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Delay-dependent Robust Stability of Neutral-type Neural Networks with Time Delays*

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Abstract: In this paper, the problem of the global robust asymptotic stability (GRAS) for a class of interval neural networks described by nonlinear delayed differential equations of the neutral type is investigated. A series of sufficient criteria for such problems are obtained by employing Lyapunov-Krasovskii functional and LMI technique. These conditions are dependent on the size of the time delay, which are usually less conservative than delay-independent ones. Moreover, the activate functions in system are generalized without assuming the boundedness and differentiability. Finally, the effectiveness of the present results is demonstrated by numerical example.

Keywords: Neural networks; Delay-dependent; Neutral-type; Robust stability; Linear matrix inequality.

1. INTRODUCTION

Recently, the problem of the global stability for delayed neural networks has attracted much attention due to its applicability in solving some image processing, signal processing, optimization and pattern recognition problems. Many important results on the global asymptotic or exponential stability have been reported in the literature, see, e.g., ([1]-[4], [7]) and the references therein. However, the stability may be destroyed by some unavoidable uncertainties caused by the existence of modelling errors, external disturbance and parameter fluctuation during the implementation on very-large-scale-integration chips. Therefore, the investigation on robustness of the networks against such errors and fluctuation is very important and significant. Many attentions have been devoted to the investigation of robust stability for CNNs and DCNNs, such as Liao et al. in [8] investigated the global robust stability of delayed interval Hopfield neural networks; Cao, Huang and Qu in [10] gave a new sufficient condition on the existence, uniqueness, and global robust stability of equilibria for interval neural networks with time delays by constructing Lyapunov functional and using matrix-norm inequality; Singh in [11] gave a novel global robust stability criterion for neural networks with delay, and other results ([??0], [18]).

On the other hand, due to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamic behavior for such complex neural reactions. These systems are called neutral-type neural networks. For example, in the biochemistry experiments, neural information may transfer across chemical reactivity, which results in a neutral-type process [13]. The stability analysis of neutral systems with delay has received considerable attention over the decades, see, e.g., ([6], [14]-[17]) and the references therein.

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Motivated by the above discussions, the main aim of this paper is to investigate global robust asymptotic stability (GRAS) for a class of interval delayed neural networks described by a nonlinear differential equation of neutral type. By constructing appropriate Lyapunov-Krasovskii functional and using the linear matrix inequality (LMI) optimization approach, some sufficient conditions are obtained to ensure the existence, uniqueness and global asymptotical stability of the equilibrium point of such a kind of delayed neural networks. The obtained conditions are dependent on the size of the time delay, which are usually less conservative than delay-independent ones. It is worth pointing out that these conditions do not require the activate functions are bounded and differentiable. Also, it should be noted that the proposed LMI condition can be checked numerically efficiently by resorting to recently developed interior-point methods. To the best of our knowledge, if any, few authors have considered global robust asymptotic stability for this class of neural networks with time delays. The work will have significance impact on the design and applications of globally stable neural networks with time delays.

The remaining of our paper is organized as follows: In Section 2 the model formulation and some preliminaries are given. The main results are stated in Section 3. In Section 4, an example is provided to show the validity of the stability conditions. In the end, our paper is closed with a conclusion.

Notation Throughout this paper, R^n denotes the n-dimensional Euclidean space. For vector $X \in R^n$, its norm is defined as $\| X \| = \sqrt{X^T X}$. E denotes identity matrix. The notation $A > B$ (respectively, $A \geq B$) means that the matrix $A - B$ is symmetric positive definite (respectively, positive semi-definite), where A and B are matrices of the same dimensions. A^T and A^{-1} denote the transpose and inverse of the matrix A. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the maximum and minimum eigenvalues of matrix A, respectively. For an arbitrarily real matrix B and two real symmetric matrices A and C, $\begin{pmatrix} A & * \\ B & C \end{pmatrix}$ denotes a real symmetric block matrix, where $*$ represents the elements above the main diagonal of a symmetric matrix.

2. PRELIMINARIES

In this paper, we consider a class of neural networks with time delays described by a nonlinear delayed differential equation of neutral type:

$$\dot{u}_i(t) = -a_i u_i(t) + \sum_{j=1}^{n} w_{ij1} \widetilde{f}_j(u_j(t)) + \sum_{j=1}^{n} w_{ij2} \widetilde{g}_j(u_j(t-\tau)) + \sum_{j=1}^{n} v_{ij} \dot{u}_j(t-\tau) + I_i, \tag{1}$$

$$u_i(t) = \phi_i(t), \ -\tau \leq t \leq 0, \tag{2}$$

where $i = 1, 2, \cdots, n$ and n denotes the number of neurons in a neural network; $u_i(t)$ denotes the state of the ith neuron at time t; $\widetilde{f}_j$, $\widetilde{g}_j$ are activation functions of the j th neuron; the scalar $a_i > 0$ is the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t; $w_{ij1}, w_{ij2}, v_{ij}, i, j = 1, 2, \cdots, n$ are known constants denoting the strength of the ith neurons on the j th neurons; τ is non-negative constant, which correspond to finite speed of axonal signal transmission delay; I_i denote the i th component of an external input source introduced from outside the network to the cell i at time. The initial $\phi_i(s, x), i = 1, 2, \cdots, n$ are bounded and first order continuous differentiable on $(-\tau, 0]$.

Throughout this paper we assume that the activation function satisfies the following assumption.

(*H*) The neurons activation functions $\tilde{f}_j$, $\tilde{g}_j$ are Lipscthiz continuous, that is, there exist constants $l_j > 0, k_j > 0$ such that

$$0 \le \frac{\tilde{f}_j(\xi_1) - \tilde{f}_j(\xi_2)}{\xi_1 - \xi_2} \le l_j; \quad 0 \le \frac{\tilde{g}_j(\xi_1) - \tilde{g}_j(\xi_2)}{\xi_1 - \xi_2} \le k_j,$$

for any $\xi_1, \xi_2 \in R, \xi_1 \ne \xi_2, j = 1, 2, \cdots, n.$

In the practical implementation of neural networks, the values of the constant and weight coefficients depend on the resistance and capacitance values which are subject to uncertainties. This may lead to some deviations in the values of a_i, w_{ij1}, w_{ij2} and v_{ij}. Hence, it is important to ensure the global robust asymptotic stability of the designed network against such parameter deviations. Since these deviations are bounded in practice, the quantities a_i, w_{ij1}, w_{ij2} and v_{ij} may be intervalized as follows:

$$\begin{cases} A_I := [\underline{A}, \overline{A}] = \{A = diag(a_i) : \underline{A} \le A \le \overline{A}, i.e., \underline{a}_i \le a_i \le \overline{a}_i, i = 1, 2, \cdots, n\} (3) \\ W_{1I} := [\underline{W}_1, \overline{W}_1] = \{W_1 = (w_{ij1})_{n\times n} : \underline{W}_1 \le W_1 \le \overline{W}_1, i.e., \underline{w}_{ij1} \le w_{ij1} \le \overline{w}_{ij1}, i = 1, 2, \cdots, n\} (4) \\ W_{2I} := [\underline{W}_2, \overline{W}_2] = \{W_2 = (w_{ij2})_{n\times n} : \underline{W}_2 \le W_2 \le \overline{W}_2, i.e., \underline{w}_{ij2} \le w_{ij2} \le \overline{w}_{ij2}, i = 1, 2, \cdots, n\} (5) \\ V_I := [\underline{V}, \overline{V}] = \{V = (v_{ij})_{n\times n} : \underline{V} \le V \le \overline{V}, i.e., \underline{v}_{ij} \le v_{ij} \le \overline{v}_{ij}, i = 1, 2, \cdots, n\} \end{cases} \tag{6}$$

Let $U^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ be an equilibrium point of model (1) for a given I_i. To simplify proofs, we will shift the equilibrium point U^* of system (1) to the origin by using the transformation

$$y_i(t) = u_i(t) - u_i^*, i = 1, 2, \cdots, n.$$

It is easy to see that system (1) can be transformed into

$$\dot{y}_i(t) = -a_i y_i(t) + \sum_{j=1}^{n} w_{ij1} f_j(y_j(t)) + \sum_{j=1}^{n} w_{ij2} g_j(y_j(t-\tau)) + \sum_{j=1}^{n} v_{ij} \dot{y}_j(t-\tau), i = 1, 2, \cdots, n, \tag{7}$$

where $f_j(y_j(t)) = \tilde{f}_j(y_j(t) + u_j^*) - \tilde{f}_j(u_j^*)$, $g_j(y_j(t-\tau)) = \tilde{g}_j(y_j(t-\tau) + u_j^*) - \tilde{g}_j(u_j^*)$, $j = 1, 2, \cdots, n.$ Then it is easy to see that $f_j(0) = 0,\ g_j(0) = 0,$ and $f_j(\cdot),\ g_j(\cdot)$ satisfy assumption (*H*).

Denote

$$\begin{aligned} Y(t) &= (y_1(t), y_2(t), \cdots, y_n(t))^T, \\ Y(t-\tau) &= (y_1(t-\tau), y_2(t-\tau), \cdots, y_n(t-\tau))^T, \\ F(Y(t)) &= (f_1(y_1(t)), f_2(y_2(t)), \cdots, f_n(y_n(t)))^T, \\ G(Y(t-\tau)) &= (g_1(y_1(t-\tau)), g_2(y_2(t-\tau)), \cdots, g_n(y_n(t-\tau)))^T, \end{aligned}$$

A = diag (a_1, a_2, ..., a_n), $W_1 = (w_{ij1})_{n\times n}$, $W_2 = (w_{ij2})_{n\times n}$ and $V = (V_{ij})_{n\times n}$ (where we do not assume that matrices W_1, W_2, V to be symmetric). Then system (4) can be rewritten in the following vector-matrix form

$$\dot{Y}(t) = -AY(t) + W_1F(Y(t)) + W_2G(Y(t-\tau)) + V\dot{Y}(t-\tau). \tag{8}$$

Definition: The neural network model given by (1) with the parameter ranges defined by (3) is globally robust asymptotically stable if the unique equilibrium point $U^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ of the model is globally asymptotically stable for all $A \in A_I, W_1 \in W_{1I}, W_2 \in W_{2I}$ and $V \in V_I$.

Before we develop the delay-dependent criteria, we note that the following two facts and two lemmas.

Fact 1. [2] Suppose W, U are any matrices, ε is a positive number and matrix $H = H^T > 0$, then the following inequality holds

$$W^TU + U^TW \le \varepsilon W^THW + \varepsilon^{-1}U^TH^{-1}U.$$

Fact 2. (Schur complement) The following linear matrix inequality (LMI)

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, and $S(x)$ depend affinely on x, is equivalent to

$$(1)\ R(x) > 0,\ \ Q(x) - S(x)R^{-1}(x)S^T(x) > 0,$$

$$(2)\ Q(x) > 0,\ \ R(x) - S(x)Q^{-1}(x)S^T(x) > 0.$$

Moreover, for convenience, we define ($i, j = 1, 2, \cdots, n,$ and $k = 1, 2$)

$$w_{ijk}^* = \max\{|\,\underline{w}_{ijk}\,|, |\,\overline{w}_{ijk}\,|\},\ \ v_{ij}^* = \max\{|\,\underline{v}_{ij}\,|, |\,\overline{v}_{ij}\,|\},$$

and

$$b_{ik} = \sum_{j=1}^{n}\left(w_{ijk}^* \sum_{l=1}^{n} w_{ljk}^*\right),\ \ c_i = \sum_{j=1}^{n}\left(v_{ij}^* \sum_{l=1}^{n} v_{lj}^*\right),$$

$$B_k = diag(b_{1k}, b_{2k}, \cdots, b_{nk}),\ \ C = diag(c_1, c_2, \cdots, c_n). \tag{9}$$

Lemma 1.[12] For any constant matrix $W_1 \in W_{1I}$, $W_1 \in W_{1I}$ and $V \in V_I$

$$W_1W_1^T \le B_1,\ \ W_2W_2^T \le B_2,\ \ VV^T \le C,$$

where the diagonal matrices B_1, B_2 and C were defined in Eq. (6).

Lemma 2.[5] The equilibrium of the system is globally asymptotically stable if there exists a C^1 function $V : R^n \to R$ such that (i) V is a positive definite, decrescent and radially unbounded, and (ii) $-\dot{V}$ is positive definite.

3. MAIN RESULTS

Theorem 1. Under the assumption (*H*), the equilibrium point U^* of system (1) is globally robust asymptotically stable if there exist positive definite matrix *P*, positive definite diagonal matrix *Q* and positive constants $\alpha, \beta, \varepsilon_1, \varepsilon_2, \varepsilon_3$ such that

$$\Psi = \begin{pmatrix} \Sigma & * & * & * & * & * & *(10) \\ PB_1^{\frac{1}{2}} & \varepsilon_1 E & * & * & * & * & *(11) \\ PB_2^{\frac{1}{2}} & 0 & \varepsilon_2 E & * & * & * & *(12) \\ PC^{\frac{1}{2}} & 0 & 0 & \varepsilon_3 E & * & * & *(13) \\ 0 & 0 & 0 & 0 & \Gamma_1 & * & *(14) \\ 0 & 0 & 0 & 0 & 0 & \Gamma_2 & *(15) \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau\beta \end{pmatrix} > 0, \tag{16}$$

where

$$\begin{aligned} \Sigma &= P\underline{A} + \underline{A}P - KQK - \varepsilon_1 L^2 - 4(\alpha + \tau\beta)(\overline{A}^2 + LB_1L), \\ \Gamma_1 &= Q - \varepsilon_2 E - 4(\alpha + \tau\beta)B_2, \\ \Gamma_2 &= \alpha E - \varepsilon_3 E - 4(\alpha + \tau\beta)C, \\ L &= diag(l_1, l_2, \cdots, l_n), \\ K &= diag(k_1, k_2, \cdots, k_n), \end{aligned}$$

and $l_i > 0, k_i > 0, i = 1, 2, \cdots, n$ are given as in assumption (H).

Proof: In order to show under what condition the origin is globally robust asymptotically stable for the system (1), we consider the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{17}$$

where

$$V_1(t) = Y^T(t)PY(t), \tag{18}$$

$$V_2(t) = \int_{t-\tau}^{t} G^T(Y(\xi))QG(Y(\xi))d\xi, \tag{19}$$

$$V_3(t) = \alpha\int_{t-\tau}^{t} \dot{Y}(\xi)^T \dot{Y}(\xi)d\xi, \tag{20}$$

$$V_4(t) = \beta\int_{-\tau}^{0}\int_{t+\xi}^{t} \dot{Y}(\xi)^T \dot{Y}(\xi)d\xi d\xi. \tag{21}$$

Computing the time-derivative of $V_i(t)\ (i=1,2,3,4)$ along the trajectory of system (5), we have

$$\begin{aligned} V_1(t) &= 2Y^T(t)P\dot{Y}(t) \\ &= 2Y^T(t)P[-AY(t)+W_1F(Y(t))+W_2G(Y(t-\tau))+V\dot{Y}(t-\tau)] \\ &= Y^T(t)[-PA-AP]Y(t)+2Y^T(t)PW_1F(Y(t)) \\ &\quad +2Y^T(t)PW_2G(Y(t-\tau))+2Y^T(t)PV\dot{Y}(t-\tau), \end{aligned} \tag{22}$$

$$V_2(t) = G^T(Y(t))QG(Y(t)) - G^T(Y(t-\tau))QG(Y(t-\tau)), \tag{23}$$

$$V_3(t) = \alpha\dot{Y}^T(t)\dot{Y}(t) - \alpha\dot{Y}^T(t-\tau)\dot{Y}(t-\tau), \tag{24}$$

$$V_4(t) = \tau\beta\dot{Y}^T(t)\dot{Y}(t) - \beta\int_{t-\tau}^{t}\dot{Y}^T(\xi)\dot{Y}(\xi)d\xi. \tag{25}$$

Then we have

$$\begin{aligned} \dot{V}(t) &= Y^T(t)[-PA-AP]Y(t)+2Y^T(t)PW_1F(Y(t))+2Y^T(t)PW_2G(Y(t-\tau)) \\ &\quad +2Y^T(t)PV\dot{Y}(t-\tau)+G^T(Y(t))QG(Y(t))-G^T(Y(t-\tau))QG(Y(t-\tau)) \\ &\quad +(\alpha+\tau\beta)\dot{Y}^T(t)\dot{Y}(t)-\alpha\dot{Y}^T(t-\tau)\dot{Y}(t-\tau)-\beta\int_{t-\tau}^{t}\dot{Y}^T(\xi)\dot{Y}(\xi)d\xi. \end{aligned} \tag{26}$$

By Fact 1, Lemma 1 and assumption (H), we have

$$G^T(Y(t))QG(Y(t)) \le Y^T(t)KQKY(t), \tag{27}$$

$$\begin{aligned} 2Y^T(t)PW_1F(Y(t)) &\le \varepsilon_1^{-1}Y^T(t)PW_1W_1^TPY(t)+\varepsilon_1F^T(Y(t))F(Y(t)) \\ &\le Y^T(t)[\varepsilon_1^{-1}PB_1^{\frac{1}{2}}B_1^{\frac{1}{2}T}P+\varepsilon_1L^2]Y(t), \end{aligned} \tag{28}$$

$$\begin{aligned} 2Y^T(t)PW_2G(Y(t-\tau)) &\le \varepsilon_2^{-1}Y^T(t)PW_2W_2^TPY(t)+\varepsilon_2G^T(Y(t-\tau))G(Y(t-\tau)) \\ &\le \varepsilon_2^{-1}Y^T(t)PB_2^{\frac{1}{2}}B_2^{\frac{1}{2}T}PY(t)+\varepsilon_2G^T(Y(t-\tau))G(Y(t-\tau)), \end{aligned} \tag{29}$$

$$\begin{aligned} 2Y^T(t)PV\dot{Y}(t-\tau) &\le \varepsilon_3^{-1}Y^T(t)PVV^TPY(t)+\varepsilon_3\dot{Y}^T(t-\tau)\dot{Y}(t-\tau) \\ &\le \varepsilon_3^{-1}Y^T(t)PC^{\frac{1}{2}}C^{\frac{1}{2}T}PY(t)+\varepsilon_3\dot{Y}^T(t-\tau)\dot{Y}(t-\tau), \end{aligned} \tag{30}$$

$$\begin{aligned} \dot{Y}^T(t)\dot{Y}(t) &\le 4Y^T(t)A^2Y(t)+4F^T(Y(t))W_1^TW_1F(Y(t)) \\ &\quad +4G^T(Y(t-\tau))W_2^TW_2G(Y(t-\tau))+4\dot{Y}^T(t-\tau)V^TV\dot{Y}(t-\tau) \\ &\le 4Y^T(t)[\overline{A}^2+LB_1L]Y(t)+4G^T(Y(t-\tau))B_2G(Y(t-\tau)) \\ &\quad +4\dot{Y}^T(t-\tau)C\dot{Y}(t-\tau). \end{aligned} \tag{31}$$

Substituting (18)-(22) into (17), we have

$$\dot{V}(t) \le Y^T(t)[-P\underline{A} - \underline{A}P + KQK + \varepsilon_1^{-1}PB_1^{\frac{1}{2}}B_1^{\frac{1}{2}T}P + \varepsilon_1 L^2 + \varepsilon_2^{-1}PB_2^{\frac{1}{2}}B_2^{\frac{1}{2}T}P$$

$$+\varepsilon_3^{-1}PC^{\frac{1}{2}}C^{\frac{1}{2}T}P + 4(\alpha+\tau\beta)(\overline{A}^2 + LB_1L)]Y(t)$$

$$+G^T(Y(t-\tau))[-Q+\varepsilon_2 E + 4(\alpha+\tau\beta)B_2]G(Y(t-\tau))$$

$$+\dot{Y}^T(t-\tau)[-\alpha E + \varepsilon_3 E + 4(\alpha+\tau\beta)C]\dot{Y}(t-\tau) - \beta\int_{t-\tau}^{t}\dot{Y}^T(\xi)\dot{Y}(\xi)d\xi$$

$$= \frac{1}{\tau}\int_{t-\tau}^{t}\Big\{Y^T(t)[-P\underline{A} - \underline{A}P + KQK + \varepsilon_1^{-1}PB_1^{\frac{1}{2}}B_1^{\frac{1}{2}T}P + \varepsilon_1 L^2 + \varepsilon_2^{-1}PB_2^{\frac{1}{2}}B_2^{\frac{1}{2}T}P$$

$$+\varepsilon_3^{-1}PC^{\frac{1}{2}}C^{\frac{1}{2}T}P + 4(\alpha+\tau\beta)(\overline{A}^2 + LB_1L)]Y(t)$$

$$+G^T(Y(t-\tau))[-Q+\varepsilon_2 E + 4(\alpha+\tau\beta)B_2]G(Y(t-\tau))$$

$$+\dot{Y}^T(t-\tau)[-\alpha E + \varepsilon_3 E + 4(\alpha+\tau\beta)C]\dot{Y}(t-\tau) - \tau\beta\dot{Y}^T(\xi)\dot{Y}(\xi)\Big\}d\xi$$

$$= \frac{1}{\tau}\int_{t-\tau}^{t}\Lambda^T(t,\xi)\begin{pmatrix} \tilde{\Sigma} & 0 & 0 & 0(32) \\ 0 & \tilde{\Gamma}_1 & 0 & 0(33) \\ 0 & 0 & \tilde{\Gamma}_2 & 0(34) \\ 0 & 0 & 0 & -\tau\beta \end{pmatrix}\Lambda(t,\xi)d\xi, \tag{35}$$

where

$$\tilde{\Sigma} = -P\underline{A} - \underline{A}P + KQK + \varepsilon_1^{-1}PB_1^{\frac{1}{2}}B_1^{\frac{1}{2}T}P + \varepsilon_1 L^2$$
$$+\varepsilon_2^{-1}PB_2^{\frac{1}{2}}B_2^{\frac{1}{2}T}P + \varepsilon_3^{-1}PC^{\frac{1}{2}}C^{\frac{1}{2}T}P + 4(\alpha+\tau\beta)(\overline{A}^2 + LB_1L),$$
$$\tilde{\Gamma}_1 = -Q + \varepsilon_2 E + 4(\alpha+\tau\beta)B_2,$$
$$\tilde{\Gamma}_2 = -\alpha E + \varepsilon_3 E + 4(\alpha+\tau\beta)C,$$
$$\Lambda = \left(Y^T(t), G^T(Y(t-\tau)), \dot{Y}(t-\tau))^T, \dot{Y}(\xi))^T\right)^T.$$

By Fact 2 and the condition (7),

$$\dot{V}(t) < 0.$$

From Lemma 2, we derive that the equilibrium of system (1) is globally asymptotically stable when $A \in A_I, W_1 \in W_{1I}, W_2 \in W_{2I}$ and $V \in V_I$. This completes the proof.

Corollary 1. Under the assumption (*H*), the equilibrium point U^* of system (1) is globally robust asymptotically stable if there exist positive definite matrix *P*, positive definite diagonal matrix *Q* and positive constants $\alpha, \beta, \varepsilon_1, \varepsilon_2, \varepsilon_3$ such that

$$\widehat{\Psi}=\begin{pmatrix} \hat{\Sigma} & * & * & * & * & * & *(36) \\ P & \varepsilon_1 E & * & * & * & * & *(37) \\ P & 0 & \varepsilon_2 E & * & * & * & *(38) \\ P & 0 & 0 & \varepsilon_3 E & * & * & *(39) \\ 0 & 0 & 0 & 0 & \widehat{\Gamma_1} & * & *(40) \\ 0 & 0 & 0 & 0 & 0 & \widehat{\Gamma_2} & *(41) \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau\beta \end{pmatrix} > 0, \tag{42}$$

where

$$\hat{\Sigma}=P\underline{A}+\underline{A}P-KQK-\varepsilon_1 LB_1L-4(\alpha+\tau\beta)(\overline{A}^2+LB_1L),$$
$$\hat{\Gamma}_1=Q-\varepsilon_2B_2-4(\alpha+\tau\beta)B_2,\ \hat{\Gamma}_2=\alpha E-\varepsilon_3C-4(\alpha+\tau\beta)C.$$

Proof: The proof is similar to that of Theorem 1, and omitted here.

When $v_{ij}=0\ (i,j=1,2,\cdots,n)$, system (1) degenerated into the following model

$$\dot{u}_i(t)=-a_iu_i(t,x)+\sum_{j=1}^{n}w_{ij1}\widetilde{f}_j(u_j(t))+\sum_{j=1}^{n}w_{ij2}\widetilde{g}_j(u_j(t-\tau))+I_i, i=1,2,\cdots,n. \tag{43}$$

Corollary 2. Under the assumption (*H*), the equilibrium point U^* of system (25) is globally robust asymptotically stable if there exist positive definite matrix *P*, positive definite diagonal matrix *Q*, and positive constants $\varepsilon_1,\varepsilon_2$ such that

$$\widetilde{\Psi}=\begin{pmatrix} \widetilde{\Gamma}_1 & * & * & *(44) \\ PB_1^{\frac{1}{2}} & \varepsilon_1 E & * & *(45) \\ PB_2^{\frac{1}{2}} & 0 & \varepsilon_2 E & *(46) \\ 0 & 0 & 0 & Q-\varepsilon_2 E \end{pmatrix} > 0, \tag{47}$$

where

$$\widetilde{\Gamma}_1=P\underline{A}+\underline{A}P-KQK-\varepsilon_1L^2.$$

Proof: Consider the Lyapunov functional

$$V(t)=Y^T(t)PY(t)+\int_{t-\tau}^{t}G^T(Y(\xi))QG(Y(\xi))d\xi.$$

Similarly, calculate the derivative of $V(t)$ along the degenerated system (25). By a minor modification of the proof of Theorem 1, we can easily obtain that the equilibrium point U^* of system (25) is globally robust asymptotically stable.

4. EXAMPLE

Example: Let $\tilde{f}_j = \tilde{g}_j$, $j = 1, 2$, with the Lipschtiz constant $l_1 = l_2 = 1$. Consider the following delayed neural network in (1) with parameters

$$\underline{A} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \overline{A} = \begin{pmatrix} 2.1 & 0 \\ 0 & 2.1 \end{pmatrix}, \underline{W}_1 = \begin{pmatrix} 0.4 & 0.2 \\ 0 & 0.5 \end{pmatrix}, \overline{W}_1 = \begin{pmatrix} 0.5 & 0.2 \\ 0.1 & 0.6 \end{pmatrix},$$

$$\underline{W}_2 = \begin{pmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{pmatrix}, \overline{W}_2 = \begin{pmatrix} 0.1 & 0.16 \\ 0.05 & 0.1 \end{pmatrix}, \underline{V} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \overline{V} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}.$$

By Theorem 1, we can conclude that this delayed neural network is globally robust stable for all $0 < \tau \le 2.0173$. When $\tau = 2.0173$, we use the Matlab LMI Control Toolbox to solve the LMI in (7), and obtain the solution as follows

$$\begin{gathered} \varepsilon_1 = 15.3706, \varepsilon_2 = 7.8443, \varepsilon_3 = 1.5388, \alpha = 2.1561, \beta = 0.0881, \\ P = \begin{pmatrix} 33.1647 & 0 \\ 0 & 33.3376 \end{pmatrix}, \quad Q = \begin{pmatrix} 8.5530 & 0 \\ 0 & 9.1925 \end{pmatrix}. \end{gathered}$$

5. CONCLUSION

In this paper, we have studied the global robust asymptotic stability for a class of interval delayed neural networks of neutral type. A series of delay-dependent sufficient conditions in the form of LMI have been established to ensure global robust stability of this model by using Lyapunov method. The obtained criteria improve and extend several earlier works greatly.

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