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Periodic Oscillation for Cohen-Grossberg-type Neural Networks with Neutral Time-varying Delays

Chuanzhi Bai*

Abstract: In this paper, a Cohen-Grossberg-type model of neural networks with neutral time-varying delays is investigated by using the continuation theorem of Mawhin's coincidence degree theory and some analysis techniques. Without assuming the continuous differentiability of time-varying delays, sufficient conditions for the existence of the periodic solutions are given. The result of this paper is new and extent previously known result.

Keywords: Cohen-Grossberg type neural networks; Neutral delay; Coincidence degree theory; Periodic solution

1. INTRODUCTION

In recent years, the cellular neural networks have been extensively studied and applied in many different fields such as signal and image processing, pattern recognition and optimization. In implementation of networks, time delays are inevitably encountered because of the finite switching speed of amplifiers. Thus, it is very important to investigate the dynamics of delay neural networks. In theory and application, the existence of periodic solutions of neural networks model is of great interest [1-9]. Very recently, Gui, Ge and Yang [10] investigated the existence of periodic solutions of Hopfield networks model with neutral delay by means of an abstract continuous theorem of k-set contractive operator and some analysis techniques. But they do require the time-varying delays $\tau_{ij}(t)$ and $\sigma_{ij}(t)$ are continuously differentiable. Furthermore, the criterion for the existence of periodic solutions of Hopfield neural networks model in [10] depends on the $\dot{\tau}_{ij}$ and $\dot{\sigma}_{ij}$.

In this paper, we consider the following Cohen-Grossberg type neural networks with neutral time-varying delays:

$$\dot{x}_i(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) - \sum_{j=1}^{n} b_{ij}(t) g_j(\dot{x}_j(t-\sigma_{ij}(t))) - J_i(t)\right], \tag{1}$$

where $i = 1, 2, \ldots, n$, $x_i(t)$ denotes the potential (or voltage) of cell i at time t; $a_i(x_i(t)))$ represent an amplification function; $b_i(x_i(t))$ be include a constant term indicating a fixed input the network; $a_{ij}(t)$ and $b_{ij}(t)$ represent the delayed strengths of connectivity and neutral delayed strengths of connectivity between cell i and j at time t, respectively; f_j and g_j are the activation functions in system (1); $J_i(t)$ is an external input on the ith unit at time t, in which $J_i : R \to R$, $i = 1, \ldots, n$, are continuous periodic functions with period ω; $\tau_{ij}(t)$, $\sigma_{ij}(t) \geq 0$ correspond to the transmission delays.

Obviously, system (1) is a generalization of the following model for an Hopfield neural networks system with neutral delays in [10]:

$$\dot{x}_i(t) = -b_i x_i + \sum_{j=1}^{n} a_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) + \sum_{j=1}^{n} b_{ij}(t) g_j(\dot{x}_j(t-\sigma_{ij}(t))) + J_i(t). \tag{2}$$

* Department of Mathematics, Huaiyin Teachers College, Huaian, Jiangsu 223300, P R China, *E-mail: czbai8@sohu.com*

In this paper, by using the continuation theorem of coincidence degree theory and some analysis technique, we obtain some new sufficient conditions for the existence of the periodic solutions of system (1). The conditions imposed on the time-varying delays $\tau_{ij}(t)$ and $\sigma_{ij}(t)$ do not need the assumptions of continuously differentiable. Our works in this paper are new and extend previous result in [10].

2. PRELIMINARIES

In this section, we state some notations, definitions and some Lemmas.

Let $A = (a_{ij})_{n\times n}$ be a real $n \times n$ matrix, $A > 0$ ($A \geq 0$) denotes each element a_{ij} is positive (nonnegative, respectively). Let $x = (x_1, x_2, ..., x_n)^T \in R^n$ be a vector, $x > 0$ ($x \geq 0$) denotes each element x_i is positive (nonnegative, respectively). For matrices or vectors A and B, $A \geq B$ ($A > B$) means that $A - B \geq (A - B > 0)$.

Definition 2.1 [11]: Matrix $A = (a_{ij})_{n\times n}$ is said to be a nonsingular M-matrix, if

(i) $a_{ii} > 0$, $i = 1, 2, ..., n$;

(ii) $a_{ij} \leq 0$, for $i \neq, j$, $i, j = 1, 2, ..., n$;

(iii) $A^{-1} \geq 0$.

Let X and Y be normed vector spaces, Let $L : \mathrm{dom}L \subset X \to Y$ be a linear mapping, L will be called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L = \mathrm{codim}\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Y. If L is a Fredholm mapping of index zero, there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$. It follows that mapping $L|_{\mathrm{dom}L\cap\mathrm{Ker}P} : (I - P)X \to \mathrm{Im}L$ is invertible. We denote the inverse of the mapping by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I- Q)N : \bar{\Omega}\to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Now, we introduce Mawhin's continuation theorem [12, p.40] as follows.

Lemma 2.1: Let X and Y be two Banach spaces, $L : \mathrm{dom}L \to Y$ be a Fredholm operator with index zero. Assume that Ω is a open bounded set in X, and N is L-compact on $\bar{\Omega}$. If all the following conditions hold:

(a) for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \mathrm{Dom}L$, $Lx \neq Nx$;

(b) $QNx \neq 0$ for each $x \in \partial\Omega \cap \mathrm{Ker}L$, and $\deg(JNQ, \Omega \cap \mathrm{Ker}L, 0) \neq 0$,

where J is an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$. Then equation $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap \mathrm{Dom}L$.

The following lemmas will be useful to prove our main result in Section 3.

Lemma 2.2 [11]: Assume that A is a nonsingular M-matrix and $Aw \leq d$, then $w \leq A^{-1}d$.

Lemma 2.3 [13]: Let $A = (a_{ij})$ with $a_{ij} \leq 0$, $i, j = 1, 2, ..., n$, $i \neq j$. Then the following statements are equivalent.

(1) A is an M-matrix.

(2) There exists a row vector $\eta = (\eta_1, \eta_2, ..., \eta_n) > (0, 0, ..., 0)$ such that $\eta A > 0$.

(3) There exists a column vector $\xi = (\xi_1, \xi_2, ..., \xi_n)^T > (0, 0, ..., 0)^T$ such that $A\xi > 0$.

Throughout this paper, we assume that

(H_1) a_{ij}, b_{ij}, $J_j \in C(\mathbb{R}, \mathbb{R})$, τ_{ij}, $\sigma_{ij} \in C(\mathbb{R}, \mathbb{R}^+)$ ($\mathbb{R}^+ = [0, \infty)$) are periodic functions with a common period $\omega(> 0)$, $i, j = 1, ..., n$.

(H_2) $a_i \in C(\mathbb{R}, \mathbb{R})$. Furthermore, there exist positive constants a_{i*} and a_i^* such that

$$a_{i*} \le a_i(u) \le a_i^*, \ \forall u \in \mathbb{R}, i = 1, \dots, n.$$

(H_3) $b_i \in C(\mathbb{R}, \mathbb{R})$. Moreover, there exist positive constants b_{i*} and b_i^* such that

$$b_{i*}u^2 \le ub_i(u) \le b_i^* u^2, \ \forall u \in \mathbb{R}, i = 1, \dots, n.$$

(H_4) $f_j, g_j \in C(\mathbb{R}, \mathbb{R})$ are Lipschitzian with Lipschitz constants L_j and l_i respectively, i.e.,

$$|f_j(x) - f_j(y)| \le L_j|x - y|, \ |g_j(x) - g_j(y)| \le l_j|x - y|, \ \forall x, y \in \mathbb{R}, j = 1, \dots, n.$$

3. EXISTENCE OF PERIODIC SOLUTION

In this section, we will use the continuation theorem of coincidence degree theory to obtain the existence of an ω-periodic solution to system (1).

For convenience, we introduce the following notations:

$$a_{ij}^+ := \max_{t\in[0,\omega]} |a_{ij}(t)|, \quad b_{ij}^+ := \max_{t\in[0,\omega]} |b_{ij}(t)|, \quad J_i^+ := \max_{t\in[0,\omega]} |J_i(t)|, \quad i, j = 1,2,\dots,n.$$

Theorem 3.1: Let (H_1)-(H_4) hold. Suppose that C and $A - B(C^{-1}D)$ are two nonsingular M-matrix, then system (1) has at least one ω-periodic solution, where

$$A = (\bar{a}_{ij})_{n\times n}, \ \bar{a}_{ij} = b_{i*}\delta_{ij} - a_{ij}^+ L, \ B = (\bar{b}_{ij})_{n\times n}, \ \bar{b}_{ij} = b_{ij}^+ l_j,$$
$$C = (\bar{c}_{ij})_{n\times n}, \ \bar{c}_{ij} = \delta_{ij} - a_i^* b_{ij}^+ l_j, \ D = (\bar{d}_{ij})_{n\times n}, \ \bar{d}_{ij} = a_i^* (b_i^* \delta_{ij} + a_{ij}^+ L_j), \ \delta_{ij} = \begin{cases} 1, & \text{for} \quad i = j, \\ 0, & \text{for} \quad i \ne j. \end{cases}$$

Proof: Take

$$C_\omega = \{x(t) = (x_1(t), \dots, x_n(t))^T \in C(R, R^n) : x_i(t + \omega) \equiv x_i(t), i = 1, \dots, n\},$$
$$C_\omega^1 = \{x(t) = (x_1(t), \dots, x_n(t)) \in C^1(R, R^n) : x_i(t + \omega) \equiv x_i(t), i = 1, \dots, n\}.$$

Then C_ω is a Banach space with the norm $\|x\|_0 = \max_{1\le i\le n}\{|x_i|_0\}$, $|x_i|_0 = \max_{t\in[0,\omega]} |x_i(t)|$, C_ω^1 is also a Banach space with the norm $\|x\| = \max\{\|x\|_0, \|x'\|_0\}$.

For each $x = (x_1, \dots, x_n) \in C_\omega^1$, $L : C_\omega^1 \to C_\omega$ and $N : C_\omega^1 \to C_\omega$ defined by

$$(Lx)(t) = \tfrac{dx}{dt} = (\dot{x}_1(t), \dots, \dot{x}_n(t))^T, \text{ and}$$

$$(Nx)(t) = \begin{bmatrix} -a_1(x_1(t))\Big(b_1(x_1(t)) - \sum_{j=1}^n a_{1j}(t) f_j(x_j(t-\tau_{1j}(t))) - \sum_{j=1}^n b_{1j}(t) g_j(\dot{x}_j(t-\sigma_{1j}(t))) - J_1(t)\Big) \\ \vdots \\ -a_n(x_n(t))\Big(b_n(x_n(t)) - \sum_{j=1}^n a_{nj}(t) f_j(x_j(t-\tau_{nj}(t))) - \sum_{j=1}^n b_{nj}(t) g_j(\dot{x}_j(t-\sigma_{nj}(t))) - J_n(t)\Big) \end{bmatrix}.$$

It is easy to see that $\mathrm{Ker}L = \mathbb{R}^n$, $\mathrm{Im}L = \{(x_1(t), \dots, x_n(t))^T \in C_\omega : \int_0^\omega x_i(s)ds = 0, i = 1, \dots, n\}$ is closed in C_ω, and L is a Fredholm mapping of index zero. Define two projectors

$$Px = \frac{1}{\omega}\int_0^\omega x(t)\,dt = \left(\frac{1}{\omega}\int_0^\omega x_1(t)dt, \dots, \frac{1}{\omega}\int_0^\omega x_n(t)\,dt\right)^T, \quad x(t) = (x_1(t), \dots, x_n(t))^T \in C_\omega^1,$$

$$Qy = \frac{1}{\omega}\int_0^{\omega} y(t)dt = \left(\frac{1}{\omega}\int_0^{\omega} y_1(t)dt, ..., \frac{1}{\omega}\int_0^{\omega} y_n(t)\, dt\right)^T, y(t) = (y_1(t), ..., y_n(t))^T \in C_{\omega}.$$

Obviously, P, Q are continuous and satisfy

$$\mathrm{Im}P = \mathrm{Ker}L, \ \mathrm{Ker}Q = \mathrm{Im}L.$$

Similar to [4], we can define the generalized inverse $K_p : \mathrm{Im}L \to \mathrm{Ker}P \cap \mathrm{dom}L$ of L and show that N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset C_{\omega}^1$.

Now we are in a position to show that there exists an appropriate open, bounded subset Ω, which satisfies all the requirements given in the continuation theorem. According to the operator equation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$\dot{x}_i(t) = -\lambda a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) - \sum_{j=1}^{n} b_{ij}(t) g_j(\dot{x}_j(t-\sigma_{ij}(t))) - J_i(t)\right], \tag{3}$$

where $i = 1, ..., n$. Suppose that $x(t) = (x_1(t), ..., x_n(t))^T \in C_{\omega}^1$ is a solution of system (3) for some $\in (0, 1)$. Hence, there exist $\xi_i \in [0, \omega]$ $(i = 1, ..., n)$ such that $|x_i(\xi_i)| = \max_{t\in[0, \omega]} |x_i(t)| = |x_i|_0$. Thus, $x_i'(\xi_i) = 0$ for $i = 1, ..., n$. By (3), we have

$$b_i(x_i(\xi_i)) = \sum_{j=1}^{n} a_{ij}(\xi_i) f_j(x_j(\xi_i - \tau_{ij}(\xi_i))) + \sum_{j=1}^{n} b_{ij}(\xi_i) g_j(\dot{x}_j(\xi_i - \sigma_{ij}(\xi_i))) + J_i(\xi_i), \quad i = 1, ..., n. \tag{4}$$

In view of (H_3), (H_4) and (4), we have

$$\begin{aligned} b_{i*}|x_i|_0 = b_{i*}\,|\,x_i(\xi_i)\,| &\le |\,b_i(x_i(\xi_i))\,| \le \sum_{j=1}^{n} |\,a_{ij}(\xi_i)\,|\,|\,f_j(x_j(\xi_i - \tau_{ij}(\xi_i)))\,| \\ &+ \sum_{j=1}^{n} |\,b_{ij}(\xi_i)\,|\,|\,g_j(\dot{x}_j(\xi_i - \sigma_{ij}(\xi_i)))\,| + |\,J_i(\xi_i)\,| \\ &\le \sum_{j=1}^{n} a_{ij}^{+}(L_j\,|\,x_j(\xi_i - \tau_{ij}(\xi_i))\,| + |\,f_j(0)\,|) + \sum_{j=1}^{n} b_{ij}^{+}(l_j\,|\,\dot{x}_j(\xi_i - \sigma_{ij}(\xi_i))\,| + |\,g_j(0)\,|) + J_i^{+} \\ &\le \sum_{j=1}^{n} (a_{ij}^{+} L_j |x_j|_0 + b_{ij}^{+} l_j |\dot{x}_j|_0) + \sum_{j=1}^{n} (a_{ij}^{+}\,|\,f_j(0)\,| + b_{ij}^{+}\,|\,g_j(0)\,|) + J_i^{+}, \quad i = 1, ..., n, \end{aligned}$$

which implies that

$$\sum_{j=1}^{n} (b_{i*}\delta_{ij} - a_{ij}^{+} L_j)\,|\,x_j\,|_0 \le \sum_{j=1}^{n} b_{ij}^{+} l_j |\dot{x}_j|_0 + \sum_{j=1}^{n} (a_{ij}^{+}\,|\,f_j(0)\,| + b_{ij}^{+}\,|\,g_j(0)\,|) + J_i^{+}, \quad i = 1, ..., n. \tag{5}$$

The formulas (5) may be rewritten in the form

$$AX \le BY + h, \tag{6}$$

where $X = (|x_1|_0, \ldots, |x_n|_0)^T$, $Y = (|\dot{x}_1|_0, \ldots, |\dot{x}_n|_0)^T$, $h = (h_i)_{n\times 1}$, $h_i = \sum_{j=1}^{n}(a_{ij}^{+}\left|f_j(0)\right| + b_{ij}^{+}\left|g_j(0)\right|) + J_i^{+}$.

Let $\eta_i \in [0, \omega]$ $(i = 1, \ldots, n)$ such that $|\dot{x}_i \eta_i| = \max_{t\in[0,\omega]} |\dot{x}_i(t)| = |\dot{x}_i|_0$. From (3), ($H_2$), ($H_3$) and ($H_4$), we get

$$\left|\dot{x}_i\right|_0 = \left|\dot{x}_i(\eta_i)\right| \le a_i(x_i(\eta_i))\left[\left|b_i(x_i(\eta_i))\right| + \sum_{j=1}^{n}\left|a_{ij}(\eta_i)\right|\left|f_j(x_j(\eta_i - \tau_{ij}(\eta_i)))\right|\right.$$

$$\left. + \sum_{j=1}^{n}\left|b_{ij}(\eta_i)\right|\left|g_j(\dot{x}_j(\eta_i - \sigma_{ij}(\eta_i)))\right| + \left|J_i(\eta_i)\right|\right]$$

$$\le a_i^{*}\left[b_i^{*}\left|x_i(\eta_i)\right| + \sum_{j=1}^{n}a_{ij}^{+}(L_j\left|x_j(\eta_i - \tau_{ij}(\eta_i))\right| + \left|f_j(0)\right|) + \sum_{j=1}^{n}b_{ij}^{+}(l_j\left|\dot{x}_j(\eta_i - \sigma_{ij}(\eta_i))\right| + \left|g_j(0)\right|) + J_i^{+}\right]$$

$$\le a_i^{*}\left[b_i^{*}\left|x_i\right|_0 + \sum_{j=1}^{n}(a_{ij}^{+}L_j\left|x_j\right|_0 + b_{ij}^{+}l_j\left|\dot{x}_j\right|_0) + \sum_{j=1}^{n}(a_{ij}^{+}\left|f_j(0)\right| + b_{ij}^{+}\left|g_j(0)\right|) + J_i^{+}\right], \quad i = 1,\ldots,n,$$

that is

$$\sum_{j=1}^{n}(\delta_{ij} - a_i^{*}b_{ij}^{+}l_j)\left|\dot{x}_j\right|_0 \le a_i^{*}\sum_{j=1}^{n}(b_i^{*}\delta_{ij} + a_{ij}^{+}L_j)\left|x_j\right|_0 + a_i^{*}\sum_{j=1}^{n}(a_{ij}^{+}\left|f_j(0)\right| + b_{ij}^{+}\left|g_j(0)\right|) + a_i^{*}J_i^{+}, \tag{7}$$

where $i = 1, \ldots, n$. It is easy to know that (7) may be rewritten as

$$CY \le DX + k. \tag{8}$$

where $k = (k_i)_{n\times 1}$, $k_i = a_i^{*}\sum_{j=1}^{n}(a_{ij}^{+}\left|f_j(0)\right| + b_{ij}^{+}\left|g_j(0)\right|) + a_i^{*}J_i^{+}$. Since C is a nonsingular M-matrix, we have by (8) and Lemma 2.2 that

$$Y \le C^{-1}DX + C^{-1}k. \tag{9}$$

Substituting (9) into (6), we get

$$(A - B(C^{-1}D))X \le BC^{-1}k + h := w = (w_1, w_2, \ldots, w_n)^T. \tag{10}$$

Since $A - B(C^{-1}D)$ is a nonsingular M-matrix, we have by (10) and Lemma 2.2 that

$$X \le (A - B(C^{-1}D))^{-1}w := (R_1, R_2, \ldots, R_n)^T. \tag{11}$$

Substituting (11) into (9), we obtain

$$Y \le C^{-1}D(R_1, \ldots, R_n)^T + C^{-1}k := (r_1, r_2, \ldots, r_n)^T. \tag{12}$$

Since $A - B(C^{-1}D)$ is an M-matrix, we have from Lemma 2.3 that there exists a vector $\varsigma = (\varsigma_1, \varsigma_2, \ldots, \varsigma_n)^T > (0, 0, \ldots, 0)^T$ such that

$$(A - B(C^{-1}D))\varsigma > (0, 0, \ldots, 0)^T,$$

which implies that we can choose a constant $p > 1$ such that

$$p(A - B(C^{-1}D))\varsigma > w, \text{ and } p\varsigma_i > R_i,\ i = 1, 2, \ldots, n. \tag{13}$$

Combine (11) with (8), we get

$$CY \le D(R_1, \ldots, R_n)^T + k := v = (v_1, v_2, \ldots, v_n)^T. \tag{14}$$

Noticing that C is an M-matrix, we have from Lemma 2.3 that there exists a vector $\zeta = (\zeta_1, \ldots, \zeta_n)^T > (0, 0, \ldots, 0)^T$ such that

$$C\zeta > (0, 0, \ldots, 0)^T,$$

which implies that we can choose a constant $q > 1$ such that

$$qC\zeta > v, \text{ and } q\zeta_i > r_i,\ i = 1, 2, \ldots, n. \tag{15}$$

Set

$$\overline{\varsigma} = (\overline{\varsigma}_1, \overline{\varsigma}_2, \ldots, \overline{\varsigma}_n)^T := p\varsigma,\ \ \overline{\zeta} = (\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_n)^T := q\zeta.$$

Then, by (13) and (15), we have

$$\overline{\varsigma}_i > R_i,\ (A - B(C^{-1}D))\,\overline{\varsigma} > w,\ \overline{\varsigma}_i > r_i, \text{ and } C\,\overline{\zeta} > v,\ i = 1, 2, \ldots n. \tag{16}$$

Now we take

$$\Omega = \{x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in C^1_\omega : |x_i|_0 < \overline{\varsigma}_i,\ |\dot{x}_i|_0 < \overline{\zeta}_i,\ i = 1, 2, \ldots, n\}.$$

Obviously, the condition (a) of Lemma 2.1 is satisfied. If $x \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap \mathbb{R}^n$, then $x(t)$ is a constant vector in $\mathbb{R}^n$, and there exists some $i \in \{1, 2, \ldots, n\}$ such that $|x_i| = \overline{\varsigma}_i$. It follows that

$$(QNx)_i = -\frac{a_i(x_i)}{\omega}\int_0^\omega \left[b_i(x_i) - \sum_{j=1}^n a_{ij}(t) f_j(x_j) - \sum_{j=1}^n b_{ij}(t) g_j(0) - J_i(t) \right] dt. \tag{17}$$

We claim that

$$|(QNx)_i| > 0. \tag{18}$$

In fact, if $|(QNx)_i| = 0$, i.e.,

$$\int_0^\omega \left[b_i(x_i) - \sum_{j=1}^n f_j(x_j) a_{ij}(t) - \sum_{j=1}^n g_j(0) b_{ij}(t) - J_i(t) \right] dt = 0.$$

Then, there exists some $t_* \in [0, \omega]$ such that

$$b_i(x_i) - \sum_{j=1}^n f_j(x_j) a_{ij}(t_*) - \sum_{j=1}^n g_j(0) b_{ij}(t_*) - J_i(t_*) = 0,$$

which implies that

$$b_{i*}\overline{\varsigma}_i = b_{i*}|x_i| \le |b_i(x_i)| \le \sum_{j=1}^n |f_j(x_j)| a_{ij}^+ + \sum_{j=1}^n |g_j(0)| b_{ij}^+ + J_i^+$$

$$\leq \sum_{j=1}^{n}\left(\left|f_j(x_j)-f_j(0)\right|+\left|f_j(0)\right|\right)a_{ij}^{+}+\sum_{j=1}^{n}\left|g_j(0)\right|b_{ij}^{+}+J_i^{+}$$

$$\leq \sum_{j=1}^{n} a_{ij}^{+}L_j\bar{\varsigma}_j+\sum_{j=1}^{n}\left(a_{ij}^{+}\left|f_j(0)\right|+b_{ij}^{+}\left|g_j(0)\right|\right)+J_i^{+}.$$

This means that

$$(A\,\bar{\varsigma})_i \leq h_i. \tag{19}$$

It is easy to know that $D\,\bar{\varsigma}\,(0, 0, ..., 0)^T$. Since C is a nonsingular M-matrix, we have from $C^{-1} \geq 0$ (Lemma 2.2) that $C^{-1}D\,\bar{\varsigma} \geq (0, 0, ..., 0)^T$. Thus, we obtain

$$B(C^{-1}D)\,\bar{\varsigma} \geq (0, ..., 0)^T. \tag{20}$$

Similarly, we have

$$BC^{-1}k \geq (0, ..., 0)^T. \tag{21}$$

From (19), (20) and (21), we get

$$(A\bar{\varsigma})_i \leq h_i + (BC^{-1}k)_i + (B(C^{-1}D)\,\bar{\varsigma})_i = w_i + (B(C^{-1}D)\,\bar{\varsigma})_i.$$

This implies that

$$(A - B(C^{-1}D)\,\bar{\varsigma})_i \leq w_i.$$

which contradicts (16). Hence, (18) holds. Consequently, $QNx, \neq 0$ for each $x \in \partial\Omega \cap \mathrm{Ker}L$.

Furthermore, let $\psi(x, \mu) = \mu(-x)+(1-\mu)JQNx$ ($\mu \in [0, 1]$), then for any $x = (x_1, x_2, ..., x_n)^T \in \partial\Omega \cap \mathrm{Ker}L$, $(x_1, x_2, ..., x_n)^T$ is a constant vector in $\mathbb{R}^n$ with $|x_i| = \bar{\varsigma}_i$ for some $i \in \{1, ..., n\}$. It follows that

$$(\Psi(x,\mu))_i = -\mu x_i - (1-\mu)\frac{1}{\omega}a_i(x_i)\int_0^{\omega}\left[b_i(x_i)-\sum_{j=1}^{n}f_j(x_j)a_{ij}(t)-\sum_{j=1}^{n}g_j(0)b_{ij}(t)-J_i(t)\right]dt. \tag{22}$$

We claim that

$$|(\psi(x, \mu))_i| > 0. \tag{23}$$

If this is not true, then $|(\psi(x, \mu))_i| = 0$, i.e.,

$$\mu x_i + (1-\mu)\frac{a_i(x_i)}{\omega}\int_0^{\omega}\left[b_i(x_i)-\sum_{j=1}^{n}f_j(x_j)a_{ij}(t)-\sum_{j=1}^{n}g_j(0)b_{ij}(t)-J_i(t)\right]dt = 0.$$

Therefore, there exists some $t^* \in [0, \omega]$ such that

$$\mu x_i + (1-\mu)\frac{a_i(x_i)}{\omega}\left[b_i(x_i)-\sum_{j=1}^{n}f_j(x_j)a_{ij}(t^*)-\sum_{j=1}^{n}g_j(0)\,b_{ij}(t^*)-J_i(t^*)\right] = 0, \tag{24}$$

which implies that

$$x_i\left[b_i(x_i)-\sum_{j=1}^{n}f_j(x_j)\,a_{ij}(t^*)-\sum_{j=1}^{n}g_j(0)b_{ij}(t^*)-J_i(t^*)\right] \leq 0.$$

Thus, we get

$$b_{i*}|x_i|^2 \le x_i b_i(x_i) \le x_i\left[\sum_{j=1}^{n} f_j(x_j)a_{ij}(t^*) + \sum_{j=1}^{n} g_j(0)b_{ij}(t^*) + J_i(t^*)\right]$$

$$\le |x_i|\left[\sum_{j=1}^{n}\left(\left|f_j(x_j) - f_j(0)\right| + \left|f_j(0)\right|\right)a_{ij}^{+} + \sum_{j=1}^{n} g_j(0)b_{ij}^{+} + J_i^{+}\right]$$

$$\le |x_i|\left[\sum_{j=1}^{n} a_{ij}^{+}L_j\overline{\varsigma}_j + \sum_{j=1}^{n}\left(a_{ij}^{+}\left|f_j(0)\right| + b_{ij}^{+}\left|g_j(0)\right|\right) + J_i^{+}\right],$$

this means that $(A\overline{\varsigma})_i \le h_i$. By (20) and (21) we obtain

$$((A - BC^{-1}D)\,\overline{\varsigma})_i \le w_i.$$

which contradicts (16). Hence, (23) holds. By the homotopy invariance theorem, we get

$$\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} = \deg\{-x, \Omega \cap \mathrm{Ker}L, 0\} \ne 0.$$

So, condition (b) of Lemma 2.1 is also satisfied. Therefore, from Lemma 2.1 we conclude that system (1) has at least one ω-periodic solution. The proof is complete.

Remark 3.1: If $a_i(t) \equiv b_i(x_i) = b_i x_i$ and $f = g$, then the system (1) reduces to system (2) in [10]. In Theorem 3.1, we remove the continuously differentiable assumptions of the time-varying delays $\tau_{ij}(t)$ and $\sigma_{ij}(t)$.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we give an example to illustrate the effectiveness of our result.

Example 4.1: Consider the following Cohen-Grossberg type neural networks with neutral time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1(x_1(t))\,[5x_1(t) - 0.5 f_2(x_2(t - \tau_{12}(t))) - 0.4 \sin t \cdot g_1(\dot{x}_1(t - \sigma_{11}(t))) - J_1(t), \\ \dot{x}_2(t) = -a_2(x_2(t))\,[4x_2(t) - 0.3 \sin t \cdot f_1(x_1(t - \tau_{21}(t))) - 0.2 g_1(\dot{x}_1(t - \sigma_{21}(t))) - J_2(t), \end{cases} \tag{25}$$

where

$$a_1(u) = 1 + \frac{1}{10}\sin u, \quad a_2(u) = 1 + \frac{1}{5}\sin u, \quad f_1(u) = f_2(u) = \frac{1}{2}\sin u, \quad g_1(u) = \frac{2}{3}u,$$

$$a_{11}(t) = 0,\ a_{12}(t) = 0.5,\ a_{21}(t) = 0.3 \sin t,\ a_{22}(t) = 0,\ b_{11}(t) = 0.4 \sin t,$$

$$b_{12}(t) = b_{22}(t) = 0,\ b_{21}(t) = 0.2,\ J_1(t) = 3 \sin t,\ J_2(t) = 0.75 \sin t,$$

$\tau_{12}(t)$, $\tau_{21}(t)$, $\sigma_{11}(t)$ and $\sigma_{21}(t)$ can be any positive continuous bounded 2π-periodic functions. Obviously, $f_i(u)$ $(i = 1, 2)$ and $g_1(u)$ satisfy the Lipschitz condition (H_4) with constants $L_i = \frac{1}{2}$ and $l_1 = \frac{2}{3}$, respectively. By the direct calculation, we have

$$A = \begin{pmatrix} 5 & -0.25 \\ -0.15 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 0.2667 & 0 \\ 0.1334 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0.7067 & 0 \\ -0.16 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} 5.5 & 0.275 \\ 0.18 & 4.8 \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} 1.415 & 0 \\ 0.2264 & 1 \end{pmatrix}, \quad A - B(C^{-1}D) = \begin{pmatrix} 2.9244 & -0.3538 \\ -1.1882 & 3.9481 \end{pmatrix},$$

and

$$(A - B(C^{-1}D))^{-1} = \begin{pmatrix} 0.3549 & 0.0318 \\ 0.1068 & 0.2629 \end{pmatrix},$$

which implies that C and $A - B(C^{-1}D)$ are two nonsingular M-matrix. Hence, all the conditions of Theorem 3.1 are satisfied. So, by Theorem 3.1, system (25) has at least one 2π-periodic solution.

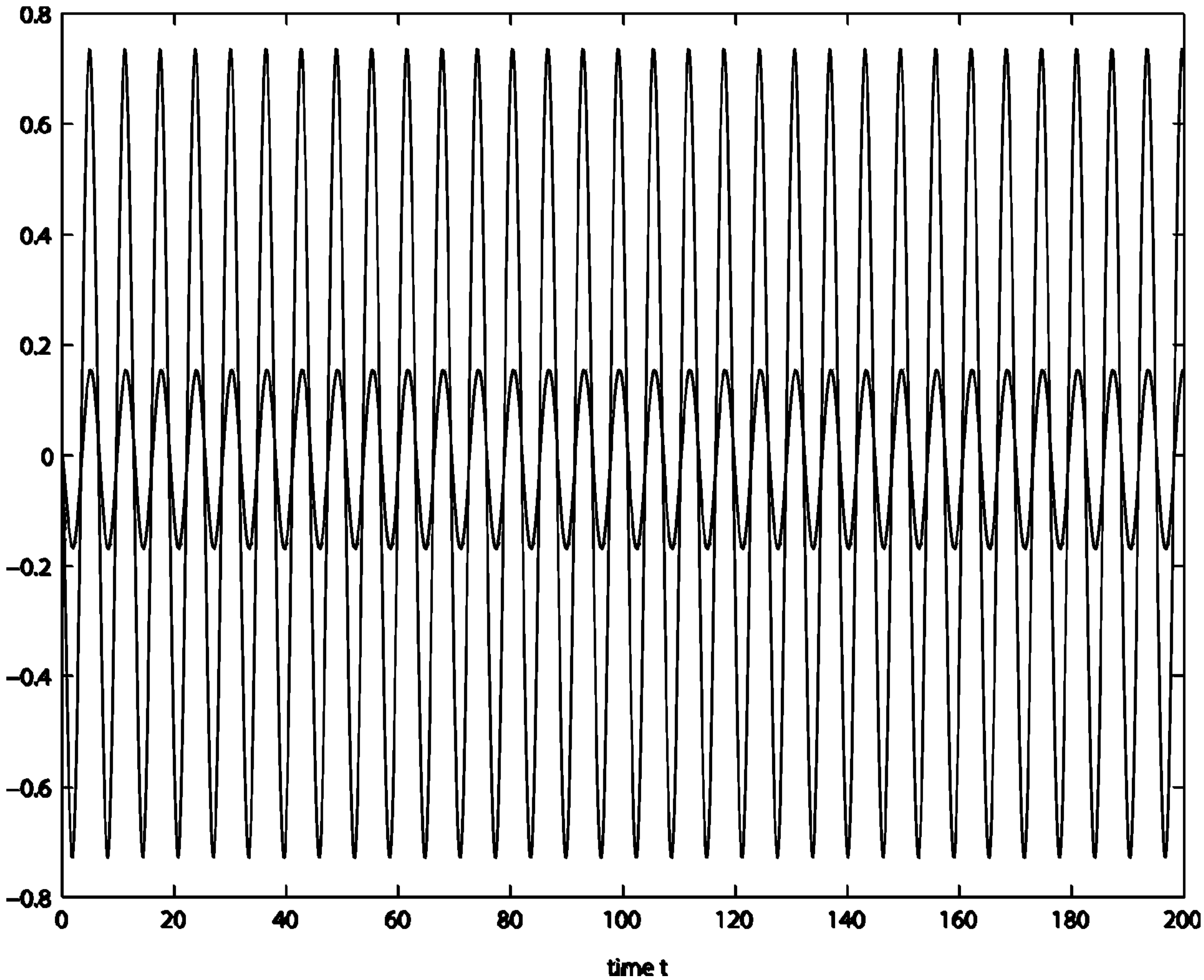

Figure 1: Numerical Solution $x_1(t)$, $x_2(t)$ of system (25), where $\tau_{12}(t) = 1$, $\tau_{21}(t) = 0.8$, $\sigma_{11}(t) = 0.6$ and $\sigma_{21}(t) = 0.7$, $x_1(s) = x_2(s) = 0$ for $s \in [-1, 0]$

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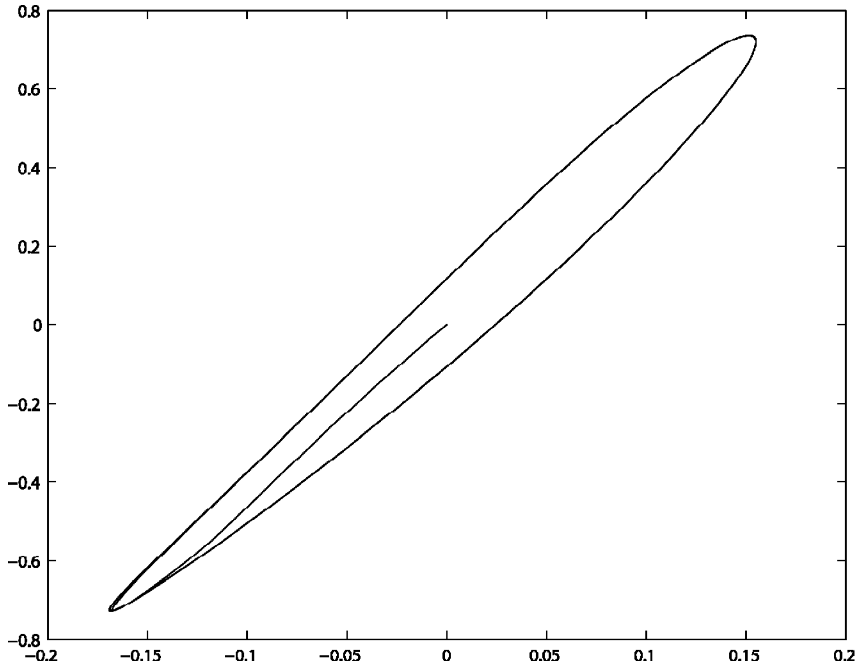

Figure 2: Phase Trajectories of System (25), where $\tau_{12}(t) = 1$, $\tau_{21}(t) = 0.8$, $\sigma_{11}(t) = 0.6$ and $\sigma_{21}(t) = 0.7$, $x_1(s) = x_2(s) = 0$ for $s \in [-1, 0]$

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