Journal of Mathematical Control Science and Applications (JMCSA)
Vol. 3 No. 2 (July-December, 2017), ISSN : 0974-0570

Exponential Stability for a Kind of Delayed Neural Networks with Unbounded Activation Functions

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Abstract: Without assumption of the boundedness of the activation functions, using Lyapunov-Krasovskii functional, relationships among the state vectors of the neural networks together with homeomorphism map, a linear matrix inequality (LMI) approach is developed to derive a new delay-dependent sufficient condition for (i) existence (ii) uniqueness and (iii) global exponential stability of equilibrium point, of a class of time-varying delay neural networks. Examples are provided to demonstrate the reduced conservativeness and effectiveness of the proposed result.

Keywords: Global exponential stability, Time-varying delay, Linear matrix inequality, Equilibrium point.

MR(2000) Subject Classification: 34D 40

1. INTRODUCTION

Neural networks have aroused a tremendous surge of investigation in these years[1-6, 8-31]. Due the finite switching speed of neurons and amplifiers, time delays inevitably exist in biological and artificial neural networks. A time delay in the response of a neuron can influence the stability of a network and deteriorate the dynamical performance creating oscillatory and unstable characteristics. One of the most investigated problems for the time delay neural networks (DNNs) is that of the existence, uniqueness, and global exponential or asymptotic stability of the equilibrium of DNNs.

To embed and solve many problems in applications of neural networks to parallel computations, signal processing and other problems involving the optimization, the dynamic neural networks have to be designed to have only a unique equilibrium point which is global asymptotic stable or global exponential stable to avoid the risk of spurious responses or the problem of local minima. In fact, earlier applications of neural networks of optimization problems have suffered from the existence of a complicated set of equilibria [2]. Thus, the global exponential stability of a unique equilibrium for DNNs is of great importance from a theoretical and an application point of view in several fields. Thus, the primary purpose in this paper is to establish a new sufficient condition ensuring that a class of neural networks with time-varying delay has a unique equilibrium which is global exponentially stable.

Some existing results on existence, uniqueness and global asymptotic stability or global exponential stability of the equilibrium concern the case where the activation functions are all bounded and strictly increasing. These assumptions make the results inapplicable to some important engineering problems. When neural networks are designed for solving optimization problems in the presence of constraints (linear, quadratic, or more general programming problems), unbounded activations modelled by diode-like exponential-type functions are needed to impose constraints satisfaction. Different from the bounded case where the existence of an equilibrium point

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is always guaranteed [3], for unbounded activations, it may happen that there are no equilibrium points [4]. When considering the widely employed piecewise linear neural networks [5], infinite intervals with zero slope are present in activations, making it of interest to drop the assumptions of strict increase and continuous first derivative for the activation. Forti and Tesi [4] studied the global asymptotic stability for neural networks with unbounded monotonic activation function. In many electronic circuits, amplifiers, which have neither monotonically increasing nor continuously differentiable input-output functions, are frequently adopted. Therefore, it seems that for some purposes, non-monotonic(and not necessarily smooth) functions might be better candidates for neuron activation in designing and implementing an artificial neural network. Xu and Lam [6] obtained a new delay-independent exponential stability condition for neural networks with time-varying delays. But they assumed the activation functions are bounded and globally Lipschtz continuous. The stability criterion in [6] requires that the derivative of the delay function $\tau(t)$ is less than 1, which is very restrictive.

In this paper, we develop a new delay-dependent exponential stability condition for a class of neural networks with time-varying delay by utilizing Lyapunov functional. We only assume that the activation functions are globally Lipschitz continuous. Under this assumption, both the existence of a unique equilibrium point and the global exponential stability of a given delayed neural networks are proved. The derived condition is expressed in term of a linear matrix inequality (LMI), which can be checked numerically very efficiently by resorting to recently developed standard algorithms such as interior-point methods, and no tuning of parameters will be involved [7]. Furthermore, our delay-dependent stability criterion removes the unreasonable restriction that the derivative of the delay function $\tau(t)$ is less than 1. Examples are provided to demonstrate the reduced conservatism and effectiveness of the proposed condition.

Notation: Through this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (or $X > Y$, respectively) means that the matrix $X - Y$ is positive semi-definite (or positive definite, respectively). The superscript T represents the transpose. The notation $\|\cdot\|$ refers to the Euclidean vector norm. R^n denotes n-dimensional Euclidean space. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions. In symmetric block matrices or long matrix expressions, we use * to represent a term that is induced by symmetry.

2. PRELIMINARIES

The neural networks with time-varying delay can be described by the following delay differential equation

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + Wg(x(t)) + W_1 g(x(t-\tau(t))) + u, \\ x(t) &= \phi(t), \quad t \in [-2h, 0], \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T$ is the neuron state vector, $g(x(t)) = [g_1(x_1(t)), \ldots, g_n(x_n(t))]^T$ is the activation function vector, and $u = [u_1, \ldots, u_n]^T$ is a constant external input vector. In the neural networks (1), matrices $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$, where the scalar $a_i > 0$ is the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t. Matrices W and W_1 are the connection weighting matrix and the delayed weighting matrix, respectively. $\tau(t)$ is a continuous function describing the time-varying transmission delays in the neural networks and satisfies $0 \leq \tau(t) \leq h$, $\dot{\tau}(t) \leq \mu$ for all $t \geq 0$, with h and μ being two nonnegative constants. The initial condition function $\phi(t) : [-2h, 0] \rightarrow R^n$ is assumed to be a continuous function. Throughout this paper, it is assumed that each neuron activation function g_j satisfies the following assumption:

Assumption 1:

$$|g_j(s) - g_j(t)| \leq k_j\, |s - t|, \ \forall s, t \in R, j = 1, \ldots, n. \tag{2}$$

Denote $K = \mathrm{diag}(k_1, \dots, k_n)$, obviously, $K \geq 0$.

Definition 1: The vector $x^* = [x_1^*, \dots, x_n^*]^T$ is said to be an equilibrium point of the DNN in Eqn (1) if $-Ax^* + Wg(x^*) + W_1 g(x^*) + u = 0$.

Definition 2: DNN in Eqn (1) is said to be exponentially stable if there is scalars $\alpha > 0$ and $\gamma > 0$ such that

$$|x(t)| \leq \gamma e^{-\alpha t} \sup_{-2h \leq \theta \leq 0} |x(\mu)|.$$

Definition 3: A map $H : R^n \to R^n$ is a homeomorphism of R^n onto itself, if $H \in C^0$, H is one-to-one, H is onto and the inverse map $H^{-1} \in C^0$, where C^0 represents the set of all continuous functions from R^n to R^n.

Since each neuron activation function g_i may not be bounded, the equilibrium point of the DNN in Eqn (1) may not exist. In order to analyze the global exponential stability of the DNN in Eqn (1), we will have to first prove the existence and uniqueness of the equilibrium point using the following lemma.

Lemma 1: If $H(x) \in C^0$ and satisfies the following conditions.

1. $H(x)$ is injective on R^n,

2. $\lim_{\|x\| \to +\infty} \|H(x)\| = +\infty$.

then $H(x)$ is a homeomorphism of R^n [8].

3. MAIN RESULT

Theorem 1: For given scalars $h > 0$ and $\mu \geq 0$, the delayed neural networks in (1) satisfying Assumption 1 has the unique equilibrium point which is globally exponentially stable for any delay $0 < \tau(t) \leq h$ if there exist matrices $D > 0$, $P > 0$, $Q > 0$, $R > 0$, P_k, $k = 1, 2, \dots, 6$, T_i, $i = 1, 2, \dots, 5$ and diagonal matrices $S_j > 0$ and $Y_j > 0$, $j = 1, 2$ such that the following LMI holds:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & P_1W_1 - A^TP_4^T + T_4^T & P - P_1 - A^TP_5^T + T_5^T & -A^TP_6^T - T_1 \\ * & \Phi_{22} & \Phi_{23} & S_2 + P_2W_1 - T_4^T & -P_2 - T_5^T & -T_2 \\ * & * & \Phi_{33} & P_3W_1 + W^TP_4^T & -P_3 + W^TP_5^T & W^TP_6^T - T_3 \\ * & * & * & -(1-\mu)Q - Y_2 + 2P_4W_1 & -P_4 + W_1^TP_5^T & W_1^TP_6^T - T_4 \\ * & * & * & * & hD - 2P_5 & -P_6^T - T_5 \\ * & * & * & * & * & -\frac{D}{h} \end{bmatrix} < 0 \tag{3}$$

where

$$\begin{aligned}
\Phi_{11} &= R + KY_1K + 2KS_1 - 2P_1A + 2T_1, \\
\Phi_{12} &= -A^TP_2^T - T_1 + T_2^T, \\
\Phi_{13} &= S_1 + P_1W - A^TP_3^T + T_3^T, \\
\Phi_{22} &= -(1-\mu)R + KY_2K + 2KS_2 - 2T_2, \\
\Phi_{23} &= P_2W - T_3^T, \\
\Phi_{33} &= Q - Y_1 + 2P_3W.
\end{aligned}$$

Proof: Denote

$$H(x) = -Ax + Wg(x) + W_1 g(x) + u. \tag{4}$$

In the rest of this Section, we prove this new sufficient condition in Theorem 1 which guarantees the existence, uniqueness and global exponential stability of the equilibrium point of the DNN given in Eqn (1) by three steps.

1. First we prove that $H(x)$ is an injective map on R^n.
2. We then prove that $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$.
3. Finally we prove that the unique equilibrium point x^* obtained from Steps 1 and 2 is globally exponentially stable.

Denote $\zeta(x, y) = [(x-y)^T, (x-y)^T, (g(x)-g(y))^T, (g(x)-g(y))^T, (H(x)-H(y))^T, 0]^T$. Then

$$\begin{aligned}
&\zeta(x, y)^T \Phi \zeta(x, y)\\
&= (x-y)^T [R + KY_1K + 2KS_1 - 2P_1A + 2T_1 - 2A^TP_2^T - 2T_1 + 2T_2^T\\
&\quad -(1-\mu)R + KY_2K + 2KS_2 - 2T_2](x-y)\\
&\quad +2(x-y)^T [S_1 + P_1W - A^TP_3^T + T_3^T + P_1W_1 - A^TP_4^T + T_4^T + P_2W - T_3^T + S_2\\
&\quad +P_2W_1 - T_4^T](g(x) - g(y)) + 2(x-y)^T [P - P_1 - A^TP_5^T + T_5^T - P_2 - T_5^T](H(x) - H(y))\\
&\quad +(g(x) - g(y))^T [Q - Y_1 + 2P_3W + 2P_3W_1 + 2W^TP_4^T - (1-\mu)Q - Y_2 + 2P_4W_1](g(x)\\
&\quad - g(y)) + 2(g(x) - g(y))^T [-P_3 + W^TP_5^T - P_4 + W_1^T P_5^T](H(x) - H(y))\\
&\quad +(H(x) - H(y))^T [hD - 2P_5](H(x) - H(y))\\
&= (x-y)^T [\mu R + K(Y_1 + Y_2)K + 2K(S_1 + S_2) - 2(P_1 + P_2)A](x-y)\\
&\quad +2(x-y)^T [S_1 + S_2 - A^T (P_3^T + P_4^T)](g(x) - g(y))\\
&\quad +2(x-y)^T (P_1 + P_2)[(H(x) - H(y)) + A(x-y)]\\
&\quad +2(x-y)^T [P - P_1 - P_2 - A^TP_5^T](H(x) - H(y))\\
&\quad +(g(x) - g(y))^T [\mu Q - Y_1 - Y_2](g(x) - g(y))\\
&\quad +2(g(x) - g(y))^T (P_3 + P_4)[(H(x) - H(y)) + A(x-y)]\\
&\quad +2(g(x) - g(y))^T [-P_3 - P_4](H(x) - H(y))\\
&\quad +2((H(x) - H(y)) + A(x-y))^TP_5^T (H(x) - H(y))\\
&\quad +(H(x) - H(y))^T [hD - 2P_5](H(x) - H(y))\\
&= (x-y)^T [\mu R + K(Y_1 + Y_2)K + 2K(S_1 + S_2)](x-y)\\
&\quad +2(x-y)^T [S_1 + S_2](g(x) - g(y)) + 2(x-y)^TP(H(x) - H(y))\\
&\quad +(g(x) - g(y))^T [\mu Q - Y_1 - Y_2](g(x) - g(y)) + h(H(x) - H(y))^TD(H(x) - H(y)).
\end{aligned} \tag{5}$$

Step 1: We prove that $H(x)$ is an injective map on R^n by showing that assuming otherwise leads to a contradiction.

We suppose that vectors x, y exist in R^n such that $x \neq y$ while $H(x) = H(y)$, then $x-y = (x_1 - y_1, \dots, x_n - y_n) \neq 0$, $A(x-y) = (W + W_1)(g(x)-g(y))$ and $H(x)-H(y) = 0$.

From Inequality (3), we have

$$\zeta(x, y)^T \Phi \zeta(x, y) < 0. \tag{6}$$

On the other hand, from Eqn (5), we have

$$\zeta(x, y)^T \Phi \zeta(x, y)$$

$$
\begin{aligned}
&= (x-y)^T[\mu R + K(Y_1+Y_2)K + 2K(S_1+S_2)](x-y) \\
&\quad +2(x-y)^T[S_1+S_2](g(x)-g(y)) + 2(x-y)^T P(H(x)-H(y)) \\
&\quad +(g(x)-g(y))^T[\mu Q - Y_1 - Y_2](g(x)-g(y)) + h(H(x)-H(y))^T D(H(x)-H(y)) \\
&= (x-y)^T[\mu R + K(Y_1+Y_2)K + 2K(S_1+S_2)](x-y) \\
&\quad +2(x-y)^T[S_1+S_2](g(x)-g(y)) \\
&\quad +(g(x)-g(y))^T[\mu Q - Y_1 - Y_2](g(x)-g(y)).
\end{aligned} \tag{7}
$$

Since matrices $S_i \geq 0$ and $Y_i \geq 0$, $i = 1, 2$ are diagonal, from Inequality (2), we have

$$(x-y)^T S_i(g(x)-g(y)) \leq (x-y)^T K S_i(x-y), \tag{8}$$

$$-(x-y)^T S_i(g(x)-g(y)) \leq (x-y)^T K S_i(x-y), \tag{9}$$

$$(g(x)-g(y))^T Y_i(g(x)-g(y)) \leq (x-y)^T K Y_i K(x-y). \tag{10}$$

Noting that $\mu \geq 0$, $R > 0$, and $Q > 0$ and Inequalities (8)-(10), we have

$$
\begin{aligned}
&(x-y)^T[\mu R + K(Y_1+Y_2)K + 2K(S_1+S_2)](x-y) \\
&+2(x-y)^T[S_1+S_2](g(x)-g(y)) \\
&+(g(x)-g(y))^T[\mu Q - Y_1 - Y_2](g(x)-g(y)) \\
\geq\ & 0.
\end{aligned} \tag{11}
$$

So,

$$\zeta(x, y)^T \Phi \zeta(x, y) \geq 0. \tag{12}$$

which contradicts Inequality (6), and hence implies that $H(x)$ is an injective map on R^n.

Step 2: We prove that

$$\lim_{\|x\|\to+\infty} \|H(x)\| = +\infty .$$

Since $H(x) = -Ax + [W + W_1]g(x) + u$ and u a constant external input vector, $H(0) = [W + W_1]g(0) + u$ is a constant vector. It suffices to show that

$$\lim_{\|x\|\to+\infty} \|\tilde{H}(x)\| = +\infty$$

where $\tilde{H}(x) = H(x) - H(0)$.

Since $\zeta(x, 0) = [x^T, x^T, (g(x)-g(0))^T, (g(x)-g(0))^T, \tilde{H}(x)^T, 0]^T$, from Inequality (3), there exist a small positive real number $a > 0$ such that

$$
\zeta(x,0)^T
\begin{bmatrix}
\Phi_{11}+aI & \Phi_{12} & \Phi_{13} & P_1W_1 - A^T P_4^T + T_4^T & P - P_1 - A^T P_5^T + T_5^T & -A^T P_6^T - T_1 \\
* & \Phi_{22} & \Phi_{23} & S_2 + P_2W_1 - T_4^T & -P_2 - T_5^T & -T_2 \\
* & * & \Phi_{33} & P_3W_1 + W^T P_4^T & -P_3 + W^T P_5^T & W^T P_6^T - T_3 \\
* & * & * & -(1-\mu)Q - Y_2 + 2P_4W_1 & -P_4 + W_1^T P_5^T & W_1^T P_6^T - T_4 \\
* & * & * & * & hD - 2P_5 & -P_6^T - T_5 \\
* & * & * & * & * & -\frac{D}{h}
\end{bmatrix}
\zeta(x,0) \leq 0. \tag{13}
$$

So

$$\zeta(x, 0)^T \Phi \zeta(x, 0) \le -a x^T x. \tag{14}$$

Noting $\tilde{H}(x) = H(x) - H(0)$ in Eqn (5), we get

$$\begin{aligned}\zeta(x, 0)^T \Phi \zeta(x, 0) &= x^T [\mu R + K(Y_1 + Y_2)K + 2K(S_1 + S_2)]x \\ &\quad + 2x^T [S_1 + S_2](g(x) - g(0)) + 2x^T P \tilde{H}(x) \\ &\quad + (g(x) - g(0))^T [\mu Q - Y_1 - Y_2](g(x) - g(0)) + h\tilde{H}(x)^T D \tilde{H}(x).\end{aligned} \tag{15}$$

Replacing y with 0 in Inequality (11) yields

$$\begin{aligned}& x^T [\mu R + K(Y_1 + Y_2)^K + 2K(S_1 + S_2)]x \\ & + 2x^T [S_1 + S_2](g(x) - g(0)) \\ & + (g(x) - g(0))^T [\mu Q - Y_1 - Y_2](g(x) - g(0)) \\ & \ge 0.\end{aligned} \tag{16}$$

Obviously,

$$h\tilde{H}(x)^T D \tilde{H}(x) \ge 0. \tag{17}$$

Combing Eqn (5)with Inequalities (16) and (17) gives that

$$2x^T P \tilde{H}(x) \le -a x^T x. \tag{18}$$

Since P is positive definite matrix, we have

$$\begin{aligned} a x^T x &\le |2\tilde{H}(x)^T P x| \\ &\le 2\lambda_{\max}(P) \|\tilde{H}(x)\| \, \|x\|,\end{aligned} \tag{19}$$

i.e.,

$$\|\tilde{H}(x)\| \ge \frac{a}{2\lambda_{\max}(P)} \|x\|. \tag{20}$$

Therefore,

$$\lim_{\|x\| \to +\infty} \|\tilde{H}(x)\| = +\infty$$

From Steps 1 and 2, the map $H(x)$ is a homeomorphism of R^n. Thus the DNN given in Eqn (1) has a unique equilibrium point. We denote the unique equilibrium point by x^*.

In order to simplify our proof of this theorem, we transform the DNN given in Eqn (1) to a new DNN using translation, and consider the global exponential stability of the transformed DNN. From Steps 1 and 2, existence and uniqueness of an equilibrium point x^* are guaranteed for the DNN given in Eqn (1). We shift the equilibrium point x^* of the DNN given in Eqn (1) to the origin 0, using the transformation $z(t) = x(t) - x^*$.

We can now put the DNN given in Eqn (1) into the following form:

$$\dot{z}(t) = -Az(t) + Wf(z(t)) + W_1 f(z(t - \tau(t))) \tag{21}$$

$$z(t) = \Phi(t) - x^*, \qquad t \in [-2h, 0]$$

where $z(t) = [z_1(t), \dots, z_n(t)]^T$, $f(z(t)) = [f_1(z_1(t)), \dots, f_n(z_n(t))]^T$, $f_j(z_j(t)) = g_j(z_j(t) + x_j^*) - g_j(x_j^*)$, $j = 1, \dots, n$. Note that the functions $f_j(z_j(t))$ satisfy the following conditions:

$$|f_j(s)| \leq k_j |s|, f_j(0) = 0, \quad \forall s \in R, j = 1, \dots, n. \tag{22}$$

If the vector $z^* = [z_1^*, \dots, z_n^*]^T$ is an equilibrium point of the DNN in Eqn (21), then $-Az^*+Wf(z^*)+W_1f(z^*) = 0$. Since the above transformation is a translation, the existence, uniqueness and stability properties of the DNN given in Eqn (1) and the transformed DNN are identical. The origin is the unique equilibrium point of the DNN in Eqn (21). Hence, the globally exponential stability of unique equilibrium point for the DNN given in Eqn (1) is equivalent to the globally exponential stability of origin of transformed DNN given in Eqn (21).

By construction, the origin is the unique equilibrium points of the DNN given in Eqn (21). Next, we will prove the origin is exponential stable.

Step 3: We choose a Lyapunov-Krasovskii functional candidate for this DNN with time-varying delay such that:

$$V(z_t) = z^T(t)Pz(t) + \int_{t-\tau(t)}^{t} [z^T(s)Rz(s) + f(z(s))^T Qf(z(s))]ds$$

$$+\int_{-h}^{0}\int_{t+r}^{t} y(s)^T Dy(s)ds\, dr \tag{23}$$

where $y(s) = \dot{z}(s)$, $D > 0$, $P > 0$, $Q > 0$ and $R > 0$.

Noting $\dot{\tau}(t) \leq \mu$ and

$$\begin{aligned}
-z^T(t)S_1f(z(t)) &\leq z^T(t)KS_1z(t),\\
f(z(t))^T Y_1f(z(t)) &\leq z^T(t)KY_1Kz(t),\\
-z^T(t-\tau(t))S_2f(z(t-\tau(t))) &\leq z^T(t-\tau(t))KS_2z(t-\tau(t)),\\
f(z(t-\tau(t)))^T Y_2f(z(t-\tau(t))) &\leq z^T(t-\tau(t))KY_2Kz(t-\tau(t)),
\end{aligned}$$

calculating the derivative of $V(z_t)$ along the solution of Eqn (21) yields

$$\begin{aligned}
\dot{V}(z_t) =\ & 2z^T(t)P\dot{z}(t) + z^T(t)Rz(t) - (1-\dot{\tau}(t))z^T(t-\tau(t))Rz(t-\tau(t))\\
&+f(z(t))^TQf(z(t)) - (1-\dot{\tau}(t))f(z(t-\tau(t)))^TQf(z(t-\tau(t)))\\
&+hy(t)^TDy(t) - \int_{t-h}^{t} y(s)^T Dy(s)ds\\
\leq\ & 2z^T(t)Py(t) + z^T(t)Rz(t) - (1-\mu)z^T(t-\tau(t))Rz(t-\tau(t))\\
&+f(z(t))^TQf(z(t)) - (1-\mu)f(z(t-\tau(t)))^TQf(z(t-\tau(t)))\\
&+hy(t)^TDy(t) - \int_{t-\tau(t)}^{t} y(s)^T Dy(s)ds\\
&+2z^T(t)S_1f(z(t)) + f(z(t))^T Y_1f(z(t))\\
&-2z^T(t)S_1f(z(t)) - f(z(t))^T Y_1f(z(t))\\
&+2z^T(t-\tau(t))S_2f(z(t-\tau(t))) + f(z(t-\tau(t)))^T Y_2f(z(t-\tau(t)))\\
&-2z^T(t-\tau(t))S_2f(z(t-\tau(t))) - f(z(t-\tau(t)))^T Y_2f(z(t-\tau(t)))\\
\leq\ & 2z^T(t)Py(t) + z^T(t)Rz(t) - (1-\mu)z^T(t-\tau(t))Rz(t-\tau(t))\\
&+f(z(t))^TQf(z(t)) - (1-\mu)f(z(t-\tau(t)))^TQf(z(t-\tau(t)))\\
&+hy(t)^TDy(t) - \int_{t-\tau(t)}^{t} y(s)^T Dy(s)ds
\end{aligned}$$

$$+2z^T(t)S_1f(z(t)) + z(t)^TKY_1Kz(t)$$
$$+2z^T(t)KS_1z(t) - f(z(t))^T Y_1f(z(t))$$
$$+2z^T(t-\tau(t))S_2f(z(t-\tau(t))) + z(t-\tau(t))^TKY_2Kz(t-\tau(t))$$
$$+2z^T(t-\tau(t))KS_2z(t-\tau(t)) - f(z(t-\tau(t)))^T Y_2f(z(t-\tau(t)))$$

$$=\xi(t)^T\begin{bmatrix} R+KY_1K+2KS_1 & 0 & S_1 & 0 & P \\ * & -(1-\mu)R+KY_2K+2KS_2 & 0 & S_2 & 0 \\ * & * & Q-Y_1 & 0 & 0 \\ * & * & * & -(1-\mu)Q-Y_2 & 0 \\ * & * & * & * & hD \end{bmatrix}\xi(t)$$

$$-\int_{t-\tau(t)}^{t} y(s)^T Dy(s)ds$$

$$\le\xi(t)^T\begin{bmatrix} R+KY_1K+2KS_1 & 0 & S_1 & 0 & P \\ * & -(1-\mu)R+KY_2K+2KS_2 & 0 & S_2 & 0 \\ * & * & Q-Y_1 & 0 & 0 \\ * & * & * & -(1-\mu)Q-Y_2 & 0 \\ * & * & * & * & hD \end{bmatrix}\xi(t)$$

$$-\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\tau(t)y(s)^T\frac{D}{h}\tau(t)y(s)ds \tag{24}$$

where $\zeta(t) = [z^T(t), z^T(t-\tau(t)), f(z(t))^T, f(z(t-\tau(t)))^T, y^T(t)]^T$.

Since

$$2[z^T(t)P_1 + z^T(t-\tau(t))P_2 + f(z(t))^TP_3 + f(z(t-\tau(t)))^TP_4 + y^T(t)P_5$$
$$+\int_{t-\tau(t)}^{t} y(s)^T dsP_6][-y(t) - Az(t) + Wf(z(t)) + W_1f(z(t-\tau(t)))] = 0, \tag{25}$$

$$2\xi(t)^T\begin{bmatrix} -P_1A & 0 & P_1W & P_1W_1 & -P_1 \\ -P_2A & 0 & P_2W & P_2W_1 & -P_2 \\ -P_3A & 0 & P_3W & P_3W_1 & -P_3 \\ -P_4A & 0 & P_4W & P_4W_1 & -P_4 \\ -P_5A & 0 & P_5W & P_5W_1 & -P_5 \end{bmatrix}\xi(t)+2\xi(t)^T\begin{bmatrix} -A^TP_6^T \\ 0 \\ W^TP_6^T \\ W_1^TP_6^T \\ -P_6^T \end{bmatrix}\int_{t-\tau(t)}^{t} y(s)ds=0, \tag{26}$$

i.e.,

$$\xi(t)^T\begin{bmatrix} -P_1A-A^TP_1^T & -A^TP_2^T & P_1W-A^TP_3^T & P_1W_1-A^TP_4^T & -P_1-A^TP_5^T \\ -P_2A & 0 & P_2W & P_2W_1 & -P_2 \\ -P_3A+W^TP_1^T & W^TP_2^T & P_3W+W^TP_3^T & P_3W_1+W^TP_4^T & -P_3+W^TP_5^T \\ -P_4A+W_1^TP_1^T & W_1^TP_2^T & P_4W+W_1^TP_3^T & P_4W_1+W_1^TP_4^T & -P_4+W_1^TP_5^T \\ -P_5A-P_1^T & -P_2^T & P_5W-P_3^T & P_5W_1-P_4^T & -P_5-P_5^T \end{bmatrix}\xi(t)$$

$$+2\xi(t)^T \begin{bmatrix} -A^T P_6^T \\ 0 \\ W^T P_6^T \\ W_1^T P_6^T \\ -P_6^T \end{bmatrix} \int_{t-\tau(t)}^{t} y(s)ds = 0. \tag{27}$$

Similarly,

$$\begin{aligned} &2[z^T(t)T_1 + z^T(t-\tau(t))T_2 + f(z(t))^T T_3 + f(z(t-\tau(t)))^T T_4 \\ &+ y^T(t)T_5][z(t) - z(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds] = 0, \end{aligned} \tag{28}$$

so

$$\xi(t)^T \begin{bmatrix} T_1 + T_1^T & -T_1 + T_2^T & T_3^T & T_4^T & T_5^T \\ T_2 - T_1^T & -T_2 - T_2^T & -T_3^T & -T_4^T & -T_5^T \\ T_3 & -T_3 & 0 & 0 & 0 \\ T_4 & -T_4 & 0 & 0 & 0 \\ T_5 & -T_5 & 0 & 0 & 0 \end{bmatrix} \xi(t)$$

$$+2\xi(t) \begin{bmatrix} -T_1 \\ -T_2 \\ -T_3 \\ -T_4 \\ -T_5 \end{bmatrix} \int_{t-\tau(t)}^{t} y(s)ds = 0. \tag{29}$$

Combining Equalities (27) with Equalities (29) gives

$$\xi(t)^T \begin{bmatrix} -2P_1A + 2T_1 & -A^T P_2^T - T_1 + T_2^T & P_1W - A^T P_3^T + T_3^T & P_1W_1 - A^T P_4^T + T_4^T & -P_1 - A^T P_5^T + T_5^T \\ * & -2T_2 & P_2W - T_3^T & P_2W_1 - T_4^T & -P_2 - T_5^T \\ * & * & 2P_3W & P_3W_1 + W^T P_4^T & -P_3 + W^T P_5^T \\ * & * & * & 2P_4W_1 & -P_4 + W_1^T P_5^T \\ * & * & * & * & -2P_5 \end{bmatrix} \xi(t)$$

$$+2\xi(t)^T \begin{bmatrix} -A^T P_6^T - T_1 \\ -T_2 \\ W^T P_6^T - T_3 \\ W_1^T P_6^T - T_4 \\ -P_6^T - T_5 \end{bmatrix} \int_{t-\tau(t)}^{t} y(s)ds = 0. \tag{30}$$

From Inequality (24) and Eqn (30), we have

$$V(z_t) \le \xi(t)^T \begin{bmatrix} R+KY_1K+2KS_1-2P_1A+2T_1 & -A^TP_2^T-T_1+T_2^T & S_1+P_1W-A^TP_3^T+T_3^T & P_1W_1-A^TP_4^T+T_4^T & P-P_1-A^TP_5^T+T_5^T \\ * & -(1-\mu)R+KY_2K+2KS_2-2T_2 & P_2W-T_3^T & S_2+P_2W_1-T_4^T & -P_2-T_5^T \\ * & * & Q-Y_1+2P_3W & P_3W_1+W^TP_4^T & -P_3+W^TP_5^T \\ * & * & * & -(1-\mu)Q-Y_2+2P_4W_1 & -P_4+W_1^TP_5^T \\ * & * & * & * & hD-2P_5 \end{bmatrix} \xi(t) + 2\xi(t)^T \begin{bmatrix} -A^TP_6^T-T_1 \\ -T_2 \\ W^TP_6^T-T_3 \\ W_1^TP_6^T-T_4 \\ -P_6^T-T_5 \end{bmatrix} \int_{t-\tau(t)}^{t} y(s)ds$$

$$-\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \tau(t)y(s)^T \frac{D}{h}\tau(t)y(s)ds. \tag{31}$$

Hence,

$$\dot V(z_t) \le \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \eta(t,s)^T \Phi \eta(t,s)ds \tag{32}$$

where Φ is defined by the left side of Inequality (3) and

$$\eta(t, s) = [z^T(t), z^T(t-\tau(t)), f(z(t))^T, f(z(t-\tau(t)))^T, y^T(t), \tau(t)y(s)^T]^T.$$

By Inequality (3), it is easy to see that there exist scalars $a > 0$, $c > 0$ such that

$$\begin{bmatrix} \Phi_{11}+aI & \Phi_{12} & \Phi_{13} & P_1W_1-A^TP_4^T+T_4^T & P-P_1-A^TP_5^T+T_5^T & -A^TP_6^T-T_1 \\ * & \Phi_{22} & \Phi_{23} & S_2+P_2W_1-T_4^T & -P_2-T_5^T & -T_2 \\ * & * & \Phi_{33} & P_3W_1+W^TP_4^T & -P_3+W^TP_5^T & W^TP_6^T-T_3 \\ * & * & * & -(1-\mu)Q-Y_2+2P_4W_1 & -P_4+W_1^TP_5^T & W_1^TP_6^T-T_4 \\ * & * & * & * & hD-2P_5+cD & -P_6^T-T_5 \\ * & * & * & * & * & -\frac{D}{h} \end{bmatrix} < 0. \tag{33}$$

Inequality (33) together with Inequality (32)implies that

$$\dot V(z_t) \le -a|z(t)|^2 - cy(t)^T Dy(t). \tag{34}$$

Now, we choose a scalar $b > 0$ satisfying

$$b\lambda_{\max}(P) + bhe^{bh}(\lambda_{\max}(R) + k^2\lambda_{\max}(Q)) - a \le 0 \tag{35}$$

$$bh^2e^{bh} - c \le 0 \tag{36}$$

where $k = \max_{i=1,\ldots,n} k_i$.

Note that

$$\begin{aligned} V(z_t) &\le \lambda_{\max}(P)z^T(t)z(t) + [\lambda_{\max}(R) + k^2\lambda_{\max}(Q)] \int_{t-\tau(t)}^{t} z^T(s)z(s)ds \\ &+h\int_{t-h}^{t} y(s)^T Dy(s)ds. \end{aligned} \tag{37}$$

Then, for the above scalar $b > 0$, we have

$$\begin{aligned} \frac{d}{dt}(e^{bt}V(z_t)) &= e^{bt}[bV(z_t) + \dot{V}(z_t)] \qquad (38) \\ &\le e^{bt}\{[b\lambda_{\max}(P) - a]|z(t)|^2 + b[\lambda_{\max}(R) + k^2\lambda_{\max}(Q)]\int_{t-\tau(t)}^{t} z^T(s)z(s)ds \\ &+bh\int_{t-h}^{t} y(s)^T Dy(s)ds - cy(t)^T Dy(t)\}. \end{aligned}$$

Now, integrating both sides of Inequality (38) from 0 to $t > 0$, we obtain

$$\begin{aligned} e^{bt}V(z_t) - V(z_0) &\le [b\lambda_{\max}(P) - a]\int_0^t e^{bs}|z(s)|^2 ds \\ &+ b[\lambda_{\max}(R) + k^2\lambda_{\max}(Q)] \int_0^t e^{bs}\int_{s-\tau(s)}^{s} |z(r)|^2 drds \\ &+bh\int_0^t e^{bs}\int_{s-h}^{s} y(r)^T Dy(r)drds - c\int_0^t e^{bs}y(s)^T Dy(s)ds \\ &\le [b\lambda_{\max}(P) - a]\int_0^t e^{bs}|z(s)|^2 ds \\ &+b[\lambda_{\max}(R) + k^2\lambda_{\max}(Q)]\int_0^t e^{bs}\int_{s-h}^{s} |z(r)|^2 drds \\ &+bh\int_0^t e^{bs}\int_{s-h}^{s} y(r)^T Dy(r)drds - c\int_0^t e^{bs}y(s)^T Dy(s)ds \\ &\le [b\lambda_{\max}(P) - a]\int_0^t e^{bs}|z(s)|^2 ds \\ &+b[\lambda_{\max}(R) + k^2\lambda_{\max}(Q)]h\int_{-h}^{t} e^{b(r+h)}|z(r)|^2 dr \\ &+bh^2\int_{-h}^{t} e^{b(r+h)}y(r)^T Dy(r)dr - c\int_0^t e^{bs}y(s)^T Dy(s)ds \\ &\le [b\lambda_{\max}(P) + bhe^{bh}(\lambda_{\max}(R) + k^2\lambda_{\max}(Q)) - a]\int_0^t e^{br}|z(r)|^2 dr \end{aligned}$$

$$+bhe^{bh}[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]\int_{-h}^{0}|z(r)|^2\,dr$$

$$+bh^2e^{bh}\int_{-h}^{0}y(r)^T Dy(r)dr+(bh^2e^{bh}-c)\int_0^t e^{br}y(r)^T Dy(r)dr. \tag{39}$$

Combining Inequality (39) with Inequalities (35) and (36) gives

$$\begin{aligned}
e^{bt}V(z_t) \le\ & bhe^{bh}[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]\int_{-h}^{0}|z(r)|^2\,dr \\
& +bh^2e^{bh}\int_{-h}^{0}y(r)^T Dy(r)dr+V(z_0) \\
\le\ & bhe^{bh}[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]\int_{-h}^{0}|z(r)|^2\,dr \\
& +bh^2e^{bh}\int_{-h}^{0}y(r)^T Dy(r)dr+\lambda_{\max}(P)|z(0)|^2 \\
& +[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]\int_{-\tau(0)}^{0}|z(s)|^2\,ds \\
& +h\int_{-h}^{0}y(s)^T Dy(s)ds \\
\le\ & \{(bhe^{bh}+1)h[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]+\lambda_{\max}(P)\}(\sup_{-2h\le\theta\le 0}|z(\theta)|)^2 \\
& +(bh^2e^{bh}+h)\lambda_{\max}(D)\int_{-h}^{0}y(r)^T y(r)dr.
\end{aligned} \tag{40}$$

Since $r \in [-h, 0]$,

$$\begin{aligned}
y(r)^T y(r) &= |y(r)|^2 \\
&= |-Az(r)+Wf(z(r))+W_1f(z(r-\tau(r)))|^2 \\
&\le [|-Az(r)|+|Wf(z(r))|+|W_1f(z(r-\tau(r)))|]^2 \\
&\le 3|-Az(r)|^2+3|Wf(z(r))|^2+3|W_1f(z(r-\tau(r)))|^2 \\
&\le 3[\lambda_{\max}^2(A)+k^2\lambda_{\max}^2(W)+k^2\lambda_{\max}^2(W_1)](\sup_{-2h\le\theta\le 0}|z(\mu)|)^2.
\end{aligned} \tag{41}$$

$$\begin{aligned}
e^{bt}V(z_t) \le\ & |(bhe^{bh}+1)h[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]+\lambda_{\max}(P)|(\sup_{-2h\le\theta\le 0}|z(\mu)|)^2 \\
& +3(bh^2e^{bh}+h)h\lambda_{\max}(D)[\lambda_{\max}^2(A)+k^2\lambda_{\max}^2(W)+k^2\lambda_{\max}^2(W_1)](\sup_{-2h\le\theta\le 0}|z(\mu)|)^2 \\
\le\ & q(\sup_{-2h\le\theta\le 0}|z(\mu)|)^2
\end{aligned} \tag{42}$$

where

$$\begin{aligned}
q =\ & (bhe^{bh}+1)h[\lambda_{\max}(R)+k^2\lambda_{\max}(Q)]+\lambda_{\max}(P) \\
& +3(bh^2e^{bh}+h)h\lambda_{\max}(D)[\lambda_{\max}^2(A)+k^2\lambda_{\max}^2(W)+k^2\lambda_{\max}^2(W_1)].
\end{aligned}$$

Then

$$e^{bt}\lambda_{\max}(P)|z(t)|^2 < q(\sup_{-2h\le\theta\le 0}|z(\mu)|)^2. \tag{43}$$

Hence

$$|z(t)| < e^{-bt/2}\sqrt{\frac{q}{\lambda_{\max}(P)}}\sup_{-2h\le\theta\le 0}|z(\theta)|. \tag{44}$$

It follows from Inequalities (44) that the DNN in (21) is exponentially stable for any delay $0 < \tau(t) \le h$. This completes the proof.

4. EXAMPLES

In this Section, we provide examples to illustrate the reduced conservatism and effectiveness of Theorem 1 by comparing it with recently reported stability results in literature.

Example 1: Consider the DNN given in Eqn (1) with the following parameters.

$$A = \begin{bmatrix} 4.1989 & 0 & 0 \\ 0 & 0.7160 & 0 \\ 0 & 0 & 1.9985 \end{bmatrix},$$

$$W = 0,$$

$$W_1 = \begin{bmatrix} -0.1052 & -0.5069 & -0.1121 \\ -0.0257 & -0.2808 & 0.0212 \\ 0.1205 & -0.2153 & 0.1315 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.4219 & 0 & 0 \\ 0 & 3.8993 & 0 \\ 0 & 0 & 1.0160 \end{bmatrix}.$$

For this example, both of the delay-dependent asymptotically conditions in [8, 27, 28] cannot *be* satisfied for any $h > 0$. Therefore, they cannot provide any results on the maximum allowed delay h. By methods in [29, 30, 31], Xu and Lam obtained h as 1.7484, 1.7644 and 0.4121, respectively. While by Theorem 1 in our paper, it is found that LMI (3) is feasible for any arbitrarily large h, i.e, the DNN given in this example is globally exponentially stable for any large h. Therefore, it can be seen that the condition given in Theorem 1 is less conservative than those in [8, 27, 28, 29, 30, 31].

Example 2: We consider the DNN given in Eqn (1) with parameters as

$$A = \begin{bmatrix} 1.0674 & 0 & 0 \\ 0 & 2.2094 & 0 \\ 0 & 0 & 0.8352 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.4094 & 0.5719 & 0.2503 \\ -1.0645 & 0.0410 & 0.9923 \\ -0.7439 & 0.63443 & 0.1066 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.3008 & 0 & 0 \\ 0 & 0.3070 & 0 \\ 0 & 0 & 0.3068 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.4911 & 0 & 0 \\ 0 & 0.9218 & 0 \\ 0 & 0 & 0.6938 \end{bmatrix}.$$

For this example, when the activation functions are bounded and globally Lipschitz continuous and $\mu = 0.1$, Xu and Lam [6] proved this delayed neural networks is globally exponentially stable by their delay-independent exponential stability criterion, i.e., Theorem 2 in [6]. However, by Theorem 1 in this paper, it is found that LMI (3) is also feasible for any arbitrarily large h. Therefore, it can be seen that the delay-dependent condition for the global exponential stability of DNN developed in this paper is effective.

If the activation functions are unbounded, Theorem 2 in [6] fails, while, our exponential stability criterion in Theorem 1 remains applicable.

If the time-varying delay function $\tau(t)$ is differentiable, but the upper bound of the derivative of the delay function $\tau(t)$ is greater than 1, Theorem 2 in [6] fails, while, our exponential stability criterion in Theorem 1 remains applicable. For example, if $\mu = 1.2$, we find the allowable maximum delay $h = 3.5336$, under which the DNN in this example is globally exponentially stable.

Remark 1: The stability of DNN given in Eqn (1) has been extensively investigated (see, for example, [27, 28, 32, 33]). Few papers concern the existence and uniqueness of equilibrium point of DNN with unbounded activation functions [27, 32, 33, 34, 35]. He et al [36] only proved global asymptotic stability of DNN given in Eqn (1) with bounded activation functions when A is identity matrix. The criterion for asymptotic stability derived in [36] can not be applied to analyze the global asymptotic stability and the global exponential stability of systems given in Example 1 and Example 2 since the activation functions are unbounded. However, without assuming that activation functions are bounded, the existence, uniqueness and global exponential stability of equilibrium point of a class of time-varying delayed neural networks are proved in our paper. Although asymptotic stability criterion for DNN in [36] is derivative-dependent, it is delay-independent. Theorem 1 in our paper gives a new delay-dependent and derivative-dependent exponential stability criterion for DNN. The global exponential stability of DNN implies global asymptotic stability. The stability criterion in [36] requires that the derivative of the delay function $\tau(t)$ is less than 1 and the activation functions are bounded, whereas our delay-dependent and derivative-dependent exponential stability criterion removes these unreasonable restrictions. The asymptotic stability of the DNN is studied in [28, 33]. All stability criteria given in [28, 33] depend on the absolute values of the connection weights. Since these stability criteria in [28, 33] neglect the signs of the entries in the connection matrices, differences between neuronal excitatory and inhibitory effects are ignored. The condition in Theorem 1 in our paper is written in terms of LMIs. So our stability criteria take sign of the entries in the connection matrices into account, i.e. these conditions take the differences between neuronal excitatory and inhibitory effects of neurons in the DNN given in Eqn (1) into account. Furthermore, our criteria are easy to check and apply in practice by using the LMI toolbox of Matlab.

5. CONCLUSION

Without assuming that activation functions are bounded, a new condition for the global exponential stability of delayed neural networks has been obtained. This stability criterion is expressed in term of LMI, which make it computationally efficient and flexible. Numerical examples are also given to show the reduced conservatism and effectiveness of the proposed result in this paper.

ACKNOWLEDGEMENTS

The authors would like to thank the reviewers for their valuable comments and constructive suggestions.

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