Stochastic Modeling and Applications
Vol.20, No. 2 (December, 2016), 79-90

# STOCHASTIC MORTALITY MODELING AND FORECASTING FOR INDIAN POPULATION WITH APPLICATIONS IN ACTUARIAL SCIENCE

RAJENDRA NANA CHAVHAN AND RAMKRISHNA LAHU SHINDE

Abstract. Since early 1990s a number of stochastic models have been developed to analyze mortality improvements throughout the world. The most popular stochastic mortality model was proposed by Lee and Carter in 1992. Subsequently as extensions of Lee and Carter model, other stochastic mortality models have been proposed in the actuarial literature as: Renshaw and Haberman [16], Currie[8], Cairns-Blake-Dowd [4] and three extensions of Cairns-Blake-Dowd [5], two models by Plat [15] and O'Hare and Li [14]. In this paper, we have quantitatively compared these ten stochastic mortality models on Indian population for explaining improvement in the mortality rates of India. We have applied these ten models on yearly mortality data of Indian population obtained from Sample Registration System, India for the period 1999-2013. By using R-codes these models are fitted for the ages 20-99 years by keeping actuarial application in mind. On the basis of Akaike Information Criteria and Bayesian Information Criteria, we observed that first model given by Plat [15] is the best fitted model for Indian female as well as male mortality. We have forecasted the future mortality rates for Indian population using time series models for time dependent parameters of the best fitted model. These stochastic models are first time applied to Indian mortality data and compared. As an application of results derived, we have presented some actuarial applications. Derived empirical results can be useful for making governmental policies for social security schemes for whole or part of the Indian population.

## 1. Introduction

Life expectancy has increased all over the world during the last few decades. According to the World Health Organization's health statistics 2014, life expectancy at birth has increased by six years between 1990 and 2012 universally. For India, life expectancy at birth was 66 years (both sexes combined) in 2012. India gained eight years in life expectancy at birth (both sexes combined) between 1990 and 2012. Mortality improvement is good for society but it presents demographers and practitioners of actuarial science with new challenges.

Generally, life insurers model future mortality by adjusting mortality rates of some base year using a suitable reduction factor. This may lead to underestimation or overestimation in mortality improvement at different ages. Life insurers face risks

2010 *Mathematics Subject Classification.* Primary 91B70; Secondary 62P05.

*Key words and phrases.* Stochastic Mortality Models, Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Life Expectancy, Social Security Schemes.

associated with both more and less mortality improvement than predicted. In case of a life insurance contract, they incur a loss if mortality improvement falls short of its predicted value, while in case of an annuity they incur a loss if it exceeds its predicted value.

Mortality modeling and forecasting has been a major research area for researchers in the field of demography and actuarial science. There had been plenty of mathematical models suggested by different authors such as Weibull, Gompertz, Makeham, Heligman Pollard etc. But mortality modeling took a large step forward with the publication of Lee and Carter[11]. This is the first mortality model with a stochastic forecast. The Lee-Carter model became very popular due to its simplicity in the parameter estimation. It has been fitting well for almost all developed countries. After Lee-Carter, there have been several extensions of the basic Lee-Carter model by including different factors. Among them, Booth, Maindonald, and Smith [2] considered the multi factor age-period extension of Lee-Carter, Renshaw and Haberman [16] proposed a model with the cohort effect and [4] used the logit transformation in the mortality model.

The main aim of this study is to fit stochastic mortality models and determine best fitting model for forecasting Indian mortality rates. We focused on mortality modeling for ages 20 to 99 years by keeping the actuarial application in mind. We have chosen suitable model for Indian mortality on the basis of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). [5], [13], [1] and [10] have compared the mortality models with the historical data of different countries. [17], [19] and [6] modeled Indian mortality by Lee-Carter model by using life tables for periods 1970-2002 (yearly), 1981-2006 (yearly) and 1901-2011 (decade-wise) respectively. In this paper, we have considered ten stochastic mortality models([11], [16], [8], [4], [5], [15] and [14]). We have applied these ten models on yearly mortality data of Indian population obtained from Sample Registration System, India for the period 1999-2013.

The rest of the paper is organized as follows: In Section 2, we discuss about data sets and their conversion in the required format. Section 3 outlines the mortality models to be considered for the study. Model fitting results are reported in Section 4. Section 5 deals with mortality forecasting with actuarial applications. Concluding remarks are reported in Section 6.

## 2. Notations and Data Description

Following are the some notations used in this paper:

- $x$: Age of a person.
- $t$: Calendar year.
- $D_{xt}$: Number of deaths of persons aged $x$ during year $t$.
- $E_{xt}$: Average number of persons aged $x$ in the population during year $t$.
- $m_{xt}$: Central rate of mortality for age $x$ in year $t$.
- $q_{xt}$: Initial mortality rate. It is the probability that person of age $x$ in year $t$ dies before his/her next birthday.
- $d_x$: Average number of deaths of person aged $x$.
- $L_x$: Average number of persons in the population of age $x$.

Central mortality rate $m_{xt}$ is defined as,

$$m_{xt} = \frac{D_{xt}}{E_{xt}} \tag{2.1}$$

Under constant force of mortality assumption, we have by [9]

$$q_{xt} \cong 1 - exp(m_{xt}) \tag{2.2}$$

We have collected data in the form of abridged life tables of the years 1999 to 2013 for Indian female and male populations separately. These life tables are presented yearly based on previous five year Indian mortality data. That is, life table for the year 1999 is prepared based on the mortality data for the period 1995-99. Such life tables for the years 1999, 2000,, 2013 are based on Indian mortality data for the periods1995-1999, 1996-2000, , 1999-2013 respectively taken from Sample Registration System (SRS), Register General of India, India. These 15 life tables are available for the ages 0, 1-4 and 5-84 (quiquennially) and 85+. For calibration of model, we need complete age wise data of average number of deaths ($d_x$) and average number of persons ($L_x$) of age $x$. For this purpose,we have used interpolative method for initial estimates and Wittaker graduation for estimation of complete sets of age specific mortality rates for each year from age group specific death rates obtained from abridged life tables. This method is thoroughly discussed by [12]. From age specific death rates, we obtained complete form of life tables and the columns of $d_x$ and $L_x$ from complete life tables are used for calibration of models.

## 3. Mortality Models and Estimation of Parameters

In this section, we have briefly introduced different mortality models which are considered under this study, the method of estimation of parameters and constraints on model parameters as suggested by model developers.

**3.1. Mortality models.** In Table 1, we have given the models to be considered and notations used for model thereafter. In Table 1, $\beta_x^{(i)}$ denotes the age related effects, $k_t^{(i)}$ denotes the period related effects, $\gamma_{t-x}^{(i)}$ denotes the cohort related effect, $n_\alpha$ is the number of ages in the data set, $\bar{x}$ is the mean age in the range, $\hat{\sigma}_x^2$ is the mean of the $(x - \bar{x})^2$, $x_c$ is a constant parameter, $(\bar{x} - x)^+ = max(\bar{x} - x, 0)$.

**3.2. Estimation.** According to [3], the counting random variable number of deaths is modeled by Poisson model. That is,

$$D_{xt} \sim Poisson(E_{xt} m_{xt})$$

Parameters are estimated by maximizing the log likelihood function which is given by

$$l(\phi, D, E) = \sum_{x,t} D_{xt} log(E_{xt} m_{xt:\phi}) - E_{xt} m_{xt:\phi} - log(D_{xt}) \tag{3.1}$$

where $\phi$ is the full set of parameters and $m_{xt:\phi}$ shows dependence of $m_{xt}$ on parameters. For the model which involved $q_{xt}$, we have used the relation,

$$m_{xt:\phi} = -log(1 - q_{xt:\phi}) \tag{3.2}$$

Table 1. Different Stochastic Mortality Models

| Model | Notation | Formula |
|---|---|---|
| LC(1992) [11] | M1 | $log(m_{xt}) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(1)}$ |
| RH (2006) [16] | M2 | $log(m_{xt}) = \beta_x^{(1)} + \beta_x^{(2)} k_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$ |
| Currie (2006)[8] | M3 | $log(m_{xt}) = \beta_x^{(1)} + \frac{1}{n_\alpha} k_t^{(2)} + \frac{1}{n_\alpha} \gamma_{t-x}^{(3)}$ |
| CBD(2006) [4] | M4 | $logit(q_{xt}) = log\left(\frac{q_{xt}}{(1-q_{xt})}\right) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)}$ |
| Extension of CBD(2006) [5] | M5 | $logit(q_{xt}) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)} + \gamma_{t-x}^{(3)}$ |
| Extension of CBD(2006) [5] | M6 | $logit(q_{xt}) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)} + ((x-\bar{x})^2 - \hat{\sigma}_x^2) k_t^{(3)} + \gamma_{t-x}^{(4)}$ |
| Extension of CBD(2006) [5] | M7 | $logit(q_{xt}) = k_t^{(1)} + (x-\bar{x}) k_t^{(2)} + (x_c - x) \gamma_{t-x}^{(3)}$ |
| Plat(2009) [15] | M8 | $log(m_{x,t}) = \beta_x^{(1)} + k_t^{(2)} + (\bar{x}-x) k_t^{(3)} + (\bar{x}-x)^+ k_t^{(4)} + \gamma_{t-x}^{(5)}$ |
| Plat(2009) [15] | M9 | $log(m_{x,t}) = \beta_x^{(1)} + k_t^{(2)} + (\bar{x}-x) k_t^{(3)} + \gamma_{t-x}^{(4)}$ |
| OHare and Li (2012) [14] | M10 | $log(m_{x,t}) = \beta_x^{(1)} + k_t^{(2)} + (\bar{x}-x) k_t^{(3)} + \left((\bar{x}-x)^+ + [(\bar{x}-x)^+]^2\right) k_t^{(4)} + \gamma_{t-x}^{(5)}$ |

**3.3. Constraints used in estimation.** Developers of the models suggested some constraints which are imposed during the parameter estimation to avoid identifiability problem. In Table 2, we present the constrains imposed for models considered in Table 1.(where, $C$ is the set of cohort years.)

[5], [15] and [14] discussed these models thoroughly. We have used the free R code of the software package Lifemetrics for calibration which is freely available on www.lifemetrics.com. We have modified R code as per need for different models.

## 4. Model Fitting for Indian Mortality Data

In this section, we have fitted and compared the ten models from Table 1 quantitatively by means of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). We have estimated the parameters of the models by maximizing log likelihood $l(\phi, D, E)$ using R code.

**4.1. Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC).** A Model with maximum log likelihood may be preferred as being the best model but sometimes model with higher number of parameters gives maximum log likelihood and models under this study have different number of parameters. We therefore need to quantify the impact on quality of fit observed by introduction of more parameters.This can be achieved by using model selection based on AIC and BIC. AIC or BIC is the bridge between quality of fit and

Table 2. Constraint(s) imposed in the models

| **Model** | **Constraints** |
|---|---|
| M1 | $\sum_x \beta_x^{(2)} = 1$ and $\sum_t k_t^{(1)} = 0$ |
| M2 | $\sum_x \beta_x^{(2)} = 1$, $\sum_t k_t^{(2)} = 0$, $\sum_{c\in C} \gamma_c^{(3)} = 0$ and $\sum_x \beta_x^{(3)} = 1$ |
| M3 | $\sum_t k_t^{(2)} = 0$ and $\sum_{t,x} \gamma_{t-x}^{(3)} = 0$ |
| M4 | No constraints |
| M5 | $\sum_{c\in C} \gamma_c^{(3)} = 0$ and $\sum_{c\in C} c\gamma_c^{(3)} = 0$ |
| M6 | $\sum_{c\in C} \gamma_c^{(4)} = 0$, $\sum_{c\in C} c\gamma_c^{(4)} = 0$ and $\sum_{c\in C} c^2\gamma_c^{(4)} = 0$ |
| M7 | $\sum_{c\in C} \gamma_c^{(3)} = 0$ |
| M8 | $\sum_{c\in C} \gamma_c^{(5)} = 0$, $\sum_{c\in C} c\gamma_c^{(5)} = 0$ and $\sum_t k_t^{(4)} = 0$ |
| M9 | $\sum_{c\in C} \gamma_c^{(4)} = 0$, and $\sum_{c\in C} c\gamma_c^{(4)} = 0$ |
| M10 | $\sum_{c\in C} \gamma_c^{(5)} = 0$, $\sum_{c\in C} c\gamma_c^{(5)} = 0$ and $\sum_t k_t^{(4)} = 0$ |

parsimony of the model. AIC and BIC can be defined as,

$$AIC = l(\hat{\phi}) - p \text{ and } BIC = l(\hat{\phi}) - \frac{1}{2}klog(N) \qquad (4.1)$$

where $\hat{\phi}$ is the maximum likelihood estimate, $l(\hat{\phi})$ is the maximum log likelihood, $k$ is the effective number of parameters being estimated that is number of parameters less the number of identifiability constraints and $N$ is the number of observations. In this study $N$=(Number of possible ages)×(Number of data years)=80×15=1200. Large value of $N$ produces better results of mortality modeling and forecasting.

Generally, model with higher log likelihood, AIC and BIC values is treated as best model. BIC has been used in the literature for quantitative comparison of models ( [5], [13], [1] and [10]). In Table 3, we have presented the results of our model fitting which includes model wise values of effective number of parameters, maximum log likelihood, AIC and BIC with ranks. To decide the best fitted model we identified first rank models based on AIC and BIC values. Then out of these first rank models we have chosen model with maximum log likelihood as the best fitted model. This is our model selection criteria taking into account values of AIC, BIC and maximum log likelihood.

Using above mentioned model selection criteria, we have identified model M8 as the best fitted stochastic mortality model for Indian female as well as male populations. The model M8 selected by us for Indian mortality for ages 20-99 years also confirmed with the general comments by [15] about model fitting for similar age group(s).

## 5. Forecasting

For forecasting future mortality rates, we have used best fitted model M8.Using SPSS, autoregressive integrated moving average (ARIMA) models are fitted for

Table 3. Maximum log likelihood, AIC and BIC values with ranks in parenthesis

| Model | $p$ | Female Population | | | Male Population | | |
|---|---|---|---|---|---|---|---|
| | | $l(\hat{\phi})$(Rank) | AIC(Rank) | BIC(Rank) | $l(\hat{\phi})$(Rank) | AIC(Rank) | BIC(Rank) |
| M1 | 173 | -5446.7(6) | -5619.7(6) | -6058.6(5) | -5348.3(5) | -5521.3(5) | -5960.2(5) |
| M2 | 345 | -5112.8(3) | -5457.8(4) | -6332.9(7) | -5151.6(1) | -5496.6(4) | -6371.7(9) |
| M3 | 186 | -5288.1(5) | -5474.1(5) | -5945.9(4) | -5351.8(6) | -5537.8(6) | -6009.6(6) |
| M4 | 30 | -17494(10) | -17524(10) | -17600.1(10) | -9445.3(10) | -9475.3(10) | -9551.4(10) |
| M5 | 122 | -7570.8(9) | -7692.8(9) | -8002.3(9) | -5897.7(9) | -6019.7(9) | -6329.1(8) |
| M6 | 136 | -5933.3(8) | -6069.3(8) | -6414.3(8) | -5553.1(8) | -5689.1(8) | -6034.1(7) |
| M7 | 124 | -5670(7) | -5794(7) | -6108.6(6) | -5431.9(7) | -5555.9(7) | -5870.5(1) |
| M8 | 216 | -5098.8(1) | -5314.8(1) | -5862.7(2) | -5158.6(2) | -5374.6(1) | -5922.5(2) |
| M9 | 201 | -5140.9(4) | -5341.9(3) | -5851.7(1) | -5231.8(4) | -5432.8(3) | -5942.7(4) |
| M10 | 216 | -5108.3(2) | -5324.3(2) | -5872.2(3) | -5172.9(3) | -5388.9(2) | -5936.8(3) |

forecasting period effects and cohort effect. We have excluded the first and last four cohorts which are set to zero for modeling and projecting cohort effect for both populations. Table 4 presents the best fitted time series ARIMA models for period effects and cohort effect for model M8.

Table 4. Fitted time series ARIMA models for period effects and cohort effect

| Population | Best fitted stochastic mortality model according to our model selection criteria | ARIMA models fitted for period effects of M8 | | | ARIMA models fitted for period effects of M8 |
|---|---|---|---|---|---|
| | | $k_t^{(2)}$ | $k_t^{(3)}$ | $k_t^{(4)}$ | $\gamma_{t-x}^{(5)}$ |
| Female | M8 | ARIMA(0,1,0) (Common model fitted) | | | ARIMA(3,2,0) |
| Male | M8 | ARIMA(0,1,0) (Common model fitted) | | | ARIMA(1,2,10) |

In Figure 1 and 2, we have shown the estimated (in solid line), forecasted (in dashed line) and 95% confidence interval (CI) of period effects and cohort effects based on model M8 for female and male populations respectively. We have forecasted period effects and cohort effect for one by one period by adding the forecasted value in the previous period and again best fitted ARIMA model was used for forecasting values for next period and so on.

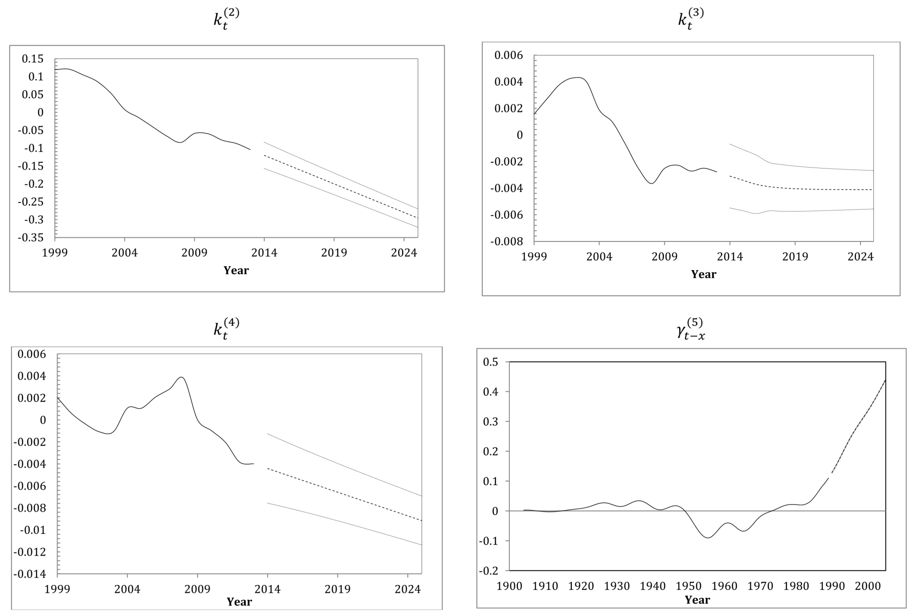


FIGURE 1. Estimated and forecasted period effects ($k_t^{(2)}$, $k_t^{(3)}$ and $k_t^{(4)}$) and cohort effect ($\gamma_{t-x}^{(5)}$) for female population based on model M8. Solid line indicates estimated values, break line indicates forecasted values and dotted line indicates 95% confidence limits.

To forecast the future mortality rates for the year 2014 to 2025, we have used estimated age effect ($\beta_x^{(1)}$), forecasted period effects ($k_t^{(2)}$, $k_t^{(3)}$ and $k_t^{(4)}$) and cohort effect ($\gamma_{t-x}^{(5)}$) for female and male populations based on model M8. We have demonstrated the nature of Indian future mortality by plotting the initial mortality rates, $q_x$ for the year 2015, 2020 and 2025 in Figure 3.

**5.1. Forecasts of Life Expectancy.** In the Table 5, we present the forecasted values of life expectancy with their CI calculated from 95% CI of period effects and cohort effects by using model M8 for both female and male populations separately for the years 2015, 2020 and 2025.

From Table 5 and Figure 4, in the next decade we expect uniform improvement of about 1.5 to 2 years in life expectancy for the ages 20 to 65 years and very small improvement for old ages for both female and male populations.

**5.2. Actuarial Application.** In this section, in order to demonstrate the impact of changes in mortality on the cost of life insurance we have obtained weighted mean and standard deviation (SD) of actuarial present value (APV) that is, net single premium (NSP) of whole life insurance at different ages and age groups.

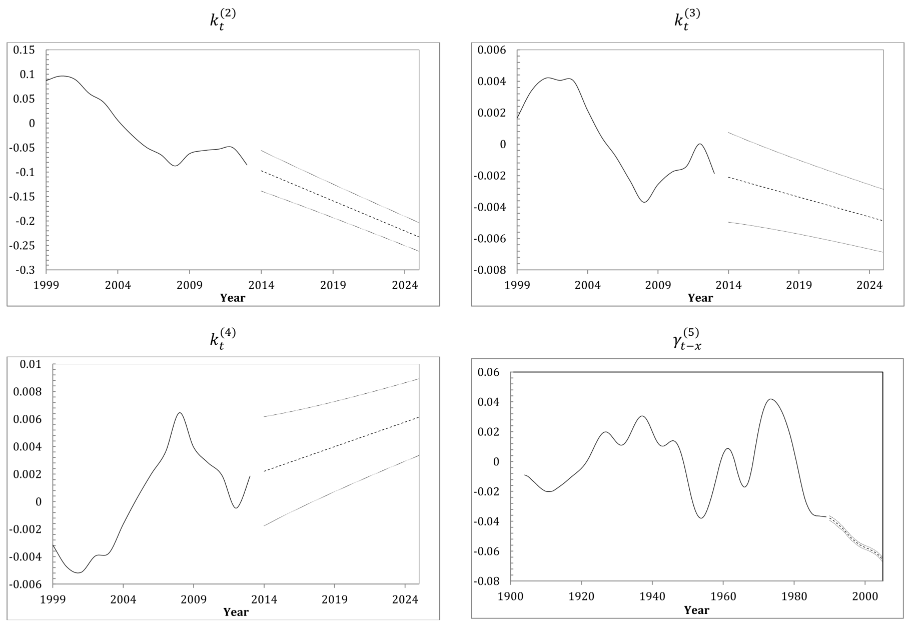


FIGURE 2. Estimated and forecasted period effects ($k_t^{(2)}$, $k_t^{(3)}$ and $k_t^{(4)}$) and cohort effect ($\gamma_{t-x}^{(5)}$) for male population based on model M8. Solid line indicates estimated values, break line indicates forecasted values and dotted line indicates 95% confidence limits.

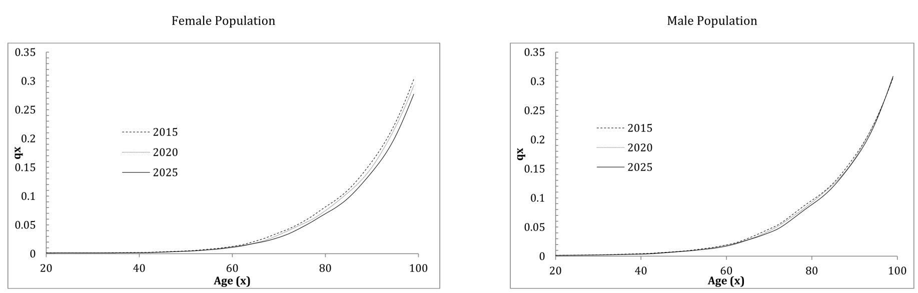


FIGURE 3. Forecasted initial mortality rates ($q_x$) for the year 2015, 2020 and 2025

Weighted mean and SD of NSP for different age groups are calculated by considering the projected population share of age groups as weights. These actuarial quantities are presented in Table 6. In Table 7, we have reported weighted average APV for deferred whole life annuity which starts payments to annuitant after attaining age 60 years. The values reported in Table 6 and Table 7 are based

TABLE 5. Forecasted Life expectancy with 95% confidence limits for some selected ages evaluated using forecasted parameters

| Population | Year | Age | | | |
|---|---|---|---|---|---|
| | | 20 | 30 | 45 | 65 |
| Female | 2015 | 54.79 (54.24,55.27) | 45.58 (45.22,45.91) | 36.33 (36.09,36.54) | 15.43 (15.40,15.45) |
| | 2020 | 55.78 (55.34,56.17) | 46.53 (46.23,46.80) | 37.2 (37.00,37.39) | 16.11 (16.10,16.12) |
| | 2025 | 56.75 (56.39,57.08) | 47.47 (47.23,47.70) | 38.13 (37.97,38.29) | 16.9 (16.90,16.90) |
| Male | 2015 | 50.5 (49.57,51.31) | 41.39 (40.71,42.00) | 32.58 (32.14,32.98) | 13.72 (13.70,13.74) |
| | 2020 | 51.16 (50.42,51.82) | 42.02 (41.47,42.51) | 33.13 (32.78,33.46) | 14.14 (14.12,14.17) |
| | 2025 | 51.81 (51.19,52.37) | 42.64 (42.18,43.05) | 33.7 (33.41,33.98) | 14.52 (14.50,14.54) |

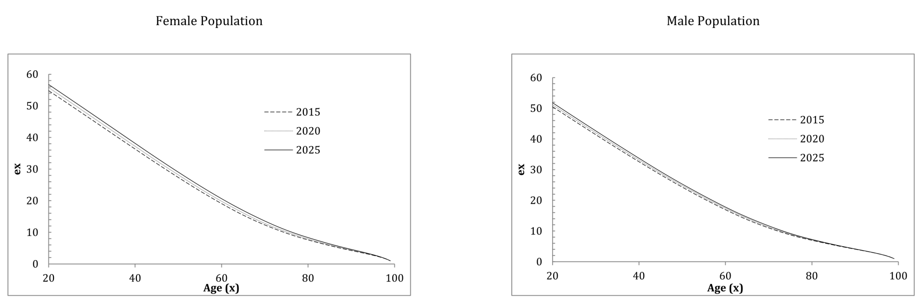


FIGURE 4. Forecasted life expectancy ($e_x$) for the year 2015, 2020 and 2025

on effective rate of interest 7.5% per annum. From Table 6, we observed that forecasted value of NSP for all ages or age groups shows decreasing trend in their average values as well as in the SD from 2015 to 2025. This is impact of change in Indian mortality in terms of future remaining life and its SD at different age groups. Reverse is observed in Table 7, for APV of deferred life annuities payable from age 60 years. That is, mean and SD of APVs of these annuities for all ages or age groups are likely to increase from 2015 to 2020 and 2020 to 2025. This is also due to improvement in life expectancy at different ages and decreased in SD of future remaining life.

TABLE 6. Age group wise weighted mean and SD of forecasted NSP for one person for whole life insurance of unit benefit payable at the end of death year

| Age/ | Female Population | | | Male Population | | |
|---|---|---|---|---|---|---|
| Age Groups | 2015 | 2020 | 2025 | 2015 | 2020 | 2025 |
| 20 | 0.0411<br>(0.0975) | 0.0385<br>(0.094) | 0.0365<br>(0.0916) | 0.0553<br>(0.109) | 0.0531<br>(0.1064) | 0.0509<br>(0.1041) |
| 30 | 0.0635<br>(0.1052) | 0.0596<br>(0.101) | 0.0566<br>(0.0987) | 0.0891<br>(0.1325) | 0.0856<br>(0.1292) | 0.0823<br>(0.1262) |
| 40 | 0.1078<br>(0.1296) | 0.102<br>(0.1256) | 0.0967<br>(0.1219) | 0.1445<br>(0.1629) | 0.14<br>(0.1605) | 0.135<br>(0.1566) |
| 50 | 0.1853<br>(0.1676) | 0.1763<br>(0.1641) | 0.1682<br>(0.162) | 0.23<br>(0.1958) | 0.2232<br>(0.1938) | 0.2177<br>(0.1929) |
| 20-39 | 0.0654<br>(0.1094) | 0.0615<br>(0.1052) | 0.0583<br>(0.1024) | 0.0902<br>(0.1355) | 0.0868<br>(0.1324) | 0.0834<br>(0.1291) |
| 40-59 | 0.1859<br>(0.1758) | 0.1772<br>(0.1722) | 0.169<br>(0.1692) | 0.2277<br>(0.2013) | 0.2215<br>(0.1995) | 0.2155<br>(0.1974) |
| 20-59 | 0.124<br>(0.1575) | 0.1179<br>(0.1532) | 0.1124<br>(0.1497) | 0.1555<br>(0.1833) | 0.151<br>(0.1807) | 0.1466<br>(0.178) |

## 6. Conclusion

We have modeled Indian mortality rates by applying ten stochastic mortality models on recent 15 mortality tables.We have compared first rank models according to AIC and BIC values and then model with maximum log likelihood has been selected as best fitted model. According to our model selection criteria, model M8 got selected as best fitted model for Indian female as well as male mortality. Based on ARIMA models fitted for period and cohort effect parameters of model M8, we have forecasted the age specific death rates and life expectancy at some ages for Indian population for the period 2014 to 2025.Actuarial quantities presented in Table 6 and Table 7 can be used for national level policy making purpose. For example, suppose Government of India wants to buy life annuity for paying pension to all members of population belonging to below poverty line from the present age group 40 to 60 years as a social security scheme,then the cost of governments investment for this purpose equals the mean APV mentioned in Table 7 for age group 40-59 multiplied by amount of annual pension and number of persons from the class of below poverty line in India.This one time investment amount can also be obtained as certain percentile (say 95th percentile) of distribution of total present value of deferred life annuity for the group of people to be insured for getting government sponsored pension.Under the law of large numbers, the probability distribution of total present value of deferred life annuity can be considered approximately normally distributed with mean equals to APV

Table 7. Age group wise weighted mean and SD of forecasted APV per person for deferred whole life annuity of unit amount payable yearly starting from age 60 years at the beginning of each year

| Age/ | Female Population | | | Male Population | | |
|---|---|---|---|---|---|---|
| Age Groups | 2015 | 2020 | 2025 | 2015 | 2020 | 2025 |
| 20 | 0.4785 (0.0586) | 0.4935 (0.0572) | 0.5072 (0.0563) | 0.4043 (0.0693) | 0.415 (0.0692) | 0.4253 (0.0687) |
| 30 | 1.0019 (0.237) | 1.0321 (0.231) | 1.0599 (0.2272) | 0.8496 (0.2862) | 0.8712 (0.2855) | 0.8921 (0.2833) |
| 40 | 2.1029 (0.9454) | 2.1621 (0.9222) | 2.2184 (0.9048) | 1.8091 (1.1511) | 1.8504 (1.1483) | 1.8915 (1.1379) |
| 50 | 4.476 (3.5124) | 4.5899 (3.4209) | 4.6969 (3.3632) | 3.9668 (4.2569) | 4.0488 (4.2267) | 4.1196 (4.1968) |
| 20-39 | 1.0473 (0.2945) | 1.079 (0.2873) | 1.1085 (0.2825) | 0.8881 (0.3546) | 0.9106 (0.3539) | 0.9326 (0.3512) |
| 40-59 | 4.6387 (3.7282) | 4.757 (3.6447) | 4.8726 (3.582) | 4.1423 (4.4402) | 4.2245 (4.4232) | 4.3018 (4.3782) |
| 20-59 | 2.7176 (1.8915) | 2.7957 (1.8543) | 2.8717 (1.8281) | 2.3794 (2.2268) | 2.433 (2.2233) | 2.4838 (2.2053) |

from Table 7 multiplied by number of people to be insured for getting pension and standard deviation obtained by using corresponding value of SD reported in Table 7. Similarly, actuarial quantities reported in Table 6 can be used to obtain the total cost of NSP for certain group of people from certain age group. Such results about estimation of total cost of insurance for whole or part of the population are demonstrated by [6]. In general, actuarial quantities from Table 6 and Table 7 can also be used for finding total allocation of amount by government for a specified social security scheme for old age such as incentives payable to senior citizens for regular health care.

**Acknowledgment.** Authors are thankful to the reviewer and editor for their valuable suggestions and comments which have substantially improved the presentation of the paper. First author would like to thank the Department of Science and Technology (INSPIRE Division), New Delhi, India for financial assistance through INSPIRE fellowship for this research.

## References

1. Biffi, P. and Clemente, G.P.: Selecting stochastic mortality models for the Italian population, *Decisions in Economics and Finance* **37(2)** (2014) 255–286.
2. Booth, H., Maindonald, J. and Smith, L.: Applying Lee-Carter under conditions of variable mortality decline, *Population studies* **56(3)** (2002) 325–336.

3. Brillinger, D.R.: The natural variability of vital rates and associated statistics (with discussion), *Biometrics* **42** (1986) 693–734.
4. Cairns, A.J., Blake, D. and Dowd, K. 2006: A Two Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration, *Journal of Risk and Insurance* **73(4)** (2006) 687–718.
5. Cairns, A.J., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A. and Balevich, I: A quantitative comparison of stochastic mortality models using data from England and Wales and the United States, *North American Actuarial Journal* **13(1)** (2009) 1–35.
6. Chavhan, R.N. and Shinde, R.L. 2013: Human mortality pattern in India and China and total cost of life insurance for whole population and subpopulations, *Journal of the Indian Statistical Association* **51(2)** (2013) 399–418.
7. Chavhan, R.N. and Shinde, R.L.: Modeling and Forecasting Mortality Using the Lee-Carter Model for Indian Population Based on Decade-wise Data, *Sri Lankan Journal of Applied Statistics* **17(1)** (2016) 51–68.
8. Currie, I.D.: Smoothing and forecasting mortality rates with P-splines, (2006).
9. David C.M. Dickson, Mary R. Hardy and Howard R. Waters: *Actuarial Mathematics for Life Contingent Risks*, Cambridge University Press, UK (2009).
10. Gogola, J.: Stochastic mortality models:Application to CR mortality data. *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, Cira Perna and Marilena Sibillo,Springer International Publishing, (2014) 113–116.
11. Lee, R.D. and Carter, L.R.: Modeling and forecasting US mortality, *Journal of the American statistical association* **87(419)** (1992) 659–671.
12. Li, S.H. and Chan, W.S.: Estimation of complete period life tables for Singaporeans, *Journal of Actuarial Practice* **11** (2004) 129–146.
13. Lovsz, E.: Analysis of Finnish and Swedish mortality data with stochastic mortality models, *European Actuarial Journal* **1(2)** (2011) 259–289.
14. OHare, C. and Li, Y.: Explaining young mortality, *Insurance: Mathematics and Economics* **50(1)** (2012) 12–25.
15. Plat, R.: On stochastic mortality modeling *Insurance: Mathematics and Economics* **45(3)** (2009) 393–404.
16. Renshaw, A.E. and Haberman, S.: A cohort-based extension to the LeeCarter model for mortality reduction factors, *Insurance: Mathematics and Economics* **38(3)** (2006) 556–570.
17. Singh, Abhishek, and Faujdar Ram: Forecasting mortality in India. *Population, Health and Development in India: Changing Perspective*, TK Roy, M. Guruswamy, and Arokiaswamy, Rawat Publication Mumbai, India, (2004) 73–100.
18. World Health Organization. 2014 World Health Statistics 2014. Geneva: WHO.
19. Yadav, A., Yadav, S. and Kesarwani, R.: Decelerating mortality rates in older ages and its prospects through Lee-Carter approach, *PloS one* **7(12)** (2012) p.e50941.

Department of Statistics, Kishinchand Chellaram College, Churchgate, Mumbai-400 020, India
*E-mail address*: chavhanrn@gmail.com

Department of Statistics and Actuarial Science, School of Mathematical Sciences,North Maharashtra University, Jalgaon-425 001, India
*E-mail address*: rlshinde@nmu.ac.in