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Geometric Conditions for Finite Horizon Noninteraction and Fault Detection Based on the Almost Controllability Subspace Algorithm

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Abstract: This work deals with the problem of noninteraction (and the dual problem of fault detection) for discrete-time linear time-invariant finite-dimensional state space systems. In particular, the standard, infinite horizon, noninteracting control requirement is replaced by a less demanding one, where noninteracting control is only sought on finite time horizons, suitably defined in connection with some structural properties of the system subblocks. A necessary and sufficient condition for solvability of the finite horizon problem thus stated is derived in terms of the almost controllability subspaces associated with the block structure of the system. The proof of the condition is constructive in the sense that it leads to a design procedure for the feedforward compensators that guarantee noninteracting control over the given time horizons. The underlying idea of the proof, i.e. the exploitation of the almost controllability subspace algorithm in the case of finite horizon noninteraction, is also compatible with a modified procedure for designing the compensators achieving infinite horizon noninteraction, which may be admissible for some specific subblocks of the system. In fact, in this latter case, it is the effective use of the controllability subspace algorithm which plays a key role. The dual counterpart in the context of fault detection introduces a structural means to identify and treat the cases where, due to the structural properties of the monitored system, some of the residuals which can be generated are only significant in a limited time. These concepts are also illustrated with a detailed numerical example.

Keywords: Geometric Approach; Almost Controllability Subspaces; FIR Systems; Noninteraction; Fault Detection.

1. INTRODUCTION

Noninteracting control is a well-known problem, deeply investigated particularly within the geometric approach. Necessary and sufficient constructive conditions for noninteracting control were first proved by Basile and Marro in [1] and, independently, by Morse and Wonham in [2]. Those conditions were transferred to the dual setting of fault detection by Massoumnia and co-workers in [3] and [4]. Henceforward, they have often been considered in the literature (see e.g. [5], [6], [7], [8], [9], [10]) and have inspired generalizations to various classes of systems: namely, nonlinear systems (see e.g. [11], [12], [13]), systems over rings (see e.g. [14], [15], [16]), linear parameter varying systems (see e.g. [17], [18]), etc.

The geometric conditions for perfect noninteraction (and the dual conditions for perfect fault detection) are sharp and easy to check by means of the standard computational tools of the geometric approach like, e.g., those made available with [19]. However, in many cases, those requirements turn out to be too restrictive, in the sense that they cannot be satisfied by certain systems with given structural properties. Hence, a big deal of research effort has been spent in devising alternative solutions, particularly resorting to some optimality criteria, like e.g. those discussed in [20], [21], [22], [23], [24], [25], [26].

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In the light of the abovementioned considerations, this work introduces a completely new approach to coping with those situations where the exact conditions for noninteraction (fault detection) are not satisfied. In fact, it is based on a relaxed formulation of the problem, where noninteraction (fault detection) is not required over an infinite time horizon, but only over finite time horizons, appropriately defined in connection with specific structural properties of the controlled (monitored) system subblocks. Hence, a necessary and sufficient condition for problem solvability is derived, which still has the advantage of being sharp and easy to check like the original geometric conditions ensuring noninteraction (fault detection) on the infinite horizon and also has the good property of guaranteeing perfect (not only optimal according to some criterion) noninteraction (fault detection) on those, appropriately defined, time intervals.

More specifically, in the context of noninteraction, the length, in terms of number of samples, of the time horizons is given by the number of steps for the almost controllability subspace algorithms (ACSAs) respectively associated with the particular system subblocks to converge and the proof of the necessary and sufficient condition for problem solvability is directly related to those algorithms. The proof is constructive in the sense that the ACSA is at the basis of the design procedure for the feedforward compensators, implemented as finite impulse response systems, achieving finite horizon noninteraction. Almost controllability subspaces were introduced by Willems in [27] and are herein considered for the first time in connection with noninteraction and fault detection issues. Besides, as was shown by Loiseau in [28], the number of steps for the ACSAs associated with the various systems subblocks to converge are directly related to some of the structural Kronecker-type indices of the system under consideration. Furthermore, a slight modification of the design procedure, consisting in the replacement of the ACSA with the controllability subspace algorithm (CSA), leads to a new procedure for the design of the feedforward dynamic units achieving infinite horizon noninteraction for those system subblocks for which it is admissible.

Duality arguments leads to a precise methodology to discriminate the situations where, due to the structural properties of the monitored systems, some of the residuals which can be generated are only meaningful for finite time intervals and, consequently, leads to a technique for devising the corresponding residual generators. The practical interest of finite horizon noninteraction is directly related to very specific applications: particularly, those where the plant lifetime is predetermined or the plant structure substantially changes at certain time instants. Nonetheless, the dual counterpart in the framework of fault detection has a broad practical impact. In fact, although the time when the generic fault input only affects the output of the corresponding residual generator (not the others) is bounded, it may be sufficient to enable detection.

Notation: The symbols $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{C}^{\odot}$ are used for the sets of real numbers, complex numbers, and complex numbers inside the unit circle, respectively. Sets, vector spaces, and subspaces are denoted by script capital letters like $\mathcal{X}$. The quotient space of a vector space $\mathcal{X}$ over a subspace $\mathcal{V} \subseteq \mathcal{X}$ is denoted by $\mathcal{X}/\mathcal{V}$. Matrices and linear maps are denoted by slanted capital letters like A. The restriction of a linear map A to an A-invariant subspace $\mathcal{J}$ is denoted by $A|_{\mathcal{J}}$. The spectrum, the image, and the kernel of A are denoted by $\sigma(A)$, im A, and ker A, respectively. The symbols A^{-1}, A^{+}, and $A^{\top}$ are used for the inverse, the generalized inverse, and the transpose of A, respectively. The symbols I and O are used for an identity matrix and a zero matrix of appropriate dimensions, respectively.

2. A REVIEW OF THE GEOMETRIC APPROACH TO INFINITE HORIZON NONINTERACTION

The discrete time-invariant linear system

$$x(t+1) = Ax(t) + Bu(t), \tag{1}$$

$$y(t) = C\,x(t), \tag{2}$$

is considered, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, and $y \in \mathbb{R}^q$ respectively denote the state, the control input, and the controlled output. The set of all admissible control input sequences is the set $\mathcal{U}_f$ of all bounded sequences with values in $\mathbb{R}^p$. The matrices B and C are full rank. The symbols $\mathcal{B}$ and $\mathcal{C}$ stand for im B and ker C, respectively. The symbol $\mathcal{V}^* = \max \mathcal{V}(A, \mathcal{B}, \mathcal{C})$ denotes the maximal $(A, \mathcal{B})$-controlled invariant contained in $\mathcal{C}$, $\mathcal{S}^* = \min \mathcal{S}(A, \mathcal{C}, \mathcal{B})$ denotes the minimal $(A, \mathcal{C})$-conditioned invariant containing $\mathcal{B}$, $\mathcal{R}_{\mathcal{V}^*}$ denotes the supremal controllability subspace contained in $\mathcal{C}$, $\mathcal{R}^*_{\mathcal{C}}$ denotes the supremal almost controllability subspace contained in $\mathcal{C}$. The subspaces $\mathcal{R}_{\mathcal{V}^*}$ and $\mathcal{R}^*_{\mathcal{C}}$ satisfy the relations $\mathcal{R}_{\mathcal{V}^*} = \mathcal{S}^* \cap \mathcal{V}^*$. and $\mathcal{R}^*_{\mathcal{C}} = \mathcal{S}^* \cap \mathcal{C}$, respectively. For any real matrix F such that $(A + BF)\,\mathcal{V}^* \subseteq \mathcal{V}^*$, also $(A + BF)\,\mathcal{R}_{\mathcal{V}^*} \subseteq \mathcal{R}_{\mathcal{V}^*}$ holds. The matrix $A + BF$ is also denoted as A_F. The subspace $\mathcal{R}_{\mathcal{V}^*}$ is the last term of the monotone nondecreasing sequence

$$\mathcal{R}^1_{\mathcal{V}^*} = \mathcal{B} \cap \mathcal{V}^*,\ \mathcal{R}^i_{\mathcal{V}^*} = (A_F \mathcal{R}^{i-1}_{\mathcal{V}^*} + \mathcal{B}) \cap \mathcal{V}^*,\ i = 2, \dots k,$$

where $k\,(\leq n)$ is the least integer such that $\mathcal{R}^{k+1}_{\mathcal{V}^*} = \mathcal{R}^k_{\mathcal{V}^*}$ (*Controllability Subspace Algorithm – CSA*). The subspace $\mathcal{R}^*_{\mathcal{C}}$ is the last term of the monotone nondecreasing sequence

$$\mathcal{R}^1_{\mathcal{C}} = \mathcal{B} \cap \mathcal{C}, \quad \mathcal{R}^i_{\mathcal{C}} = (A\mathcal{R}^{i-1}_{\mathcal{C}} + \mathcal{B}) \cap \mathcal{C},\ i = 2, \dots k,$$

where $k\,(\leq n)$ is the least integer such that $\mathcal{R}^{k+1}_{\mathcal{C}} = \mathcal{R}^k_{\mathcal{C}}$ (*Almost Controllability Subspace Algorithm – ACSA*).

Problem 1 *(Noninteraction by Dynamic Feedforward)***:** Let system Σ be ruled by (1), (2) with initial state $x(0)=0$. Let $\sigma(A) \subset \mathbb{C}^{\odot}$. Let the output $y \in \mathbb{R}^q$ be partitioned into k blocks $(y_1, \dots, y_k)$ with $2 \leq k \leq q$. Find a feedforward dynamic compensator Σ_c, modeled by

$$z(t+1) = A_c\, z(t) + B_c\, h(t), \tag{3}$$

$$u(t) = C_c\, z(t) + D_c\, h(t), \tag{4}$$

with state $z \in \mathbb{R}^{n_c}$, input $h \in \mathbb{R}^{p_c}$ $(p_c \geq k)$, output $u \in \mathbb{R}^p$, and initial state $z(0) = 0$, and an input block partition $(h_1, \dots, h_k)$ such that in the compensated system, for any $i = 1, \dots, k$, the output y_i is pointwise controllable by the corresponding input h_i with all the other outputs y_j, $j = 1, \dots, k$, $j \neq i$, identically zero.

Let the matrices $C_1, \dots, C_k$ denote the submatrices of C corresponding to the output partition $(y_1, \dots, y_k)$. Consequently, for any $i = 1, \dots, k$, $\mathcal{C}_i = \ker C_i$, $\bar{\mathcal{C}}_i = \bigcap_{j=1,\dots,k,\, j\neq i} \mathcal{C}_j$, $\bar{\mathcal{V}}^*_i = \max \mathcal{V}(A, \mathcal{B}, \bar{\mathcal{C}}_i)$, $\bar{\mathcal{S}}^*_i = \min \mathcal{S}(A, \bar{\mathcal{C}}_i, \mathcal{B})$, $\mathcal{R}_{\bar{\mathcal{V}}^*_i} = \bar{\mathcal{S}}^*_i \cap \bar{\mathcal{V}}^*_i$..

Theorem 1: Let system Σ be ruled by (1), (2) with $x(0) = 0$. Let $\sigma(A) \subset \mathbb{C}^{\odot}$. Let the output $y \in \mathbb{R}^q$ be partitioned into k blocks $(y_1, \dots, y_k)$ with $2 \leq k \leq q$. Problem 1 is solvable if and only if

$$C_i \mathcal{R}_{\bar{\mathcal{V}}^*_i} = \operatorname{im} C_i,\ i = 1, \dots, k.$$

Proof: See [19].

The necessary and sufficient condition stated in Theorem 1 is closely connected to the property of the generic i-th subspace $\mathcal{R}_{\bar{\mathcal{V}}^*_i}$ of being the maximal subspace reachable from the origin along trajectories which can be maintained in $\mathcal{R}_{\bar{\mathcal{V}}^*_i}$, hence in $\bar{\mathcal{C}}_i$, by a suitable control action. Consequently, those trajectories affect the output y_i, but are invisible at any other output y_j, $j = 1, \dots, k$, $j \neq i$.

Remark 1: Feedforward control requires that the to-be-controlled system (1), (2) be stable. This assumption is not indeed restrictive with respect to those of stabilizability of (A,B) and detectability of (A,C) which are usually considered. In fact, as is well-known, system (1), (2) can be stabilized by output dynamic feedback, if those assumptions hold. A discussion of prestabilization in connection with specific structural properties of the system can be found in [29].

3. FINITE HORIZON NONINTERACTION

This section deals with the modified formulation of the problem of noninteraction. First, the length of the finite time horizons is defined in connection with the structural properties of the subblocks of the to-be-controlled system. Then, the geometric necessary and sufficient condition for problem solvability is proved.

As in Section 2, let the output $y \in \mathbb{R}^q$ of system Σ be partitioned into k blocks $(y_1, \ldots, y_k)$ with $2 \le k \le q$. Besides the notation previously introduced, for any $i = 1, \ldots, k$, let $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$ denote the supremal almost controllability subspace contained in $\bar{\mathcal{C}}_i$, so that $\mathcal{R}^*_{\bar{\mathcal{C}}_i} = \bar{\mathcal{S}}^*_i \cap \bar{\mathcal{C}}_i$, and let $\bar{\rho}_i$ denote the number of steps for the corresponding ACSA to converge.

Problem 2 *(Finite Horizon Noninteraction by Dynamic Feedforward)***:** Let system Σ be ruled by (1), (2) with $x(0) = 0$. Let $\sigma(A) \subset \mathbb{C}^{\odot}$. Let the output $y \in \mathbb{R}^q$ be partitioned into k blocks $(y_1, \ldots, y_k)$ with $2 \le k \le q$. Find a feedforward dynamic compensator Σ_c, ruled by (3), (4) with $z(0) = 0$, and an input block partition $(h_1, \ldots, h_k)$ such that in the compensated system, for any $i = 1, \ldots, k$, the output y_i is pointwise controllable by the corresponding input h_i with all the other outputs y_j, $j = 1, \ldots, k$, $j \neq i$, identically zero until the time $t = \bar{\rho}_i$.

Theorem 2: Let system Σ be ruled by (1), (2) with $x(0) = 0$. Let $\sigma(A) \subset \mathbb{C}^{\odot}$. Let the output $y \in \mathbb{R}^q$ be partitioned into k blocks $(y_1, \ldots, y_k)$ with $2 \le k \le q$. Problem 2 is solvable if and only if

$$\mathcal{C}_i \mathcal{R}^*_{\bar{\mathcal{C}}_i} = im\, C_i,\ i = 1, \ldots, k.$$

Proof: If. This part of the proof is constructive. For any $i = 1, \ldots, k$, the condition $\mathcal{C}_i \mathcal{R}^*_{\bar{\mathcal{C}}_i} = \mathrm{im} C_i$ allows a feedforward dynamic unit Σ_{c_i}, satisfying the requirement on the block output y_i, to be designed. The precompensator Σ_{c_i} should generate a control action of $\bar{\rho}_i$ steps steering the state of Σ from the origin to any state in $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$ along a trajectory in $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$, so that it has no effect on the outputs y_j, $j = 1, \ldots, k$, $j = i$ until the time $t = \bar{\rho}_i$. Thus, Σ_{c_i}, ruled by

$$z_i(t+1) = A_{ci}\, z_i(t) + B_{ci}\, h_i(t), \tag{5}$$

$$u_i(t) = C_{ci}\, z_i(t) + D_{ci}\, h_i(t), \tag{6}$$

with $z_i(0) = 0$, is found to be an FIR system defined by the IO equation

$$u_i(t) = \sum_{\ell=0}^{\bar{\rho}_i - 1} \Phi_i(\ell) h_i(t-\ell), \qquad t = 0, 1, \ldots, \tag{7}$$

where the gain matrices $\Phi_i(\ell)$, $\ell = 0, 1, \ldots, \bar{\rho}_i - 1$, should be designed suitably. To this aim, let M_i^k, $k = 1, \ldots, \bar{\rho}_i$, denote the respective basis matrices of the subspaces subsequently generated by the ACSA of $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$ (i.e., im $M_i^k = \mathcal{R}^k_{\bar{\mathcal{C}}_i}$, $k = 1, \ldots, \bar{\rho}_i$). For any column vector of $M_i^{\bar{\rho}_i}$, representing a state in $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$, there is a control sequence of $\bar{\rho}_i$ steps steering the state of Σ from the origin to that state in $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$ along a trajectory whose intermediate states belong to the subspaces subsequently generated by the ACSA of $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$. Let $X_i(k) = M_i^k \Gamma_i(k)$ and $U_i(k)$, $k = 1, \ldots, \bar{\rho}_i - 1$, respectively denote the matrix of the states and that of the controls at the k-th step: i.e., the j-th column vectors of $X_i(k)$ and $U_i(k)$ respectively are the state and the control at the k-th step of the trajectory reaching the state represented by the j-th column vector of $M_i^{\bar{\rho}_i}$ at the time $t = \bar{\rho}_i$. Consistently, the generic $\Gamma_i(k)$ is a matrix with a number of rows equal to the number of columns of the corresponding matrix M_i^k and a number of columns equal to the number of columns of $M_i^{\bar{\rho}_i}$ (hence, also equal to the dimension of the subspace $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$, since $M_i^{\bar{\rho}_i}$ is a basis matrix of $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$). The entries of the generic j-th column of $\Gamma_i(k)$ are the coefficients of the linear combination of the column vectors of M_i^k expressing the j-th column vector of the corresponding $X_i(k)$ (in fact, as mentioned above, there is a control sequence such that the intermediate states of the corresponding state trajectory belong to the subspaces respectively generated by the ACSA). The matrix sequences $X_i(k)$, $U_i(k)$, $k = 1, \ldots, \bar{\rho}_i - 1$, are provided by the recursive relation

$$[\Gamma_i(k)^\top \; U_i(k)^\top]^\top = [AM_i^k \; B]^+ X_i(k+1),\; X_i(k) = M_i^k \Gamma_i(k), \quad k = \bar{\rho}_i - 1, \ldots, 1,\; X_i(\bar{\rho}_i) = M_i^{\bar{\rho}_i},$$

which is derived from the ACSA of $\mathcal{R}^*_{\bar{\mathcal{C}}_i}$. Moreover,

$$U_i(0) = B^+ X_i(1).$$

Hence, by linearity, the gain matrices in (7) are

$$\Phi_i(\ell) = U_i(\lambda), = \ell = 0, 1, \ldots, \bar{\rho}_i - 1,$$

and the matrices in (5), (6) are

$$A_{C_i} = \begin{bmatrix} O & I & O & \ldots & O \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & O \\ \vdots & \ddots & \ddots & \ddots & I \\ O & \ldots & \ldots & \ldots & O \end{bmatrix},\; B_{C_i} = \begin{bmatrix} O \\ \vdots \\ \vdots \\ O \\ I \end{bmatrix},\; C_{C_i} = \begin{bmatrix} \Phi_i(\bar{\rho}_i - 1) & \ldots & \Phi_i(1) \end{bmatrix},\; D_{c_i} = \Phi_i(0). \tag{8}$$

Finally, the precompensator Σ_c, with state $z(t) = [z_1(t)^\top \ldots z_k(t)^\top]^\top$, input $h(t) = [h_1(t)^\top \ldots h_k(t)^\top]^\top$, and output $u(t) = u_1(t) + \ldots + u_k(t)$, is ruled by (3), (4) with

$$A_c = \begin{bmatrix} A_{c_1} & O & \ldots & O \\ O & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ O & \ldots & O & A_{c_k} \end{bmatrix}, \quad B_c = \begin{bmatrix} B_{c_1} & O & \ldots & O \\ O & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ O & \ldots & O & B_{c_k} \end{bmatrix}, \tag{9}$$

$$C_c = [C_{c_1} \;\ldots\; \ldots\; C_{c_k}], \qquad D_c = [D_{c_1} \;\ldots\; \ldots\; D_{c_k}]. \tag{10}$$

Only if. Necessity is due to the subspace $\mathcal{R}^*_{\bar{\mathcal{C}}_i}, i = 1, \ldots, k$, being the maximal set of states reachable from the origin along trajectories in $\bar{\mathcal{C}}_i$, hence invisible at all the other outputs y_j, $j = 1, \ldots, k$, $j \neq i$.

4. FINITE HORIZON NONINTERACTION WITH STANDARD CONDITIONS FOR SOME BLOCK OUTPUTS

In noninteracting control problems, a frequent situation is that where the standard conditions are satisfied for some block outputs, while the extended conditions only are satisfied for the others. The aim of this section is to illustrate how to design the precompensators relative to the block outputs for which the more restrictive conditions (i.e., those ensuring infinite horizon noninteraction with respect to the considered block outputs) hold and how to include those precompensators in the global scheme, along with the precompensators relative to the block outputs for which the sole less restrictive conditions hold.

Whenever the standard condition is met for some block output y_i, exploiting the properties of the supremal controllability subspace $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ contained in $\bar{\mathcal{C}}_i$ is the most convenient option. The synthesis procedure for the corresponding feedforward dynamic unit Σ_{c_i} is discussed in this section. The precompensator proposed herein differs from that presented in [1] and, in revised form, in [19] since, in addition to the part replicating the dynamics of $(A + BF_i)|_{\mathcal{R}_{\bar{\mathcal{V}}_i^*}}$, where F_i is any real matrix such that $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ is $(A + BF_i)$-invariant, it includes an FIR system ensuring pointwise control of the block output y_i.

In this section, the symbol $\bar{\rho}_i$ denotes the number of steps for the CSA of $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ to converge and the symbols M_i^k, $k = 1, \dots, \bar{\rho}_i$, denote the basis matrices of the subspaces ordinately generated by the CSA of $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$. The use of the same symbols of Section 3 to address different objects is aimed at avoiding rewriting of some equations and highlighting consistency of the design procedure with that previously introduced.

Let the condition $C_i\mathcal{R}_{\bar{\mathcal{V}}_i^*} = \mathrm{im}C_i$ hold for some block output y_i, $i = 1, \dots, k$. The precompensator Σ_{c_i} should generate a control action driving the state of Σ from the origin to any state of $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ in $\bar{\rho}_i$ steps, along a trajectory in $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$, and then maintaining the state in $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$, so that the outputs y_j, $j = 1, \dots, k$, $j \neq i$, remain identically zero. Thus, Σ_{c_i}, ruled by (5), (6) with $z_i(0) = 0$, should include an FIR system forcing the state transition from the origin to any state in $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ and a standard dynamic unit steering the state along a trajectory in $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$. Let the FIR system be described by the IO equation

$$u_{i,f}(t) = \sum_{\ell=0}^{\bar{\rho}_i-1} \Phi_i(\ell) h_i(t-\ell), \qquad t = 0, 1, \dots, \tag{11}$$

or, equivalently, by

$$z_{i,f}(t+1) = A_{i,f}\, z_{i,f}(t) + B_{i,f}\, h_i(t), \tag{12}$$

$$u_{i,f}(t) = C_{i,f}\, z_{i,f}(t) + D_{i,f}\, h_i(t), \tag{13}$$

with $z_{i,f}(0) = 0$, where the matrices $A_{i,f}$, $B_{i,f}$, $C_{i,f}$, $D_{i,f}$ have the same structure of those in (8). Let the standard dynamic unit be ruled by

$$z_{i,d}(t+1) = A_{i,d}\, z_{i,d}(t) + B_{i,d}\, v_{i,d}(t), \tag{14}$$

$$u_{i,d}(t) = C_{i,d}\, z_{i,d}(t), \tag{15}$$

with $z_{i,d}(0) = 0$. The feedforward dynamic unit Σ_{c_i} should be obtained by connecting the FIR system (12), (13) and the standard dynamic unit (14), (15) so that $v_{i,d} = u_{i,f}$ and $u_i = u_{i,f} + u_{i,d}$. Therefore, the matrices in (5), (6) are

$$A_{c_i} = \begin{bmatrix} A_{i,f} & O \\ B_{i,d}C_{i,f} & A_{i,d} \end{bmatrix}, \; B_{c_i} = \begin{bmatrix} B_{i,f} \\ B_{i,d}D_{i,f} \end{bmatrix}, \; C_{c_i} = \begin{bmatrix} C_{i,f} & C_{i,d} \end{bmatrix}, \; D_{c_i} = D_{i,f}. \tag{16}$$

The design of both the FIR system and the standard dynamic unit requires that a real matrix F_i such that $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ is $(A + BF_i)$-invariant (or, briefly, A_{F_i}-invariant) be chosen.

First, the design of the FIR system is considered. As mentioned above, let M_i^k, $k = 1, \dots, \bar{\rho}_i$, denote the respective basis matrices of the subspaces subsequently generated by the CSA of $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ (i.e., $\mathrm{im}M_i^k = \mathcal{R}^k_{\bar{\mathcal{V}}_i^*}$, $k = 1, \dots, \bar{\rho}_i$). For any column vector of $M_i^{\bar{\rho}_i}$, representing a state in $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$, there is a control sequence of $\bar{\rho}_i$ steps steering the state of Σ from the origin to that state in $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$ along a trajectory whose intermediate states belong to the subspaces subsequently generated by the CSA of $\mathcal{R}_{\bar{\mathcal{V}}_i^*}$. Let $X_i(k) = M^k_i\,\Gamma_i(k)$ and $U_i(k)$, $k = 1, \dots, \bar{\rho}_i - 1$, denote the matrices of the states and controls at the k-th step, respectively. The matrix sequences $X_i(k)$, $U_i(k)$, $k = 1, \dots, \bar{\rho}_i - 1$, are provided by the recursive relation

$$[\Gamma_i(k)^{\mathrm{T}}\ U_i(k)^{\mathrm{T}}]^{\mathrm{T}} = [A_{F_i} M_i^k\ \ B]^+ X_i(k+1), \quad X_i(k) = M_i^k\,\Gamma_i(k), \quad k = \bar{\rho}_i - 1, \dots, 1,\ X_i(\bar{\rho}i) = M_i^{\bar{\rho}i},$$

which is derived from the CSA of $\mathcal{R}_{\bar{\mathcal{V}}_i}$. Moreover,

$$U_i(0) = B^+ X_i(1) \tag{1.}$$

Hence, by linearity, the gain matrices in (11) are

$$\Phi_i(\ell)=U_i(\ell), \quad \ell = 0, 1, \dots, \bar{\rho}_i - 1.$$

Then, the design procedure for the standard dynamic unit is illustrated. Let the similarity transformation $T_i = [T_{i,1}\quad T_{i,2}]$, where $\mathcal{R}_{\bar{\mathcal{V}}_i^*} = \mathrm{im}\,T_{i,1}$, be performed on the triple (A_{Fi}, B, F_i). The matrices $A'_{F_i} = T_i^{-1} A_{F_i} T_i$ $B'_i = T_i^{-1}B$, $F'_i = F_i T_i$, partitioned according to T_i, have the structures

$$A'_{F_i} = \begin{bmatrix} A'_{F_{i,11}} & A'_{F_{i,12}} \\ O & A'_{F_{i,22}} \end{bmatrix}, \quad B'_i = \begin{bmatrix} B'_{i,1} \\ B'_{i,2} \end{bmatrix}, \qquad F'_1 = \begin{bmatrix} F'_{i,1} & F'_{i,2} \end{bmatrix}.$$

Hence, the matrices in (14), (15) are

$$A_{i,d} = A'_{F_{i,11}},\ B_{i,d} = B'_{i,1},\ C_{i,d} = F'_{i,1}.$$

Moreover, if F_i is chosen such that $\sigma((A + BF_i)|_{\mathcal{R}_{\bar{\mathcal{V}}_i^*}}) \subset \mathbb{C}^{\circ}$, from the time $t = \bar{\rho}_i$ on, the state trajectory of Σ is steered to the origin asymptotically.

In conclusion, when the standard conditions are satisfied for some block outputs and the extended conditions only are satisfied for the others, the matrices in (3), (4), defining the precompensator Σ_c, still have the structure shown in (9), (10), but the matrices A_{c_i}, B_{c_i}, C_{c_i}, D_{c_i}, $i = 1, \dots, k$, either are of the type shown in (8) or of the type shown in (16).

5. IMPACT ON FAULT DETECTION ISSUES

The generalization discussed in the context of noninteraction throughout the previous sections can be transferred to the context of fault detection by a straightforward application of duality arguments. Hence, statements and proofs will not be repeated for the dual context. Nonetheless, the impact of the generalization on fault detection issues will be briefly shown referring to the, so-called, fundamental case, where the fault inputs are partitioned into two blocks only.

Let us consider the, so-called, fundamental noninteracting control problem — i.e., the problem where the output of system Σ, defined by the triple (A,B,C), is partitioned into two blocks only (y_1, y_2), corresponding to the matrices C_1, C_2 (Fig. 1a) — and the, so-called, fundamental fault detection problem — i.e., the problem where the fault inputs are partitioned into two blocks only (m_1, m_2) (Fig. 1b). In standard noninteraction, the compensation unit Σ_{c_1} should generate a control action steering the state of Σ on the supremal controllability subspace $\mathcal{R}_{\mathcal{V}_2^*}$ contained in $\mathcal{C}_2$, the kernel of $\mathcal{C}_2$. The compensation unit Σ_{c_2} should behave likewise with respect to $\mathcal{R}_{\mathcal{V}_1^*}$, the supremal controllability subspace contained in $\mathcal{C}_1$, the kernel of $\mathcal{C}_1$. In the relaxed version of noninteraction, Σ_{c_1} should generate a control action forcing the state of Σ on the supremal almost controllability

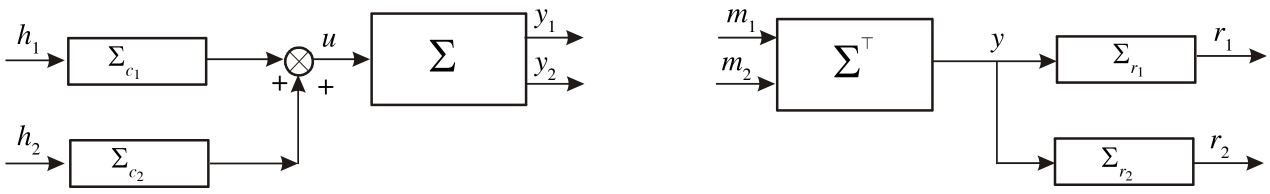

Figure 1: Block Diagrams for Fundamental Noninteraction (a) and Fault Detection (b)

subspace $\mathcal{R}^*_{\mathcal{C}_2}$ (contained in $\mathcal{C}_2$) for a certain, limited time, and Σ_{c_2} should do likewise with respect to $\mathcal{R}^*_{\mathcal{C}_1}$, the supremal almost controllability subspace contained in $\mathcal{C}_1$. Thus, a unit pulse at the input h_1 affects y_1 but not y_2 in standard noninteraction, while it also changes y_2 after a certain number of samples in finite horizon noninteraction (Fig. 2a). By duality, the residual generator Σ_{r_1} should guarantee that the residual r_1 reveals a unit pulse at m_1 but not a unit pulse at m_2 in standard fault detection, while r_1 is also perturbed by a unit pulse at m_2 after a certain number of samples in finite horizon fault detection (Fig. 2b). The residual generator Σ_{r_2} should perform similarly, with reverse roles played by m_1 and m_2.

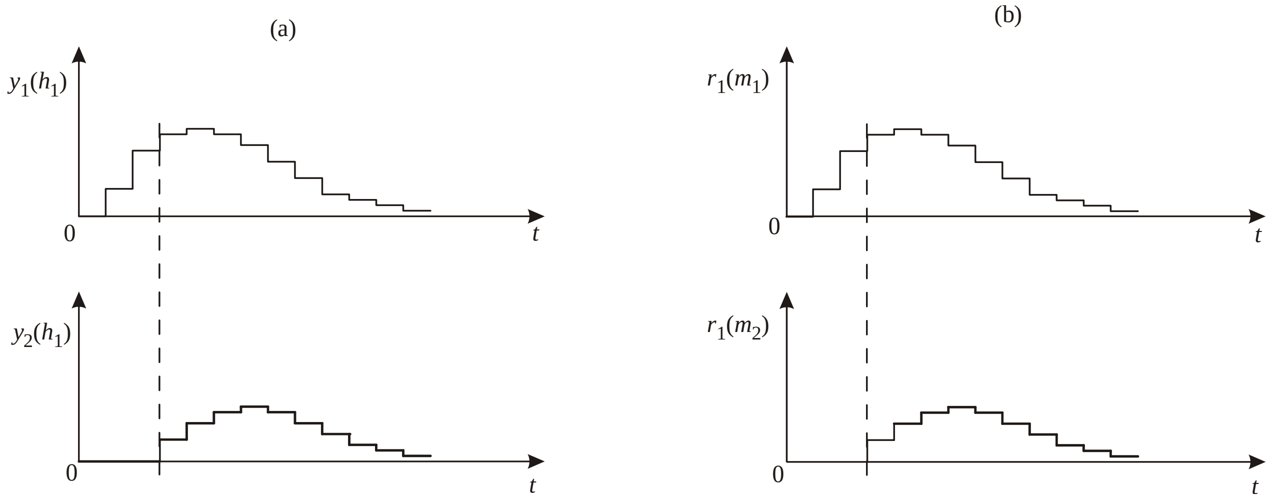

Figure 2: Outputs for Finite Horizon Noninteraction (a) and Fault Detection (b)

6. AN ILLUSTRATIVE EXAMPLE

Let the noninteracting control problem by dynamic feedforward be stated for system Σ, ruled by (1), (2) with

$$A = \begin{bmatrix} .6 & 0 & 0 & 0 & -.3 & 0 \\ 1 & 1 & -.3 & 0 & 0 & 0 \\ 0 & .4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .9 & 0 & -.4 \\ 0 & 0 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & .7 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 \\ \hline C_2 \end{bmatrix} = \left[\begin{array}{cccccc} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right],$$

where C_1, C_2 identify the output block partition (y_1, y_2). Since $\mathcal{R}_{\bar{\mathcal{V}}_1^*} = \{0\}$, the condition for standard noninteraction is not satisfied for the block output y_1. However, since

$$\mathcal{R}^1_{\bar{\mathcal{C}}_1} = \mathrm{im}\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}, \mathcal{R}^2_{\bar{\mathcal{C}}_1} = \mathrm{im}\begin{bmatrix}1 & 0\\0 & -1\\0 & 0\\0 & 0\\0 & 0\\0 & 0\end{bmatrix}, R^3_{\bar{\mathcal{C}}_1} = R^*_{\bar{\mathcal{C}}_1} = \mathrm{im}\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -.5774\\0 & 0 & 0\\0 & 0 & .5774\\0 & 0 & .5774\end{bmatrix},$$

and $C_1\mathcal{R}^*_{\bar{\mathcal{C}}_1} = \mathrm{im}C_1 = \mathbb{R}^2$, the extended condition is satisfied for y_1. Conversely, the standard condition is satisfied for the block output y_2 since

$$\mathcal{R}^1_{\mathcal{V}_2^*} = \mathrm{im}\begin{bmatrix}0\\0\\0\\0\\.7071\\.7071\end{bmatrix}, \mathcal{R}^2_{\mathcal{V}_2^*} = \mathrm{im}\begin{bmatrix}0 & 0\\0 & 0\\0 & 0\\.9428 & 0\\.2357 & .7071\\-.2357 & .7071\end{bmatrix}, \mathcal{R}^3_{\bar{\mathcal{V}}_2^*} = \mathcal{R}_{\bar{\mathcal{V}}_2^*} = \mathrm{im}\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix},$$

and $C_2\mathcal{R}_{\bar{\mathcal{V}}_2^*} = \mathrm{im}\ C_2 = \mathbb{R}^2$. Consequently, the precompensator Σ_{c_1} consists of the sole FIR system designed according to the procedure detailed in Section 3, while Σ_{c_2} includes both the FIR system and the standard dynamic unit devised as shown in Section 4.

In particular, Σ_{c_1} has the IO description (7), where $\bar{\rho}_1 = 3$ and the gain matrices $\Phi_1(\ell)$, $\ell = 0, 1, 2$, are

$$\Phi_1(0) = \begin{bmatrix}0 & 0 & -1.4434\\0 & 0 & 0\end{bmatrix} \qquad \Phi_1(1) = \begin{bmatrix}0 & -1 & 2.3094\\0 & 0 & 0\end{bmatrix}, \qquad \Phi_1(2) = \begin{bmatrix}1 & .6 & -1.4434\\0 & 0 & .5774\end{bmatrix}.$$

The precompensator Σ_{c_2} is described by (5), (6) with (16). Let F_2 be such that $\sigma((A + BF_2)|_{\mathcal{R}_{\bar{\mathcal{V}}_2^*}}) = \{.1, .2, .3\}$. The FIR system has the IO description (11), where $\bar{\rho}_1 = 3$ and the gain matrices $\Phi_2(\ell)$, $\ell = 0, 1, 2$, are

$$\Phi_2(0) = \begin{bmatrix}6.25 & -12.5 & 12.5\\-6.25 & 12.5 & -12.5\end{bmatrix}, \ \Phi_2(1) = \begin{bmatrix}1.875 & 1.25 & -1.25\\-1.875 & -1.25 & 1.25\end{bmatrix}, \ \Phi_2(2) = \begin{bmatrix}2.375 & -.75 & -.25\\-2.375 & .75 & .25\end{bmatrix}.$$

The dynamic unit is ruled by (14), (15), where the matrices

$$A_{2,d} = \begin{bmatrix}.9 & 0 & .4\\2.1 & .2 & 1.2\\-2.1 & .3 & -.5\end{bmatrix}, \quad B_{2,d} = \begin{bmatrix}0 & 0\\0 & -1\\0 & 1\end{bmatrix}, \quad C_{2,d}\begin{bmatrix}2.1 & -.6 & 1.2\\-2.1 & .3 & -1.2\end{bmatrix}$$

have been derived from the partition of the matrices A'_{F_2}, B'_2, F'_2 obtained through the state space basis transformation $T_2 = [-e_4\ -e_5\ e_6\ -e_1\ -e_2\ -e_3]$, where e_j, $j = 1, \dots, 6$, denotes the j-th vector of the main basis of $\mathbb{R}^6$.

The numerical results presented above can be better interpreted if referred to Fig. 1a: the precompensator Σ_{c_1} guarantees that a unit pulse at h_1 only affects the output y_2 from the time $t = 4$ on, while the precompensator Σ_{c_2} guarantees that a unit pulse at h_2 does not affect the output y_1. These facts are also illustrated in Fig. 3 and Fig. 4. Figure 3 shows the unit pulse response of Σ with the precompensator Σ_{c_1} : a unit pulse at the block input of Σ_{c_1} (i.e., In(1), In(2), In(3) in the plot) does not change the second block output of Σ (i.e., Out(3), Out(4)) until the time $t = 4$; meanwhile, the unit pulse at the block input of Σ_{c_1} affects the first block output of Σ (i.e., Out(1), Out(2)). Figure 4 shows the unit pulse response of Σ with the precompensator Σ_{c_2} : a unit pulse at the block input of $\Sigma c2$ (i.e., In(1), In(2), In(3) in the plot) does not affect the first block output of Σ (i.e., Out(1), Out(2)), while it affects the second block output (i.e., Out(3), Out(4)).

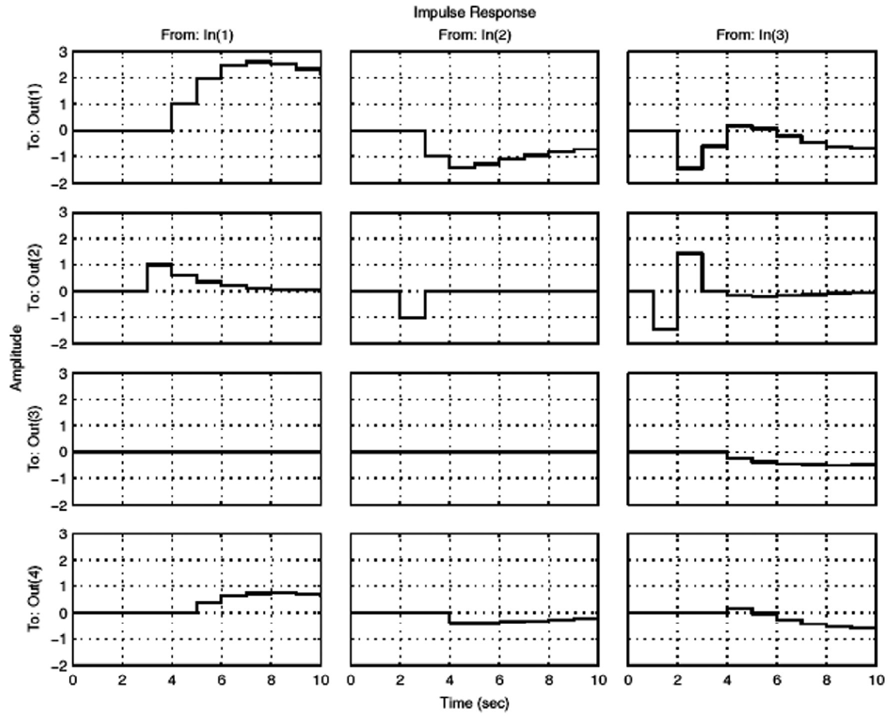

Figure 3: Unit Pulse Response of System Σ With Precompensator Σ_{c_1}

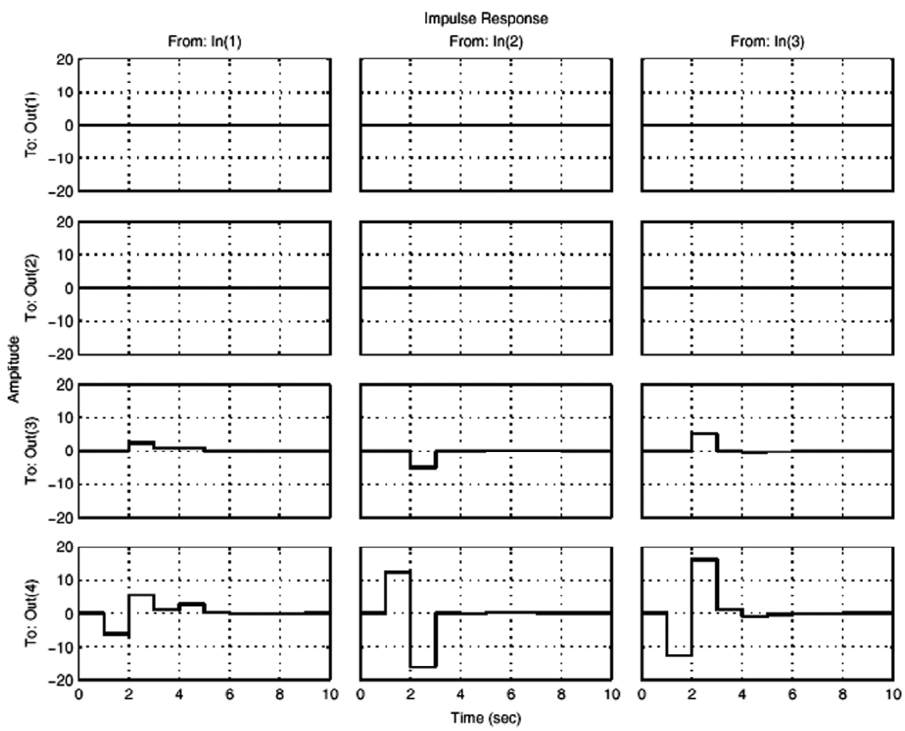

Figure 4: Unit Pulse Response of System Σ With Precompensator Σ_{c_2}

Then, let the fault detection problem be stated for system $\Sigma^\top$, ruled by

$$x(t+1) = A^\top x(t) + C^\top m(t),$$

$$y(t) = B^\top x(t),$$

where the matrices $C_1^\top$, $C_2^\top$ identify the fault input partition (m_1, m_2) shown in Fig. 1b. The residual generators Σ_{r_1} and Σ_{r_2} are respectively derived from Σ_{c_1} and Σ_{c_2} by duality. In the context of fault detection, the properties previously pointed out imply that Σ_{r_1} guarantees that the residual r_1 is affected by m_2 only from the time $t = 4$ on, while Σ_{r_2} ensures that r_2 is not affected by m_1. These facts are illustrated in Fig. 5 and Fig. 6. Figure 5 shows the unit pulse response of Σ with the residual generator Σ_{c_1} : a unit pulse at the second block input of Σ (i.e, In(3), In(4) in the plot) does not change the block output of Σ_{r_1} (i.e, Out(1), Out(2), Out(3)) until the time t=4; meanwhile, the output of Σ_{r_1} is perturbed by a unit pulse at the first block input (i.e, In(1), In(2)). Figure 6 shows the unit pulse response of $\Sigma^\top$ with the residual generator Σ_{r_1} : a unit pulse at the first block input of $\Sigma^\top$ (i.e, In(1), In(2)) does not affect the block output of Σ_{r_2} (i.e., Out(1), Out(2), Out(3)), while the latter one is affected by a unit pulse at the second block input (i.e., In(3), In(4)). Hence, in the overall scheme for fault detection, while detection of the second block fault input is possible in accordance with the standard conditions, detection of the first block

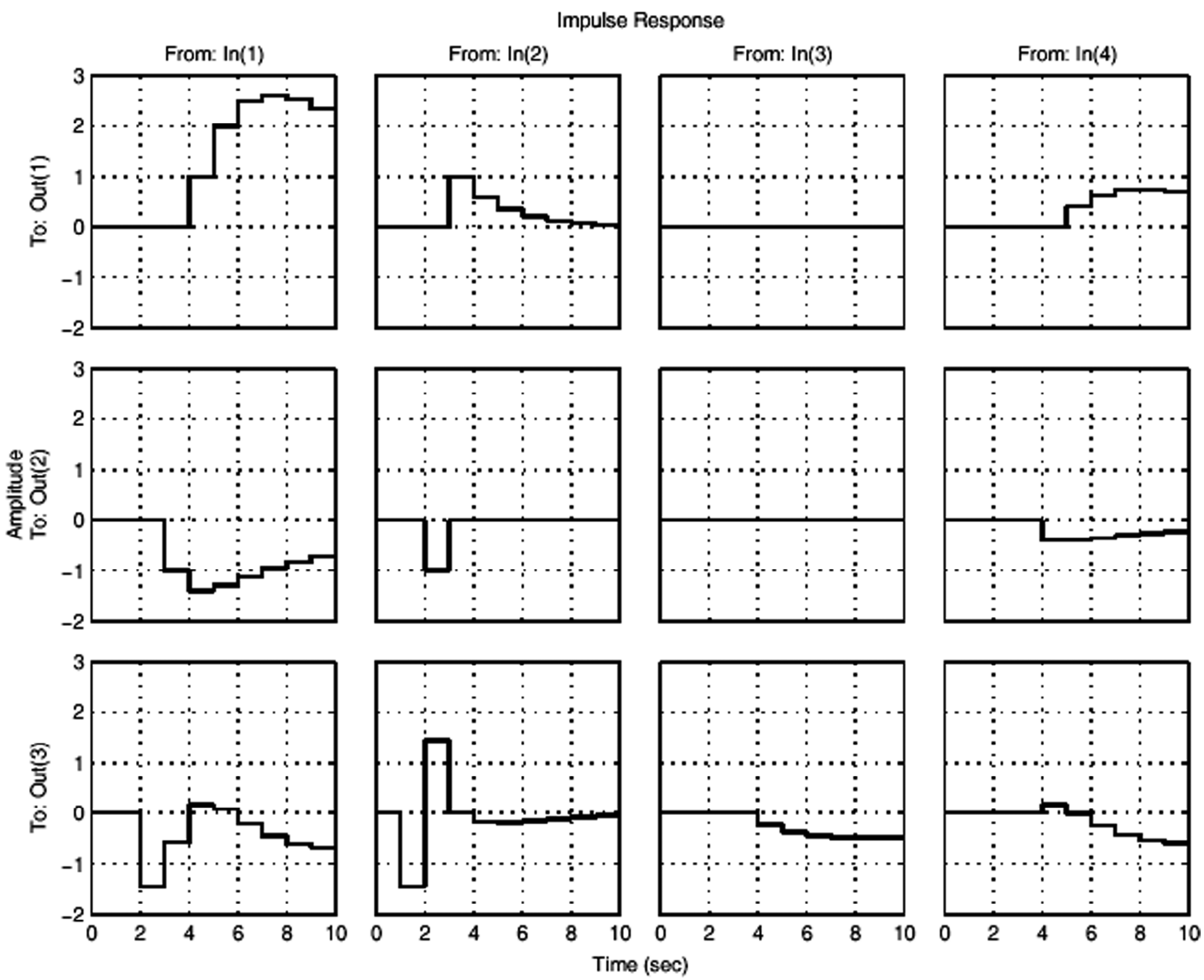

Figure 5: Unit Pulse Response of System Σ with Residual Generator Σ_{r_1}

fault input, which solely satisfies the extended version of the geometric structural conditions, is nonetheless possible by exploiting information referring to the finite time interval until $t = 4$: i.e., the time when the perturbation of the corresponding residual can only be ascribed to the first block fault input.

Consistency of the results respectively obtained for noninteraction and fault detection is evident in the comparison between Fig. 3 and Fig. 5 and between Fig. 4 and Fig. 6, respectively. In fact, if each figure is interpreted as a matrix of graphs, it is easy to see that the matrix of Fig. 5 is the transpose of the matrix of Fig. 3 and, similarly, the matrix of Fig. 6 is the transpose of the matrix of Fig. 4. In other words, the generic graph shown in the subfigure positioned at the i-th row and j-th column of Fig. 3 (Fig. 4) also appears in the subfigure positioned at the j-th row and i-th column of Fig. 5 (Fig. 6).

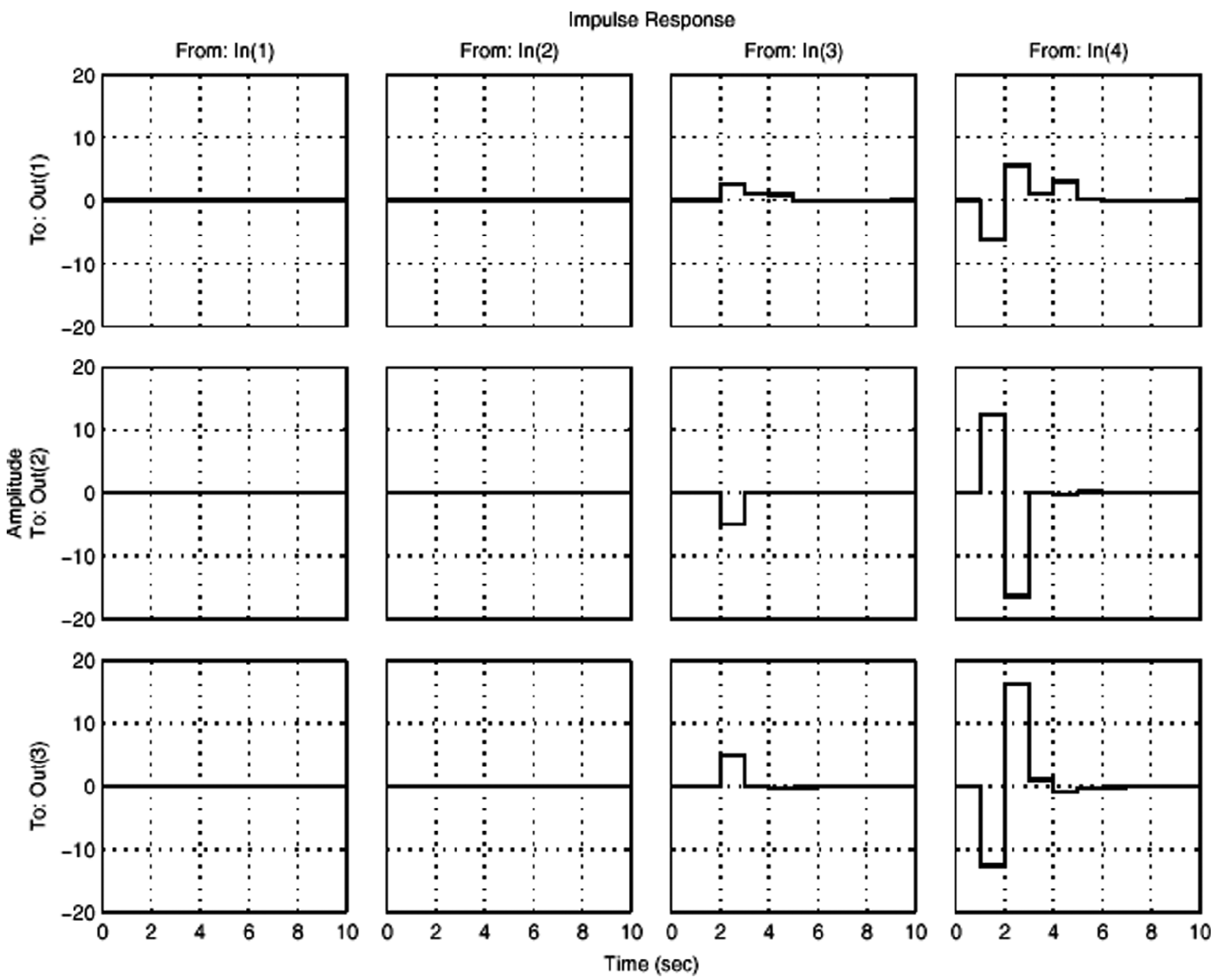

Figure 6: Unit Pulse Response of System $\Sigma^\top$ with Residual Generator Σ_{r_2}

7. CONCLUSION

In this work, the methodological approach to noninteraction based on the properties of fundamental geometric objects like controllability invariant subspaces has been extended to the case where the structural properties of the investigated systems only allow noninteraction to be achieved on finite horizons, suitably defined in connection with the number of steps for the ACSAs associated with the different system subblocks to converge. A necessary and sufficient condition for solvability of the relaxed problem has been proved. The if-part of the proof is constructive, since it directly leads to the procedure for devising the feedforward dynamic units achieving the control objective. A slight modification, consisting in replacing the ACSA with the CSA, leads to a new procedure for devising the feedforward dynamic units which guarantee the standard, infinite horizon, noninteraction for those subblocks of the system for which it is admissible. The transfer of the abovementioned results to the context of fault detection has been commented referring to the, so-called, fundamental problems. A numerical example has illustrated the proposed methodology.

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