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Effect of Various Parameters of RBC on Oxygen Concentration in Tissues

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Abstract: In order to survive, humans have to extract oxygen from the atmosphere and transport it to their cells where it is utilized for essential metabolic processes. Tissues differ in their ability to withstand anoxia (lack of oxygen). Initially a lack of oxygen affects organ function but with time, irreversible damage is done (within minutes in the case of the brain) and revival is impossible. In this paper, we studied the effect of RBC cells on oxygen concentration. It is observed that as the length of RBC, hematocrit and distance between two RBCs get increased; the oxygen concentration in tissue also increases. It has been also investigated that as the velocity of RBC get increased the concentration of oxygen in tissue decreases.

Keywords: Oxygen concentration, RBC cell, Tissue, Microcirculation, Blood flow

1. INTRODUCTION

The coupling of fluid dynamics and biology at the level of the cell is an intensive area of investigation because of its critical role in normal physiology and disease. Today, the fluid mechanics of cells and the effect of flows on biological function are active areas of research, spanning the fields of biophysics, bioengineering, physiology, and biology.

Interest in this field has been driven both by the richness of the fluid-dynamic phenomena observed and by their fundamental biologic importance. Oxygen is transported from the air that we breathe to each cell in the body. Oxygen transport within the body is a multistep processes. First, during respiration, oxygen is transported by convection to the lungs. Next, oxygen passes across the lung capillaries by diffusion and binds to hemoglobin in red blood cells. Then the oxygen in these cells moves throughout the body by convection. Once oxygen reaches the tissues, it diffuses through them and within the cells to the aerobic reactions that produce energy in the form of adenosine tri phosphate (ATP). In the normal lung, the diffusion of oxygen into the blood is very rapid and is complete, even if the cardiac output is increased (exercise) and the blood spends less time in contact with the alveolus. This may not happen when the alveolar capillary network is abnormal (pulmonary disease). However, the ability of the lung to compensate is great and problems caused by poor gas diffusion are a rare cause for hypoxia, except with diseases such as alveolar fibrosis. Blood flow is an integral part of oxygen delivery and the presence of RBCs increases the resistance to flow [10, 11, 14]. Thus, the rheological property of RBCs with varying size is important to know the effect on oxygen concentration in tissue. However, larger RBCs are more deformable than smaller ones. Differences in RBC size are the result of changes in RBC dimensions which attended changes in vessel size, to insure deformation and preserve oxygen concentration efficiency between RBCs and tissue. As a result RBC size has been a central part of the microcirculatory system. A low cardiac output, a low hemoglobin concentration (anemia) or low hemoglobin oxygen saturation will result in an inadequate delivery of oxygen, unless a compensatory change occurs in one of the other factors.

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Alternatively, if oxygen delivery falls relative to oxygen consumption the tissues extract more oxygen from the hemoglobin. In spite of the great importance of oxygen; the stores of oxygen in the body are small and would be unable to sustain life for more than a few minutes. If breathing ceases, oxygen stores are limited to the oxygen in the lung and in the blood. Intracellular energy production, a prerequisite for all physiological cellular reactions, is dependent on continual absorption of oxygen through respiration and transport from the alveoli and eventually into the mitochondria. The enormous quantity of the metabolized oxygen is utilized by the body for energy production as a vital condition for sound cell functions and regenerative processes. Several models of the oxygen exchange through the blood–gas barrier have been published [6, 7, 12]. In all these works, the oxygen partial pressure profile (PO_2) along the pulmonary capillary was determined by means of iterative procedures. In fact, since the mathematical description of the oxygen transport process from the alveoli to the capillaries is complicated by the reversible binding between hemoglobin and oxygen, it is very difficult to determine a closed form for the oxygen partial pressure profile along the pulmonary capillaries. As a consequence, in complex models of the human respiratory system [1, 3, 8] the oxygen partial pressure at the end of the pulmonary capillaries is generally assumed equal to the alveolar oxygen partial pressure profile. We try to derive expressions for the effect of length of RBC, hematocrit, velocity of RBC and distance between two RBCs on oxygen concentration, which will lead toward more physiological explanation of the flow of blood in tissues.

2. MATHEMATICAL MODEL FORMULATION

When oxygen reaches the tissue, equilibrium favors the dissociation of oxygen from the hemoglobin due to the lower oxygen levels and higher concentration of dissolved oxygen levels in tissue. Oxygen transport from the oxygenated red cells to the tissues is not simply the opposite of the process occurring in the lungs. Because the kinetics of dissociation differs from that of binding, and the tissue thickness is much greater than the thickness of the alveolar epithelium and capillary endothelium. Oxygen delivery to tissues involves the dissociation of oxygen from hemoglobin, the diffusion and convection of oxygen through the plasma, diffusion through the cells and tissue, and finally, the reaction of oxygen in the mitochondria as part of aerobic metabolism. As we know that RBCs are indeed a centerpiece in the evolution of the microcirculation. It is considered capillary to be homogeneous for oxygen transport; but, in reality, the capillaries contain discrete red blood cells (RBCs), and this discreteness (Length of RBC, velocity of RBC, Distance between two RBCs) will affect oxygen concentration in the tissue. Physiology of oxygen concentration in tissue is depicted in Fig.1.

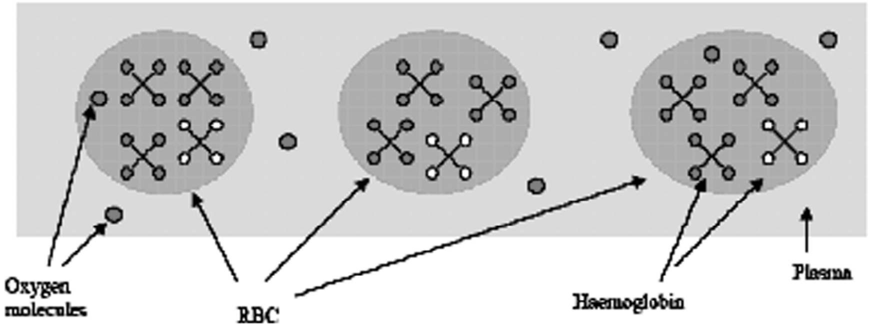


Figure 1: Oxygen Concentration in Tissue

In order to develop a model of oxygen transport to tissues, it is necessary to model a simplified organization of capillaries. The geometry of the problem is depicted in Figure 2.

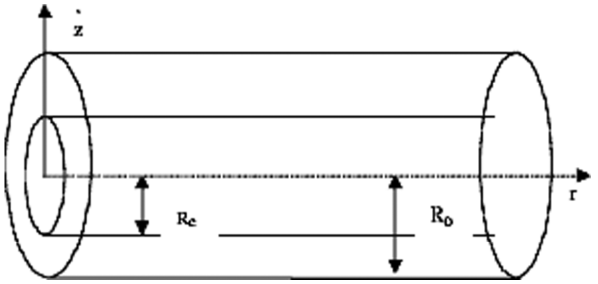


Figure 2: Geometry of the Problem

In the present paper, to consider the oxygen transport in the tissue, Krogh model is considered under the following assumptions [5, 9, 13]:

i. The capillaries are arranged in an ordered array that can be represented by a central capillary surrounded by a cylinder of tissue.

ii. The oxygen –hemoglobin dissociation reaction is at equilibrium and occurs uniformly throughout the blood.

iii. Oxygen release in plasma occurs uniformly.

iv. The mass transfer resistance offered by the cell free fluid layer and endothelium is neglected.

v. Axial diffusion in tissue and blood is negligible.

vi. Oxygen uptake in tissue can be represented by zeroth- order kinetics, and the update is uniform throughout the tissue.

vii. Other reactions of oxygen in tissue are negligible.

Let us consider, a steady state, no convection in the tissue and a mass balance on steady state. The one-dimensional oxygen diffusion and reaction in the tissue yields the following relation [4]:

$$\frac{DO_2}{r}\frac{d}{dr}\left(r\frac{dCO_2}{dr}\right)=RO_2 \tag{1}$$

Although oxygen consumption generally follows Michaelis-Menton kinetics, the reaction rate is assumed to be zero order. This assumption is based on the small value of the Michaelis constant ($K_m = 0.67$ μM to 2.7 μM) relative to average oxygen concentrations in tissue. That is, only when the oxygen concentrations are close to zero, the rate deviate from zeroth –order kinetics.

At the capillary surface, the tissue concentration is in equilibrium with the oxygen concentration in plasma:

$$R = R_c,\ CO_2 = CR_c \tag{2}$$

As is noted subsequently, the oxygen concentration in the capillary is, in general, a function of axial distance along the capillary.

At R_0, however, there is a plane of symmetry between the two cylinders. Thus, the flux is zero:

At $$r = R_0,\ -DO_2\frac{dCO_2}{dr} = 0 \tag{3}$$

The solution of equation (1) is

$$CO_2 = \frac{RO_2 r^2}{4DO_2} + A\ln r + B \tag{4}$$

where, A and B are constants.

When the boundary conditions (from equations (2) & (3)) are applied, the concentration profile in the tissue is given as:

$$\frac{CO_2}{CR_c} = 1 + \frac{RO_2 R_o^2}{4CR_c DO_2}\left[\left(\frac{r^2}{R_0^2} - \frac{R_c^2}{R_o^2}\right) - 2\ln\frac{r}{R_c}\right] \tag{5}$$

The group $\frac{RO_2 R_o^2}{4CR_c DO_2}$ represents a dimensionless reaction rate and is denoted by Ω.

So,

$$\Omega = \frac{RO_2 R_o^2}{4CR_c DO_2} \tag{6}$$

Take $r^* = r/R_0$ and $R^* = R_c/R_0$, equation (5) becomes

$$\frac{CO_2}{CR_c} = 1 + \Omega\left[(r^{*2} - R^{*2}) - 2\ln\frac{r^*}{R_c^*}\right] \tag{7}$$

Now,

$$DO_2 = \frac{kL^2}{\phi^2} \tag{8}$$

From equations (6) and (8), we find $\Omega = \frac{\text{Ø}^2 RO_2 R_0^2}{4kCR_c L^2}$ (9)

The relation between hematocrit, length of RBC and distance between two RBCs [2] is given by

$$Hct = \frac{L}{L + \Delta z} \tag{10}$$

Peclet number is defined as

$$Pe = \frac{v\Delta z}{DO_2} \tag{11}$$

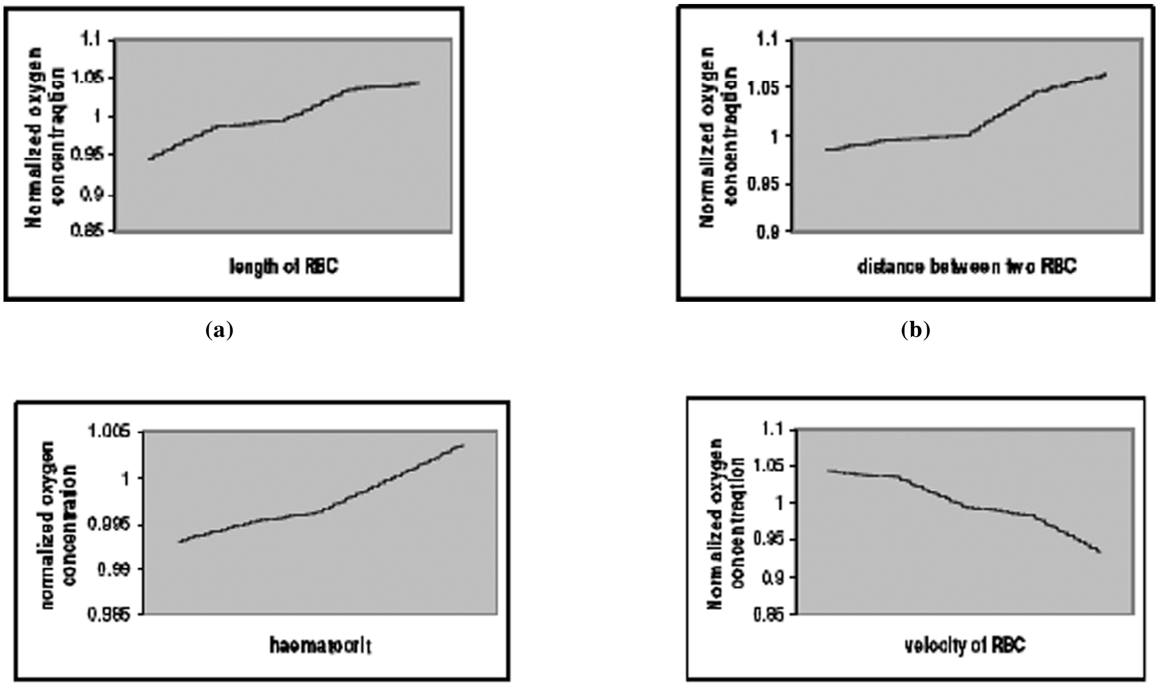
Now, from equations (5), (9), (10) and (11), we observe the effect of length of RBC, distance between two RBCs, hematocrit and velocity of RBC on concentration of oxygen.

3. DISCUSSION

It is found that RBCs are a primitive character in blood flow. Thus, we are interested to find the link between RBC size and oxygen concentration in tissue. We hypothesize that RBC size effects the oxygen concentration in tissue. An analytical solution is used to consider the effect of RBC on oxygen concentration from the capillary to the tissue.

In order to get a physiological insight into the effect of length of RBC, distance between two RBCs, hematocrit and velocity of RBC on concentration of oxygen, the following physiological values are taken:

$$r^* = r/R_0 = 0.10,\ R^* = R_c/R_0 = 0.05,\ Ø = 1,\ k = 128\ \text{sec}^{-1},\ D0_2 = 6 \times 10^{-6}\ \text{cm}^2\ \text{sec}^{-1}$$
$$\Omega = 0.01$$



(a) (b)

(c) (d)

Figure 3 (a), (b), (c), (d): Represents the Variation of Normalized Concentration of Oxygen against Length of RBC, Distance Between Two RBCs and Hematocrit and Velocity of RBC Respectively

We found a positive correlation between length of RBC, distance between two RBCs, hematocrit and the oxygen concentration in tissue. But we found a negative correlation between velocity of RBC and the concentration of oxygen in tissue. From our study, we observe that as the length of RBC, distance between two RBCs and hematocrit is increased, the oxygen concentration in tissue also increases (Fig. 3(a), Fig. 3 (b), Fig. 3(c)). Also as the velocity of RBC increases, the concentration of oxygen in tissue decreases (Fig. 3(d)).

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