4)	0.6565	10	0.6597	0.6588	0.0095	0.0081
		15	0.6599	0.6587	0.0082	0.0075
		20	0.6591	0.6576	0.0074	0.0063
		30	0.6585	0.6577	0.0061	0.0051
		35	0.6583	0.6572	0.0052	0.0047
		40	0.6574	0.6569	0.0036	0.0019
		50	0.6571	0.6562	0.0014	0.0005

Table 4. MLE and Bayes estimators and MSE for estimates of $R_{s,k}$

$\delta_1 = 0.2, \delta_2 = 0.7, \theta = 0.6, \beta = 1.1$, prior2						
(s, k)	$R_{s,k}$	n = m	$R_{s,k}^{\hat{srs}}$	$R_{s,k}^{\hat{B}}$	$MSE(R_{s,k}^{\hat{srs}})$	$MSE(R_{s,k}^{\hat{B}})$
(1, 3)	0.9627	10	0.9692	0.9691	0.0082	0.0072
		15	0.9697	0.9687	0.0059	0.0088
		20	0.9685	0.9672	0.0048	0.0053
		30	0.9674	0.9664	0.0041	0.0034
		35	0.9667	0.9643	0.0032	0.0021
		40	0.9641	0.9635	0.0025	0.0016
		50	0.9630	0.9626	0.0014	0.0008
(2, 4)	0.9104	10	0.9167	0.9149	0.0089	0.0066
		15	0.9156	0.9141	0.0074	0.0061
		20	0.9157	0.9132	0.0062	0.0054
		30	0.9145	0.9124	0.0055	0.0032
		35	0.9136	0.9118	0.0051	0.0027
		40	0.9112	0.9105	0.0027	0.0016
		50	0.9109	0.9102	0.0011	0.0002

5. Real Data Analysis

In this section, we present a real data which was reported by Xia et al (2009) and Saracoglu et al (2012). Data set I: Breaking strength of jute fiber length 10 mm (Variable X)
693.73, 704.66, 323.83, 778.17, 123.06, 637.66, 383.43, 151.48, 108.94, 50.16, 671.49, 183.16, 257.44, 727.23, 291.27, 101.15, 376.42, 163.40, 141.38, 700.74, 262.90, 353.24, 422.11, 43.93, 590.48, 212.13, 303.90, 506.60, 530.55, 177.25
Data set II: Breaking strength of jute fiber length 20 mm (Variable Y)
71.46, 419.02, 284.64, 585.57, 456.60, 113.85, 187.85, 688.16, 662.66, 45.58, 578.62, 756.70, 594.29, 166.49, 99.72, 707.36, 765.14, 187.13, 145.96, 350.70, 547.44, 116.99, 375.81, 581.60, 119.86, 48.01, 200.16, 36.75, 244.53, 83.55.
For the above data sets, we fit the new Weibull-Pareto model and also checked the validity of the model using Kolmogorov-Smirnov(K-S) test for each data set. It was found that for data set I and II, the k-s distanced are $1.8041e^{-}16$ and $6.9389e^{-}18$ with the corresponding p value are 0.9599 and 0.9634 respectively. From the result, it shows that new Weibull Pareto distribution fits better for the data sets. The maximum likelihood estimate and Bayes estimate, based on the parameters with its standard errors are $\hat{\delta}_1 = 0.06808(0.1613)$, $\hat{\delta}_2 = 14.9301(1.0026)$, $\hat{\theta} = 2.7695(4.1627)$ and $\hat{\beta} = 1.43064(0.0461)$ are obtained as $\hat{R}_{1,3}^{M} = 0.7679$, $\hat{R}_{1,3}^{B} = 0.7712$ under prior1 and $\hat{R}_{1,3}^{B} = 0.7724$ under prior2. For $s = 2$ and $k = 4$ MLE and Bayes estimators are $\hat{R}_{2,4}^{Msrs} = 0.6547$, $\hat{R}_{2,4}^{B} = 0.6585$ under prior1 and $\hat{R}_{2,4}^{B} = 0.6587$ under prior2.

6. Summary and Conclusions

The estimation of s-out-of-k system reliability under stress-strength setup is considered in the present paper when the underlying distributions follow new Weibull-Pareto. The system reliability is estimated using maximum likelihood and Bayesian approaches. Approximate Bayes estimators are obtained using Lindley's approximation technique. Simulation study results indicates that MSEs of system reliability decreases as sample size increases. Bayes estimators for both priors performs better than maximum likelihood method.

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