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AN EXPLICIT SOLUTION FOR AN INVENTORY MODEL WITH POSITIVE LEAD TIME AND BACKLOGS

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Abstract. In this paper, we consider a single server queuing system with inventory where customers arrive according to a Poisson process. Inventory is replenished according to the (s, S) policy. Here lead time is considered positive whereas service times are negligible. The lead time follow exponential distribution. We assume that the customers who arrive during stock out period remains in the system until replenishment occurs. Stability of the above system is analyzed and steady state vector is calculated explicitly. Several performance measures such as expected number of customers in the system, expected inventory level in the system, mean replenishment rate and so on were calculated numerically. The effect of different parameters on the various performance measures were discussed. An expression for the mean waiting time of an arriving customer was also obtained. Most of the papers in inventory queuing models assume that no arrival is entertained when inventory level is zero. But we consider backlog of customers and could arrive at an explicit solution for the steady state vector.

1. Introduction

Queueing inventory systems was first studied by (Melikov and Molchano 1992) and (Sigman and Simchi- Levi 1992). Later Berman and et.al [1] discussed inventory systems where time is required to process the inventory needed to serve the customer. The models they discussed were deterministic models. Queueing Inventory models with exponential/arbitrary distributions were first studied by Berman and Kim [2] and Berman and Sapna [3]. Krishnamoorthy and his co-authors [4, 6, 7, 9, 10, 11, 12, 13, 14] studied inventory models where the service time for providing the inventoried item was considered positive. They used Matrix analytic methods to discuss these models.

Padmavathi.I et al [15] studied a finite source (s,S) inventory system. In this model the idea of postponed demands and server vacation have been considered. Krishnamoorthy and Islam [8] considered an (s,S) inventory system with postponed demands ,Poisson arrivals and exponential lead time. Sivakumar and Arivarignam [16] studied a perishable inventory system with postponed demands in which the demands that occur during the stock out period enter a pool with independent Bernoulli trial.

Key words and phrases. (s,S) inventory policy, instantaneous service, backlogs, positive lead time, explicit solution for steady state vector.

2. Model description

The model we consider is described as follows. Customers arrive to a queueing system which has only one server where some inventory is served. The arrivals are in accordance with a Poisson process with parameter λ. The replenishment of the inventoried item is done according to (s,S) policy, the replenishment time being an exponential random variable with parameter δ. We assume negligible service time for this model, but backlogs are allowed in the sense that customers who join the queue when the inventory level drops to zero form a queue and remains in the system until inventory replenishment is realized.

Now we describe the queueing model mathematically. For that we use the following notations.
$N(t)$: The number of customers in the system at time t.
$L(t)$: The number of inventories in the system at time t.
Then $\Omega = \{Y(t) : t \geq 0\} = \{(N(t), L(t)) : t \geq 0\}$ is a Markov chain. The state space of this Markov chain can be described as $H = \{(0, j) : 0 \leq j \leq S\} \cup \{(i, 0) : i \geq 1\}$ and it can be partitioned into levels $\tilde{i}$ defined as $\tilde{0} = \{(0, 0), (0, 1), \ldots, (0, S)\}$ and $\tilde{i} = \{((i-1)Q+1, 0), ((i-1)Q+2, 0), \ldots, ((i-1)Q+Q, 0)\}; i \geq 1$. The Markov chain under consideration is a level independent Quasi Birth Death (QBD) process. Let $S - s = Q$, I_n denotes an identity matrix of order n and e denotes a column vector of 1's of appropriate order. The infinitesimal generator matrix of the process Ω is

$$E = \begin{bmatrix} B_0 & B_1 & 0 & 0 & - & - & - \\ B_2 & A_1 & A_0 & 0 & 0 & - & - \\ 0 & A_2 & A_1 & A_0 & 0 & 0 & - \\ 0 & 0 & A_2 & A_1 & A_0 & 0 & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \end{bmatrix}$$

Here $B_0 = [b_{ij}]_{(S+1)\times(S+1)}$, where

$$b_{ij} = \begin{cases} -(\lambda + \delta) & : j = i; 1 \leq i \leq s+1 \\ -\lambda & : j = i; s+1 \leq i \leq S+1 \\ \lambda & : j = i-1; 2 \leq i \leq S+1 \\ \delta & : j = Q+i; 1 \leq i \leq s+1 \\ 0 & : \text{otherwise} \end{cases}$$

$$B_1 = [b_{ij}]_{(S+1)\times Q},\ b_{ij} = \begin{cases} \lambda & : i = j = 1 \\ 0 & : \text{otherwise} \end{cases}$$

$$B_2 = [b_{ij}]_{Q\times(S+1)},\quad b_{ij} = \begin{cases} \delta & : i + j = Q; 1 \leq i \leq Q \\ 0 & : \text{otherwise} \end{cases}$$

$$A_2 = \delta I_Q; \quad A_0 = [a_{ij}]_{Q\times Q}; \quad a_{ij} = \begin{cases} \lambda & : i = Q, j = 1 \\ 0 & : \text{otherwise} \end{cases}$$

$$A_1 = [a_{ij}]_{Q\times Q}; \quad a_{ij} = \begin{cases} -(\lambda+\delta) & : i = j \\ \lambda & : j = i+1; 1 \le i \le Q-1 \end{cases}$$

3. Analysis of the model

Stability condition: Suppose $A = A_0 + A_1 + A_2$ and let $x = (x_1, x_2, \ldots, x_Q)$ be the steady state vector of the generator matrix A. Then $xA = 0$ gives the following equations

$$-\lambda x_1 + \lambda x_Q = 0$$
$$-\lambda x_{i+1} + \lambda x_i = 0;\ 1 \le i \le Q-1$$

Hence $x_1 = x_2 = \ldots = x_Q$. A quasi birth death (QBD) process is stable if and only if the rate of drift to the left is larger than rate of drift to the right; that is $xA_0e < xA_2e$ (see Neuts). For the model under study the above expression simplifies to $\dfrac{\lambda}{Q\delta} < 1$. We state this result below as a theorem.

Theorem 3.1. *The Markov chain Ω is stable if and only if $\dfrac{\lambda}{Q\delta} < 1$.*

4. Computation of steady state vector

We find the steady state vector of Ω explicitly. Let $x = (x_0, x_1, \ldots)$ be the steady state vector of the Markov chain Ω. Here

$$x_0 = (x_0(0,0), x_0(0,1), \ldots, x_0(0,S))$$

and $x_i = (x_i((i-1)Q+1, 0), x_i((i-1)Q+2, 0), \ldots, x_i((i-1)Q+Q, 0)); i \ge 1$. The steady state equations are given by

$$\begin{aligned} xE = 0 \implies & x_0B_0 + x_1B_2 = 0 \\ & x_0B_1 + x_1A_1 + x_2A_2 = 0 \\ & x_iA_0 + x_{i+1}A_1 + x_{i+2}A_2 = 0; i \ge 1 \end{aligned}$$

Ω is a level independent QBD process. Hence the steady state vector of Ω is given by $x_{i+1} = x_1R^i; i \ge 1$ (see Neuts), where R is the minimal non negative solution of the matrix quadratic equation $R^2A_2 + RA_1 + A_0 = 0$. All the rows of A_0 except the last are zeros. Hence the same is true for the R matrix. Assume that

$$R = [r_{ij}]_{Q\times Q}; r_{ij} = \begin{cases} r_j, & i = Q, 1 \le j \le Q \\ 0, & \textit{otherwise} \end{cases}$$

Now $R^2A_2 + RA_1 + A_0 = 0$ gives us the following equations.

$$\begin{aligned}
\lambda - (\lambda+\delta)r_1 + \delta r_1 r_Q &= 0;\\
\lambda r_1 - (\lambda+\delta)r_2 + \delta r_2 r_Q &= 0;\\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots&\\
\lambda r_{Q-2} - (\lambda+\delta)r_{Q-1} + \delta r_{Q-1} r_Q &= 0;\\
\lambda r_{Q-1} - (\lambda+\delta)r_Q + \delta r_Q^2 &= 0
\end{aligned}$$

From the above equations we easily see that $r_j = r^j$ for every $j; r = r_1$. Then any of the above equations gives $\lambda - (\lambda+\delta)r + \delta r^{Q+1} = 0$. Dividing by $r-1$ we get $\delta r^Q + \delta r^{Q-1} + \ldots + \delta r - \lambda = 0$. Let $f(y) = \delta y^Q + \delta y^{Q-1} + \ldots + \delta y - \lambda$. Since $f(0)$ and $f(1)$ have opposite sign $f(y)$ has a root r between 0 and 1. Hence

$$R = [r_{ij}]_{Q\times Q}; r_{ij} = \begin{cases} r^j, & i = Q, 1 \le j \le Q\\ 0, & otherwise\end{cases}$$

Now from $x_{i+1} = x_1 R^i; i \ge 1$ we get $x_2 = (r, r^2, \ldots, r^Q)x_1(Q,0)$; $x_3 = (r^{Q+1}, r^{Q+2}, \ldots, r^{2Q})x_1(Q,0)$ and so on. It remains to get x_0 and x_1 in terms of $x_1(Q,0)$.

$$x_0(0,0) = \left\{\left(\frac{\lambda+\delta}{\lambda}\right)^Q - r\frac{\delta}{\lambda}\left(\frac{1 - r^Q\left(\frac{\lambda+\delta}{\lambda}\right)^Q}{1 - r\left(\frac{\lambda+\delta}{\lambda}\right)}\right)\right\}x_1(Q,0)$$

$$x_1(i,0) = \left\{\left(\frac{\lambda+\delta}{\lambda}\right)^{Q-i} - r^{i+1}\frac{\delta}{\lambda}\left(\frac{1 - r^{Q-i}\left(\frac{\lambda+\delta}{\lambda}\right)^{Q-i}}{1 - r\left(\frac{\lambda+\delta}{\lambda}\right)}\right)\right\}x_1(Q,0); \quad 1 \le i \le Q-1$$

$$x_0(0,1) = \left(\frac{\lambda+\delta}{\lambda}\right)x_0(0,0) - \frac{\delta}{\lambda}x_1(Q,0)$$

$$x_0(0,i+1) = \left(\frac{\lambda+\delta}{\lambda}\right)x_0(0,i) - \frac{\delta}{\lambda}\left\{\left(\frac{\lambda+\delta}{\lambda}\right)^i - r^{Q-i+1}\frac{\delta}{\lambda}\left(\frac{1 - r^i\left(\frac{\lambda+\delta}{\lambda}\right)^i}{1 - r\left(\frac{\lambda+\delta}{\lambda}\right)}\right)\right\}x_1(Q,0); \quad 1 \le i \le s$$

$$x_0(0,i+1) = x_0(0,i) - \frac{\delta}{\lambda}\left\{\left(\frac{\lambda+\delta}{\lambda}\right)^i - r^{Q-i+1}\frac{\delta}{\lambda}\left(\frac{1 - r^i\left(\frac{\lambda+\delta}{\lambda}\right)^i}{1 - r\left(\frac{\lambda+\delta}{\lambda}\right)}\right)\right\}x_1(Q,0); \quad s+1 \le i \le Q-1$$

$$x_0(0,Q+i+1) = x_0(0,Q+i) - \frac{\delta}{\lambda}x_0(0,i);\ 0 \le i \le s-1.$$

Now $x_1(Q,0)$ is got from the condition $x_0 e_1 + \left(\sum_{i=1}^{\infty} x_i\right)e_2 = 1$ where e_1 and e_2 are column vector of one's of appropriate order.

5. System Performance Measures

We compute some performance measures numerically.

5.1. Expected waiting time of a customer in the queue. First we compute the expected waiting time in the queue of a tagged customer, who joins as the l^{th} customer in the queue, $(k-1)Q < l \leq kQ$. For that consider a Markov process $\psi = (\hat{N}(t))$, where $\hat{N}(t)$ denotes the rank, which is the position of the customer in the queue. The state space of this Markov chain is given by $\hat{F} = \{1, 2, \ldots, k\} \cup \Delta$, where Δ is an absorbing state which corresponds to the tagged customer being taken for service. The infinitesimal generator matrix of the process ψ is given by $\hat{Q} = \begin{bmatrix} T & T^0 \\ 0 & 0 \end{bmatrix}$, where T^0 is a $k \times 1$ matrix such that $T^0(i,1) = \begin{cases} \delta; & i = k \\ 0; & otherwise \end{cases}$ and $T = \begin{bmatrix} -\delta & \delta & 0 & 0 & 0 & 0 \\ 0 & -\delta & \delta & 0 & 0 & 0 \\ 0 & 0 & -\delta & \delta & 0 & 0 \\ 0 & 0 & - & - & - & - \\ 0 & 0 & 0 & - & - & - \\ 0 & 0 & 0 & 0 & - & -\delta \end{bmatrix}$. Now the waiting time W^k of the tagged customer is the time until absorption of the Markov process which is given by $W^k = -\alpha(T)^{-1}e$, where $\alpha = (1, 0, \ldots, 0, 0)$ and e is a column vector of ones of appropriate order. Since $T^{-1} = \begin{bmatrix} \frac{-1}{\delta} & \frac{-1}{\delta} & \frac{-1}{\delta} & - & - & \frac{-1}{\delta} \\ 0 & \frac{-1}{\delta} & \frac{-1}{\delta} & - & - & \frac{-1}{\delta} \\ 0 & 0 & \frac{-1}{\delta} & \frac{-1}{\delta} & - & - \\ 0 & 0 & 0 & \frac{-1}{\delta} & \frac{-1}{\delta} & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & - & \frac{-1}{\delta} \end{bmatrix}$, we have $W^k = \frac{k}{\delta}$. Hence the expected waiting time of a general customer is given by

$$E(W_L) = \left(\sum_{k=1}^{\infty} \frac{k}{\delta} x_k \right) e = \frac{\lambda}{\delta^2} \left(\frac{2}{1 - r^Q} + \frac{r^Q}{(1 - r^Q)^2} \right) x_1(Q, 0) + \frac{\lambda}{\delta^2} x_0(0, 0).$$

In a similar manner, we can find the second moment of the waiting time of a customer as

$$E(W_L^2) = \sum_{k=1}^{\infty} W_2^k x_k = \frac{2\lambda}{\delta^3} \left[\frac{(1 - r^Q)^{-3} - 1}{r^Q} \right] x_1(Q, 0) + \frac{2\lambda}{\delta^3} x_0(0, 0),$$

where $W_2^k = 2\alpha(T^{-2})e = \frac{k(k+1)}{\delta^2}$.

5.2. Other Performance Measures.

(1) The mean number of customers in the system is given by

$$L(N) = \sum_{j=1}^{Q} \sum_{i=0}^{\infty} (iQ + j) x_{i+1}(iQ + j, 0).$$

(2) The mean inventory level in the system is given by

$$INV_{mean} = \sum_{j=1}^{S} j x_0(0, j).$$

(3) The mean replenishment rate $ERR = \delta \left(1 - \sum_{j=s+1}^{S} x_0(0,j)\right)$.

(4) The probability that inventory level is zero $P(I=0) = \left(1 - \sum_{j=1}^{S} x_0(0,j)\right)$.

(5) The probability that inventory level is greater than s

$$P(I > s) = \sum_{j=s+1}^{S} x_0(0,j).$$

6. Numerical Illustration

In this section we provide numerical illustration of the system performance measures as underlying parameters vary.

6.1. Effect of reorder level s on various performance measures. In table 1 we see that as s increases the mean inventory level in the system also increases, mean number of customers in the system decreases and expected replenishment rate increases. The increase in mean inventory level is as expected since the orders are placed early. The decrease in mean number of customers in the system $L(N)$ is due to the fact that as the mean inventory level increases more customers leave the system after getting served. The increase in average replenishment rate ERR is obvious since as s increases there will be lesser number of states where order is not placed. This is clear from the formula $ERR = \delta \left(1 - \sum_{j=s+1}^{S} x_0(0,j)\right)$.

Table 2 shows a decrease in the expected waiting time of a customer with an increase in s. As the reorder level s increases, with the maximum inventory level being the same, the time between two order placements decreases. Hence it becomes less probable for a customer to encounter shortage of inventory. This leads to a decrease in average waiting time of the customer. The decrease in waiting time variance with increase in s is also in favour of the system performance.

TABLE 1. Effect of s on the various performance measures

$\lambda = 1 \quad \delta = 2 \quad S = 25$

s	INV_{mean}	$L(N)$	$P(I=0)$	ERR	$P(I>s)$
5	13.52575	0.02635	0.01318	0.1	0.9
6	14.0176	0.0185	0.00925	0.10526	0.89474
7	14.51167	0.01303	0.00651	0.11111	0.88889
8	15.00717	0.00921	0.0046	0.11765	0.88235
9	15.50347	0.00653	0.00327	0.125	0.875
10	16.00004	0.00466	0.00233	0.13333	0.86667
11	16.49637	0.00335	0.00167	0.14286	0.85714

TABLE 2. Effect of re order level s on average waiting time

$\lambda = 1 \quad \delta = 2 \quad S = 25$

s	$E(W_L)$	$E(W_L^2)$	$V(W_L)$
5	0.00879	0.01758	0.017503
6	0.00617	0.01235	0.012312
7	0.00435	0.0087	0.008681
8	0.00307	0.00615	0.006141
9	0.00218	0.00437	0.004365
10	0.00156	0.00312	0.003118
11	0.00112	0.00225	0.002249

6.2. Effect of maximum reorder level S on various performance measures. In table 3 we see that as S increases the mean inventory level increases, mean number of customers in the system decreases and average replenishment rate also decreases. The increase in mean inventory level is due to the fact that the order quantity $Q = S - s$ increases as S increases. The decrease in mean number of customers in the system $L(N)$ is due to the effect of more customers leaving the system after getting served, since expected inventory level increases. The decrease in average replenishment rate ERR is obvious since as S increases there will be more number of states where order is not placed. This is just the reverse to that with increase in s. This is clear from the formula $ERR = \delta\left(1 - \sum\limits_{j=s+1}^{S} x_0(0,j)\right)$.

Table 4 shows a decrease in waiting time of a customer with an increase in S. As maximum inventory level increases, with the re order level being the same, even though the time between two order placements increases, the order quantity $S - s$ increases. Hence more customers will be served with each replenishment. This leads to a decrease in waiting time of the customer. It is also seen that the variance of waiting time also decreases with increase in S.

TABLE 3. Effect of S on the various performance measures

$\lambda = 1 \quad \delta = 2 \quad s = 5$

S	INV_{mean}	$L(N)$	$P(I = 0)$	ERR	$P(I > s)$
11	6.37214	0.10702	0.05289	0.3333	0.66667
12	6.94287	0.08514	0.04226	0.28571	0.71429
13	7.4823	0.07122	0.03544	0.25	0.75
14	8.00485	0.06156	0.03069	0.22222	0.77778
15	8.51776	0.05444	0.02716	0.2	0.8
16	9.02499	0.04893	0.02443	0.18182	0.81818
17	9.5288	0.04452	0.02224	0.16667	0.83333

TABLE 4. Effect of the maximum inventory level S on mean waiting time

$\lambda = 1 \quad \delta = 2 \quad s = 5$

S	$E(W_L)$	$E(W_L^2)$	$V(W_L)$
11	0.03984	0.09237	0.090783
12	0.03056	0.06651	0.065576
13	0.02494	0.05246	0.051838
14	0.0212	0.04375	0.043301
15	0.01854	0.03782	0.037476
16	0.01655	0.03351	0.033236
17	0.01498	0.03021	0.029986

6.3. Effect of replenishment rate δ on various performance measures. Table 5 shows that the expected inventory level in the system INV_{mean} increases, expected number of customers in the system $L(N)$ decreases and the expected replenishment rate ERR remains constant as replenishment rate increases. The increase in INV_{mean} is obvious and decrease in expected number of customers in the system is due to fact that as δ increases, INV_{mean} increases as stated earlier and so more customers leave the system getting served. The expected replenishment rate is independent of replenishment rate for $ERR = \dfrac{\lambda}{(S-s)}$.

Table 6 shows that a decrease in expected waiting time which is expected. The variance of waiting time is also found to decrease as replenishment rate δ increases.

TABLE 5. Effect of δ on the various performance measures

$\lambda = 1 \quad s = 5 \quad S = 11$

δ	INV_{mean}	$L(N)$	$P(I=0)$	ERR	$P(I>s)$
1	6.37214	0.10702	0.05289	0.3333	0.6667
1.2	6.76228	0.05091	0.03025	0.3333	0.72222
1.4	7.02939	0.02659	0.01847	0.3333	0.7619
1.6	7.22392	0.0149	0.01185	0.3333	0.79167
1.8	7.37208	0.00882	0.00789	0.3333	0.81481
2	7.48883	0.00545	0.00543	0.3333	0.83333
2.2	7.58329	0.00349	0.00383	0.3333	0.84848

TABLE 6. Variation of waiting time with replenishment rate δ

$\lambda = 1 \quad s = 5 \quad S = 11$

δ	$E(W_L)$	$E(W_L^2)$	$V(W_L)$
1	0.03984	0.09237	0.090783
1.2	0.01717	0.03118	0.030885
1.4	0.00825	0.01243	0.012362
1.6	0.0043	0.00557	0.005552
1.8	0.00238	0.00271	0.002704
2	0.00139	0.00141	0.001408
2.2	0.00084	0.00078	0.000779

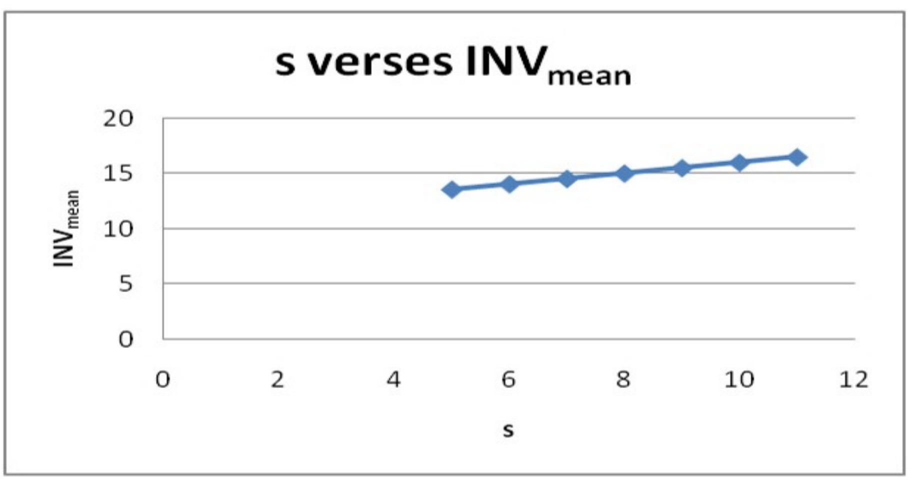

FIGURE 1. Reorder level verses Expected Inventory level

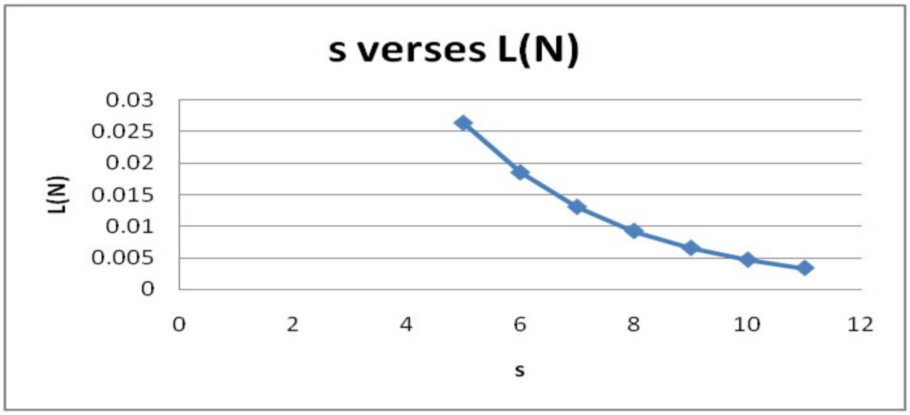

FIGURE 2. Reorder level verses Expected Number of Customers in the System

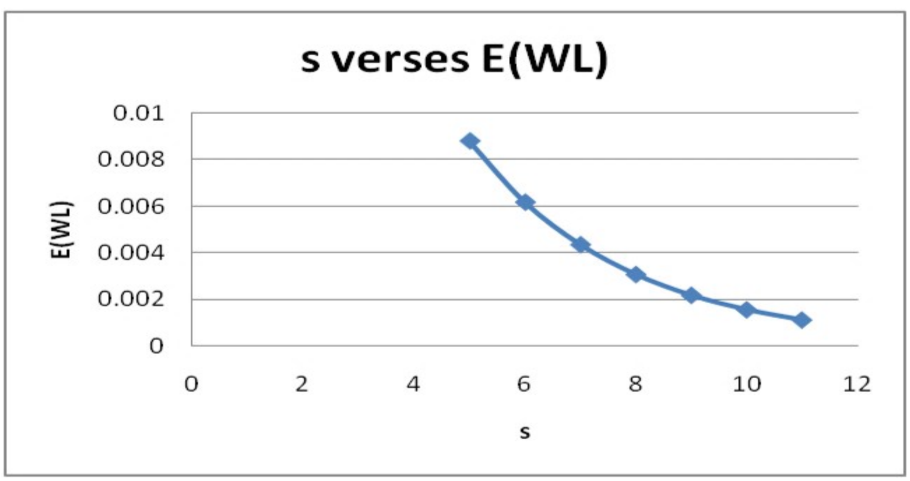

FIGURE 3. Reorder level verses Expected Waiting Time

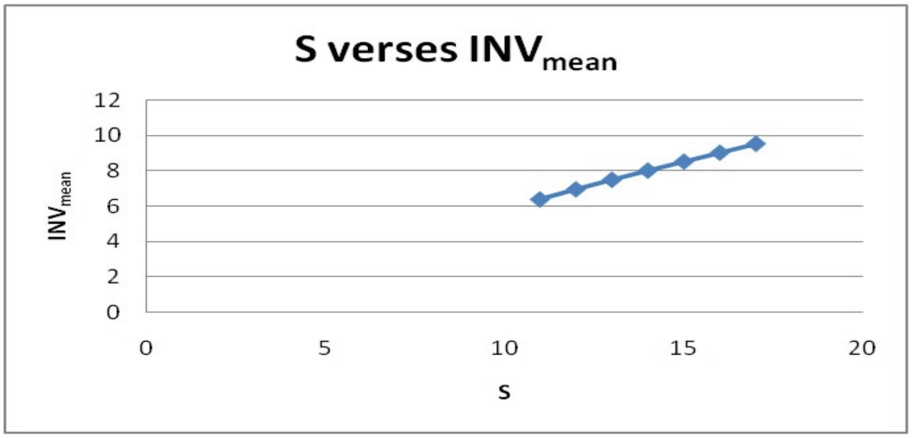

FIGURE 4. Maximum Inventory level verses Expected Inventory Level

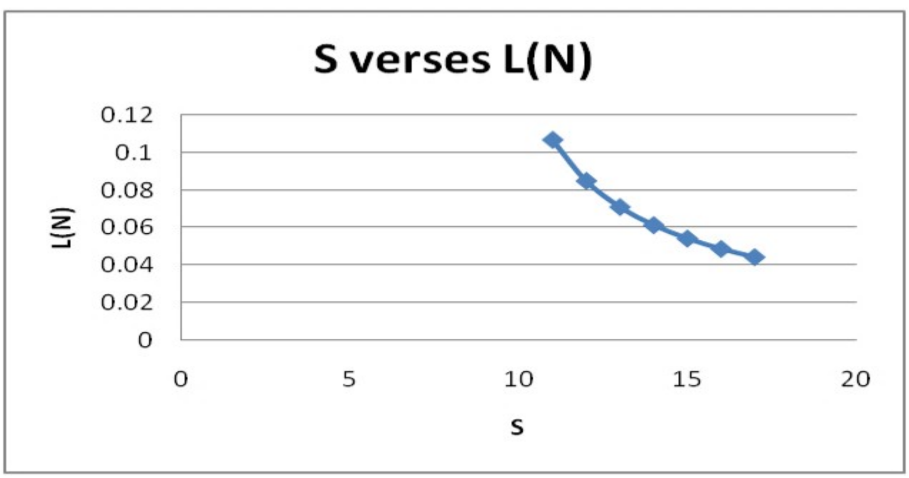

FIGURE 5. Maximum Inventory level verses Expected Number of customers in the System

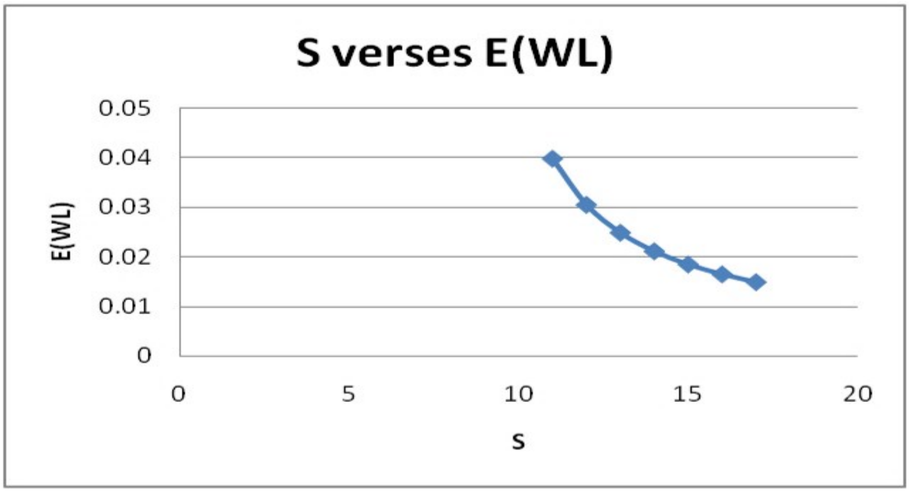

Figure 6. Maximum Inventory level verses Expected waiting time

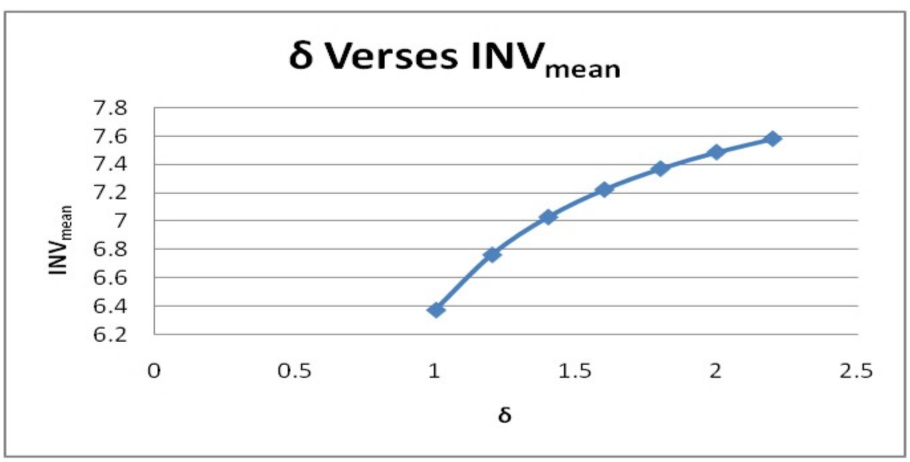

Figure 7. Replenishment Rate verses Expected Inventory Level

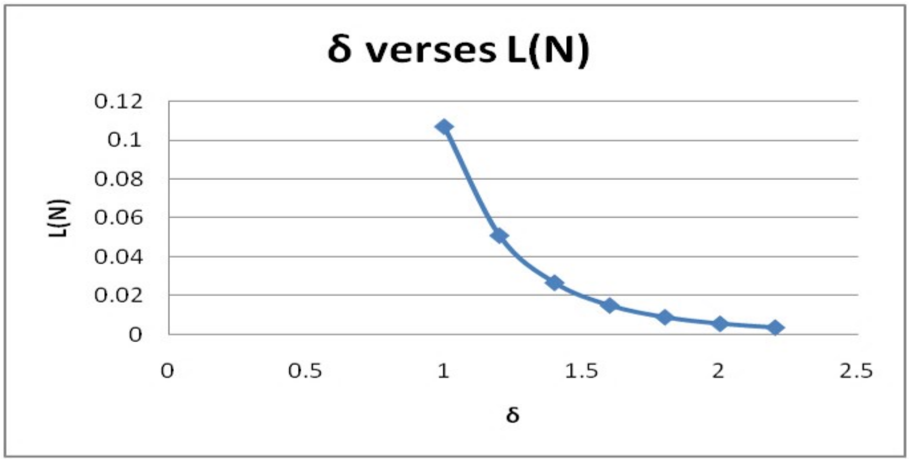

FIGURE 8. Replenishment Rate verses Expected Number of Customer in the System

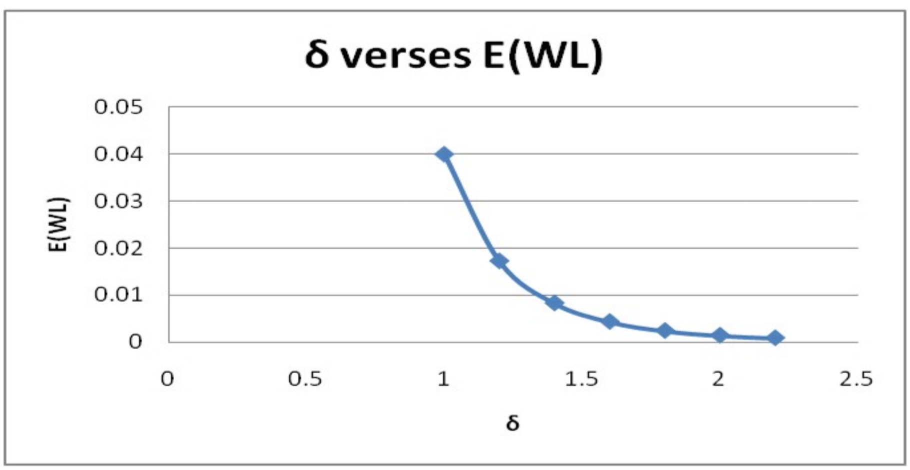

FIGURE 9. Replenishment rate verses Waiting time

Conclusion

We studied a single server queueing model with negligible service time and backlogs. We wish to extend this model to one with positive service time which may have many practical applications.

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