Stochastic Modeling and Applications
Vol. 25 No. 1 (January-June, 2021)

Received: 11th January 2021 *Revised: 12th February 2021* *Selected: 22nd February 2021*

NEW SUBCLASS OF ANALYTIC FUNCTIONS WITH NEGATIVE COEFFICIENTS DEFINED BY MITTAG- LEFFLER FUNCTION

PASUNOORI SRINIVASULU AND V.SRINIVAS

Abstract. In this work, we introduce and study a new subclass of analytic functions defined by a differential operator and obtained coefficient estimates, growth and distortion theorems, radii of starlikeness, convexity and close-to-convexity are obtained. Furthermore, we obtained integral means inequalities for the function.

2020Mathematics Subject Classification: 30C45

1. Introduction

Let A denote the class of analytic functions f defined on the unit disk $U = \{z : |z| < 1\}$ with normalization $f(0) = 0$ and $f'(0) = 1$. Such a function has the Taylor series expansion about the origin in the form

$$f(z) = z + \sum_{\eta=2}^{\infty} a_\eta z^\eta, \tag{1.1}$$

denoted by S, the subclass of A consisting of functions that are univalent in U. For $f \in A$ given by and $g(z)$ given by

$$g(z) = z + \sum_{\eta=2}^{\infty} b_\eta z^\eta$$

their convolution (or Hadamard product), denoted by $(f * g)$, is defined as

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z) \quad (z \in U).$$

Denote by T the subclass of A consisting of functions f of the form

$$f(z) = z - \sum_{\eta=2}^{\infty} a_\eta z^\eta, \tag{1.2}$$

This subclass was introduced and studied by Silverman[5] .

1991 *Mathematics Subject Classification.* Primary 30c45.
Key words and phrases. analytic, coefficient bounds, starlike, distortion.

The following defines the familiar Mittag-Leffler function $E_\upsilon(z)$ introduced by Mittag- Leffler [3] and its generalization $E_{\upsilon,\tau}(z)$ introduced by Wiman [9].

$$E_\upsilon(z) = \sum_{\eta=0}^{\infty} \frac{z^\eta}{\Gamma(\upsilon\eta + 1)}$$

and

$$E_{\upsilon,\tau}(z) = \sum_{\eta=0}^{\infty} \frac{z^\eta}{\Gamma(\upsilon\eta + \tau)}$$

where $\upsilon, \tau \in \mathbb{C}, \Re(\upsilon) > 0$ and $\Re(\tau) > 0$.
We define the function $Q_{\upsilon,\tau}(z)$ by

$$Q_{\upsilon,\tau}(z) = z\Gamma(\tau)E_{\upsilon,\tau}(z).$$

Now, for $f \in A$, we define the following differential operator$\mathfrak{D}_\lambda^m(\upsilon,\tau)f : A \to A$ by

$$\begin{aligned} \mathfrak{D}_\lambda^0(\upsilon,\tau)f(z) &= f(z) * Q_{\upsilon,\tau}(z) \\ \mathfrak{D}_\lambda^1(\upsilon,\tau)f(z) &= (1-\lambda)(f(z) * Q_{\upsilon,\tau}(z)) + \lambda z(f(z) * Q_{\upsilon,\tau}(z))' \\ &\vdots \\ \mathfrak{D}_\lambda^m(\upsilon,\tau)f(z) &= \mathfrak{D}_\lambda^1\left(\mathfrak{D}_\lambda^{m-1}(\upsilon,\tau)f(z)\right) \end{aligned}$$

If f is given by (1.1) then from the definition of the operator $\mathfrak{D}_\lambda^m f$ it is easy to see that

$$\mathfrak{D}_\lambda^m(\upsilon,\tau)f(z) = z + \sum_{\eta=2}^{\infty} \phi_\eta^m(\lambda,\upsilon,\tau)a_\eta z^\eta \tag{1.3}$$

where

$$\phi_\eta^m(\lambda,\upsilon,\tau) = \frac{\Gamma(\tau)}{\Gamma(\upsilon(\eta-1)+\tau)}[\lambda(\eta-1)+1]^m. \tag{1.4}$$

Note that

(1) when $\upsilon = 0$ and $\tau = 1$, we get Al-Oboudi operator [1].
(2) when $\upsilon = 0, \tau = 1$ and $\lambda = 1$, we get Salagean operator [5].
(3) when $m = 0$, we get $E_{\upsilon,\tau}(z)$, Srivastava et al. [9]. .

If $f \in T$ is given by then we have

$$\mathfrak{D}_\lambda^m(\upsilon,\tau)f(z) = z - \sum_{\eta=2}^{\infty} \phi_\eta^m(\lambda,\upsilon,\tau)a_\eta z^\eta$$

Now, by making use of the differential operator $\mathfrak{D}_\lambda^m(\upsilon,\tau)f$, we define a new subclass of functions belonging to the class A.
In this paper , using the operator $\mathfrak{D}_\lambda^m(\upsilon,\tau)f(z)$, we define the following new class motivated by
Murugusunderamoorthy and Magesh [4].
Definition 1. The function $f(z)$ of the form (1.1) is in the class $S_\lambda^m(\mu,\gamma,\varsigma)$ if it satisfies the inequality

$$Re\left\{\frac{z\left(\mathtt{D}_\lambda^m(\upsilon,\tau)f(z)\right)'}{(1-\mu)z+\mu\mathtt{D}_\lambda^m(\upsilon,\tau)f(z)} - \gamma\right\} > \varsigma\left|\frac{z\left(\mathtt{D}_\lambda^m(\upsilon,\tau)f(z)\right)'}{(1-\mu)z+\mu\mathtt{D}_\lambda^m(\upsilon,\tau)f(z)} - 1\right|$$

for $0 \le \lambda \le 0$, $0 \le \gamma \le 1$ and $\varsigma \ge 0$.

Further we define $TS_{\lambda}^{m}(\mu,\gamma,\varsigma) = S_{\lambda}^{m}(\mu,\gamma,\varsigma) \cap T$.
The aim of this paper is to study the coefficient bounds , radii of close-to-convex and starlikeness

convex linear combinations for the class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.Furthermore, we obtained integral means inequalities for the functions in $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.

Theorem 1: A function *f(z)* of the form (1.1) is in $S_{\lambda}^{m}(\mu,\gamma,\varsigma)$

$$\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_n| \leq 1-\gamma \tag{1.5}$$

where $0 \leq \mu \leq 1$, $0 \leq \gamma < 1$,$\varsigma \geq 0$ and $\phi_{\eta}^{m}(\lambda,\upsilon,\tau))$ is given by (1.4).
Proof: It suffices to show that

$$\varsigma\left|\frac{z\left(\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)\right)'}{(1-\mu)z+\mu\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)}-1\right|-Re\left\{\frac{z\left(\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)\right)'}{(1-\mu)z+\mu\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)}-1\right\}\leq 1-\gamma.$$

We have

$$\varsigma\left|\frac{z\left(\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)\right)'}{(1-\mu)z+\mu\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)}-1\right|-Re\left\{\frac{z\left(\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)\right)'}{(1-\mu)z+\mu\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)}-1\right\}$$

$$\leq (1+\varsigma)\left|\frac{z\left(\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)\right)'}{(1-\mu)z+\mu\mathtt{D}_{\lambda}^{m}\left(\upsilon,\tau\right)u\left(z\right)}-1\right|$$

$$\leq (1+\varsigma)\frac{\sum\limits_{n=2}^{\infty}(n-\mu)\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_n|\,|z|^{n-1}}{1-\sum\limits_{n=2}^{\infty}\mu\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_n|\,|z|^{n-1}}$$

$$\leq (1+\varsigma)\frac{\sum\limits_{n=2}^{\infty}(n-\mu)\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_n|}{1-\sum\limits_{n=2}^{\infty}\mu\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_n|}$$

The last expression is bounded above by $(1-\gamma)$ if

$$\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_n| \leq 1-\gamma$$

and the proof is complete.
Theorem 2: Let $0 \leq \mu \leq 1$, $0 \leq \gamma < 1$and $\varsigma \geq 0$ then a function f of the form (1.2) to be in the class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$ if and only if

$$\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right) \leq 1-\gamma \tag{1.6}$$

where $\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)$ are given by (1.4).

Proof: In view of Theorem 1, we need only to prove the necessity. If $f \in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$ and z is real then

$$Re\left\{\frac{1-\sum\limits_{n=2}^{\infty}n\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)a_{n}z^{n-1}}{1-\sum\limits_{n=2}^{\infty}\mu\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)a_{n}z^{n-1}}-\gamma\right\}>\varsigma\left|\frac{\sum\limits_{n=2}^{\infty}(n-\mu)\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)a_{n}z^{n-1}}{1-\sum\limits_{n=2}^{\infty}\mu\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)a_{n}z^{n-1}}\right|$$

Letting z $\to$ 1 along the real axis, we obtain the desired inequality

$$\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)|a_{n}|\leq 1-\gamma,$$

where $0\leq\mu<1$, $0\leq\gamma<1$, $\varsigma\geq 0$ and $\phi_{\eta}^{m}(\lambda,\upsilon,\tau)$ are given by (1.4).

Corollary 1. If *f(z)* $\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$, then

$$|a_{n}|\leq\frac{1-\gamma}{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}\tag{1.7}$$

where $0\leq\mu<1$, $0\leq\gamma<1$, $\varsigma\geq 0$ and $\phi_{\eta}^{m}(\lambda,\upsilon,\tau)$ are given by (1.4). Equality holds for the function

$$f(z)=z-\frac{1-\gamma}{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}z^{n}.\tag{1.8}$$

Theorem 3. Let
$f_{1}(z)=z$ and

$$f_{n}(z)=z-\frac{1-\gamma}{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}z^{n},n\geq 2.\tag{1.9}$$

Then *f(z)* $\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$, if and only if it can be expressed in the form

$$f(z)=\sum_{n=1}^{\infty}w_{n}f_{n}(z),w_{n}\geq 0,\sum_{n=1}^{\infty}w_{n}=1.\tag{1.10}$$

Proof. Suppose *f(z)* can be written as in (1.10).Then

$$f(z)=z-\sum_{n=2}^{\infty}w_{n}\frac{1-\gamma}{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}z^{n}.$$

Now,

$$\sum_{n=2}^{\infty}w_{n}\frac{(1-\gamma)[n(1+\varsigma)-\mu(\gamma+1)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}{(1-\gamma)[n(1+\varsigma)-\mu(\gamma+1)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}=\sum_{n=2}^{\infty}w_{n}=1-w_{1}\leq 1.$$

Thus *f(z)* $\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$. Conversely , let us have *f(z)* $\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.Then by using (1.7) , we get

$$w_{n}=\frac{[n(1+\varsigma)-\mu(\gamma+1)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}{(1-\gamma)}a_{n},n\geq 2$$

and $w_{1}=1-\sum_{n=2}^{\infty}w_{n}$.Then we have $f(z)=\sum_{n=1}^{\infty}w_{n}f_{n}(z)$ and hence this completes the proof of Theorem.

Theorem 4. The class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$is a convex set.

Proof. Let $f_{j}(z)=z-\sum_{n=2}^{\infty}a_{n,j}z^{n}$,$a_{n,j}\geq 0$, j =1,2

(1.10)

be in the class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$. It sufficient to show that the function $h(z)$ defined by

$$h(z)=\xi f_1(z)+(1-\xi)f_2(z), 0\le \xi<1,$$

is in the class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$. Since

$$h(z)=z-\sum_{n=2}^{\infty}\left[\xi a_{n,1}+(1-\xi)a_{n,2}\right]z^n,$$

An easy compution with the aid of of Theorem 2, gives

$$\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\ \xi\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)a_{n,1}\ +\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\ (1-\xi)\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)a_{n,2}$$

$$\le \xi(1-\gamma)+(1-\xi)(1-\gamma)$$

$$\le (1-\gamma),$$

which implies that $h\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.
Hence $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$ is convex.
Next we obtain the radii of close –to-convexity ,starlikeness and convexity for the class$TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.

Theorem 5. Let the function $f(z)$defined by (1.2) belong to the class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.Then $f(z)$ is close-to-convex of order δ $(0\le\delta<1)$ in the disc $|z|<r_1$, where

$$r_1=\inf_{n\ge 2}\left[\frac{(1-\delta)\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}{n(1-\gamma)}\right]^{1/n-1},n\ge 2. \quad (1.11)$$

The result is sharp, with the extremal function $f(z)$ is given by (1.8)
Proof. Given $f\in T$, and f is close-to-convex of order δ, we have

$$|f'(z)-1|<1-\delta \quad (1.12)$$

For the left hand side of (1.12) we have

$$|f'(z)-1|\le\sum_{n=2}^{\infty}na_n\,|z|^{n-1}$$

The last expression is less than $1-\delta$

$$\sum_{n=2}^{\infty}\frac{n}{1-\delta}a_n\,|z|^{n-1}\le 1.$$

Using the fact, that *f(z)* $\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$if and only if

$$\sum_{n=2}^{\infty}\frac{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}{(1-\gamma)}a_n\le 1,$$

We can (1.12) is true if

$$\frac{n}{1-\delta}\,|z|^{n-1}\le\frac{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}\left(\lambda,\upsilon,\tau\right)}{(1-\gamma)}$$

or, equivalently,

$$|z| \leq \left\{ \frac{(1-\delta)[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}(\lambda,\upsilon,\tau)}{n(1-\gamma)} \right\}^{1/n-1}$$

which completes the proof.

Theorem 6. Let the function $f(z)$defined by (1.3) belong to the class $TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$.Then $f(z)$ is starlike of order of order δ $(0 \leq \delta < 1)$ in the disc $|z| < r_2$, where

$$r_2 = \inf_{n\geq 2} \left[\frac{(1-\delta)\sum_{n=2}^{\infty}[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}(\lambda,\upsilon,\tau)}{(n-\delta)(1-\gamma)} \right]^{1/n-1} \tag{1.13}$$

The result is sharp,with extremal function $f(z)$ is given by (1.8).

Proof. Given $f \in T$, and f is starlike of order δ, we have

$$\left| \frac{zf'(z)}{f(z)} - 1 \right| < 1-\delta \tag{1.14}$$

For the left hand side of (1.14) we have

$$\left| \frac{zf'(z)}{f(z)} - 1 \right| \leq \sum_{n=2}^{\infty} \frac{(n-1)a_n\,|z|^{n-1}}{1-\sum_{n=2}^{\infty} a_n\,|z|^{n-1}}$$

The last expression is less than 1- if

$$\sum_{n=2}^{\infty} \frac{n-\delta}{1-\delta} a_n\,|z|^{n-1} < 1.$$

Using the fact that *f(z)* $\in TS_{\lambda}^{m}(\mu,\gamma,\varsigma)$ if and if

$$\sum_{n=2}^{\infty} \frac{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}(\lambda,\upsilon,\tau)}{(1-\gamma)} a_n \leq 1,$$

We can say (1.14) is true if

$$\sum_{n=2}^{\infty} \frac{n-\delta}{1-\delta}\,|z|^{n-1} \leq \frac{[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}(\lambda,\upsilon,\tau)}{(1-\gamma)}$$

or equilently

$$|z|^{n-1} \leq \frac{(1-\delta)[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_{\eta}^{m}(\lambda,\upsilon,\tau)}{(n-\delta)(1-\gamma)}$$

which yields the starlikeness of the family.

Integral Means Inequalities

In [6], Silverman found that the function $f_2(z) = z - \frac{z^2}{2}$ is often extremal over the family T. He applied this function to resolve his integral means inequality conjuctured [7] and settled in [8], that

$$\int_0^{2\pi} \left|f(re^{i\varphi})\right|^{\eta} d\varphi \leq \int_0^{2\pi} \left|f_2(re^{i\varphi})^{\eta}\right| d\varphi,$$

for all $f \in T$, $\eta > 0$ and $0 < r < 1$. In [6] , he also proved his conjucture for the subclasses

$T^*(\alpha)$ and $C(\alpha)$ of T.
Now, we prove Silverman 's conjecture for the class of functions $TS_\lambda^m(\mu,\gamma,\varsigma)$..
We need the concept of subordination between analytic functions and a subordination
theorem of Littlewood [2].
Two functions f and g, which are analytic in E, the function f is said to be subordinate to g in E if there exists a function w analytic in E with $w(0)=0$, $|w(z)|<1$, $(z\in E)$ Such that $f(z)=g(w(z))$, $(z\in E)$.
We denote this subordination by $f(z)\prec g(z)$. ($\prec$denotes subordination).
Lemma 1. If the functions $f$and $g$are analytic in E with $f(z)\prec g(z)$,then for $\eta>0$and$z=re^{i\varphi}$ $0<r<1$,

$$\int_0^{2\pi}\left|g(re^{i\varphi})\right|^\eta d\varphi\le\int_0^{2\pi}\left|f(re^{i\varphi})\right|^\eta d\varphi$$

Now, we discuss the integral means inequalities for functions f in $TS_\lambda^m(\mu,\gamma,\varsigma)$.

$$\int_0^{2\pi}\left|g(re^{i\varphi})\right|^\eta d\varphi\le\int_0^{2\pi}\left|f(re^{i\varphi})\right|^\eta d\varphi$$

Theorem 7. Let $f\in TS_\lambda^m(\mu,\gamma,\varsigma)$, $0\le\mu<1$, $0\le\gamma<1$, and $f_2(z)$be defined by

$$f_2(z)=z-\frac{1-\gamma}{\varphi_2(\lambda,m,\mu,\varsigma,\gamma)}z^2 \tag{1.15}$$

Proof. For $f(z)=z-\sum_{n=2}^\infty a_nz^n$, (1.15) is equivalent to

$$\int_0^{2\pi}\left|1-\sum_{n=2}^\infty a_nz^{n-1}\right|^\eta d\varphi\le\int_0^{2\pi}\left|1-\frac{1-\gamma}{\varphi_2(\lambda,m,\mu,\varsigma,\gamma)}z\right|^\eta d\varphi$$

By Lemma 1, it is enough to prove that

$$1-\sum_{n=2}^\infty a_nz^{n-1}\prec 1-\frac{1-\gamma}{\varphi_2(\lambda,m,\mu,\varsigma,\gamma)}z$$

Assuming

$$1-\sum_{n=2}^\infty a_nz^{n-1}\prec 1-\frac{1-\gamma}{\varphi_2(\lambda,m,\mu,\varsigma,\gamma)}w(z),$$

and using (1.6) we obtain

$$|w(z)|=\left|\sum_{n=2}^\infty\frac{\varphi_2(\lambda,m,\mu,\varsigma,\gamma)}{1-\gamma}a_nz^{n-1}\right|\le|z|\sum_{n=2}^\infty\frac{\varphi_2(\lambda,m,\mu,\varsigma,\gamma)}{1-\gamma}a_n\le|z|$$

where $\varphi_n(\lambda,m,\mu,\varsigma,\gamma)=[n(1+\varsigma)-\mu(\gamma+\varsigma)]\phi_\eta^m(\lambda,\upsilon,\tau)$
This completes the proof.

References

[1] F. M. Al-Oboudi, On univalent functions defined by a generalized Salagean operator, Internat. J. Math. Math.Sci., 27 (2004), 1429–1436.
[2] J.E. Littlewood, On inequalities in the theory of functions, Proc. London Math. Soc., 23, 481-519. 1925.
[3] G. M. Mittag-Leffler, Sur la nouvelle fonction E(x), CR Acad. Sci. Paris, 137(1903), 554–558.

[4].G. Murugusundarmoorthy and N. Magesh, Certain sub-classes of starlike functions of complex order involving generalized hypergeometric functions. Int. J. Math. Math. Sci., art ID 178605, 12, 2010.
[5] G. S. Salagean, Subclasses of univalent functions, Lecture Note in Math. (SpringerVerlag),1013 (1983), 362–372 .
[6] H. Silverman, Univalent functions with negative coefficients, Proc. Amer. Math. Soc. 51, 109-116, 1975.
[7] H. Silverman, A survey with open problems on univalent functions whose coefficient are negative, Rocky Mountain J. Math., 21, 1099-1125, 1991.
[8] H.Silverman, Integral means for univalent functions with negative coefficient, Houston J. Math., 23, 169-174, 1997.
[9] H. M. Srivastava, B. A. Frasin and V. Pescar, Univalence of integral operators involving Mittag- Leffler functions, Appl. Math. Inf. Sci., 11 (2017), 635–641.
[10] A. Wiman, Uber den fundamentalsatz in der teorie der funktionen E(x), Acta Math., 29 (1905), 191–201.

Pasunoori Srinivasulu: Department of Mathematics, Rajeev Gandhi University of KnowledgeTechnologies, IIIT,Basar,Nirmal-504107. T.S
E-mail address: Srinivaspasupuleti12@gmail.com

V.SRINIVAS: Department of Mathematics, Dr. B.R.Ambedkar Open University, Hyderabad -500033. T.S.