Journal of Mathematical Control Science and Applications (JMCSA)
Vol. 2 No. 1 (January-June, 2016), ISSN : 0974-0570

A Subspace to Describe Grasping Internal Forces in Robotic Manipulation Systems

Paolo Mercorelli*

Abstract: Due to technological development, robotics applications are growing in many industrial sectors and even in applications such as medical ones (micro manipulation of internal tissues or laparoscopy) or in entertainment (special effects in cinema). Robotic devices with high performance in precision and speed are required for all these advanced functions. Through a new approach to the study of robotic manipulation this paper presents some constructive matrices structures which can be used to design force controllers. It is based on the well–known geometric control of system dynamics. In such a framework some typical problems in robotics can be mathematically formalized and analyzed. A generalized linear model is used and a careful analysis is made. The main result consists of proposing an explicit new matrices structure for grasping internal forces.

Keywords: Subspaces, matrices, manipulators, internal forces.

AMS Subject Classification: 93D09, 19L64, 70Q05, 14L24.

1. INTRODUCTION

The coordinated use of the multiple fingers of a robot hand or, similarly, of multiple arms in cooperating tasks; the use of the inner links of a robot arm or finger to hold an object, and the exploitation of parallel mechanical structures, are examples of a non–conventional usage of mechanisms for manipulation. It is possible to refer to such devices as "general manipulation systems". A rigorous definition of "general manipulation system" will be given later.

In [1] the authors pointed out their interest in characterizing the structural property of the linearized model of the general manipulation system as reachability and observability. For a broad overview of the manipulation control problem, the reader is referred to [2] and the references therein. Recent contributions to the topic of manipulation marked progress in the area of geometric approach by using linear algebra. In [3] the authors showed that a geometric approach provides procedures to guarantee robustness of the system against parametric uncertainties on the model. In general, applying matrix structures to mechanical systems has many advantages as pointed out in [4]. There are theoretical and practical advantages, easy and elegant interpretation of the results as well as easy computer implementation.

The earliest use of the matrix oriented to the control of systems was by Basile and Marro ([5], [6]) and to Wonham and Morse ([7], [8]), where the authors propose a geometric approach to solve some problems as non-interacting control, observer and disturbance rejection.

In this paper a linearized model of the general mechanisms of manipulation is used. Reachable "internal" forces are structural properties of general manipulation systems. The use of linearized model dynamics in the analysis of general manipulation systems is believed to be a significant advancement with respect to the literature,

* University of Applied Sciences Wolfsburg, Dep. of Vehicles, Production and Control Techniques, Robert-Koch-Platz 8-a, 38440 Wolfsburg, Germany.

which is almost solely based on quasi–static models, and in fact provides richer results and a better insight. Furthermore, linearized analysis is considered as a fundamental preparatory step towards full non-linear analysis, which at the moment appears to be too complex to be achieved in full generality. Finally, it is worth while mentioning that there exists a subclass of Cartesian manipulators where the linearized model provides an exact model of the complete system dynamics.

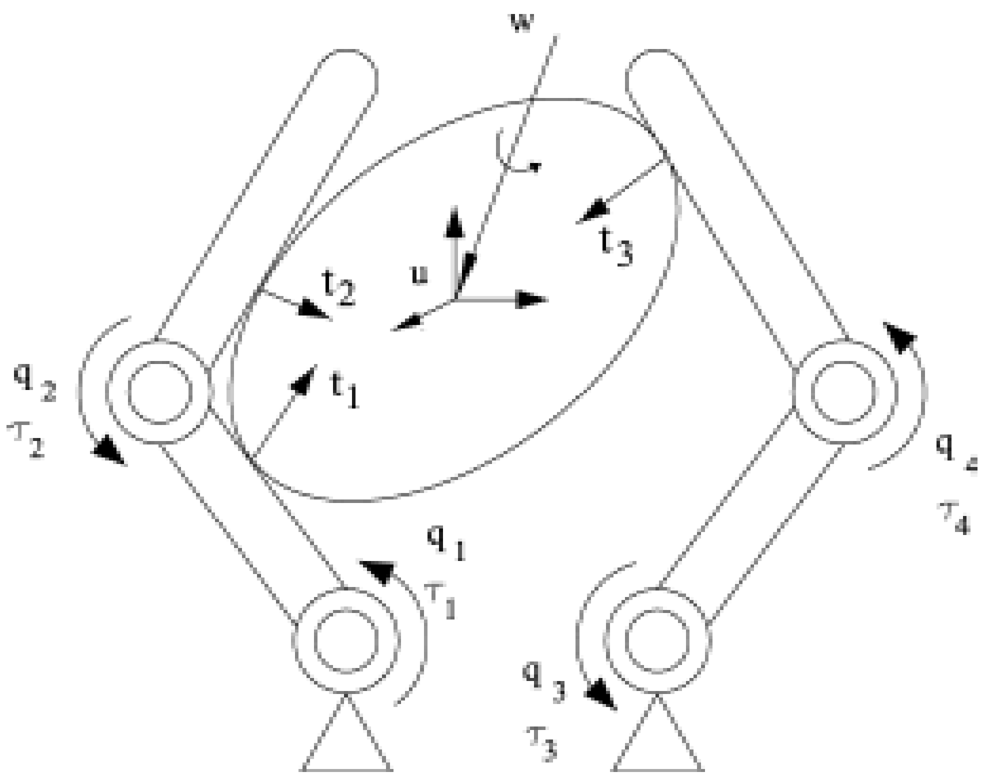

Figure 1: Vector Notation for General Manipulation System Analysis

The paper is organized as follows. Section 2 introduces some notation and provides the linearized dynamics of manipulation systems. In Section 3 the system outputs are specified in terms of object motions and contact forces.

2. DYNAMIC MODEL

In this section the linearized model of the dynamics of a general manipulation system is derived. For a detailed discussion of this model refer to [9].

The vector of manipulator joint positions is denoted by $\mathbf{q} \in \Re^q$, $\tau \in \Re^q$ the vector of joint actuator torques, $\mathbf{u} \in \Re^d$ the vector locally describing the position and the orientation of a frame attached to the object, and $\mathbf{w} \in \Re^d$ the vector of forces and torques resultant from external forces are acting directly on the object. In literature, $\mathbf{w}$ is usually referred to as the disturbance vector. The force/torque interaction $\mathbf{t}_i$ (see Fig. 1) at the

i–th contact is taken into account by using a lumped–parameter $(\mathbf{K}_i, \mathbf{B}_i)$ model of visco–elastic phenomena. According to this model, the contact force vector $\mathbf{t}_i$ is

$$\mathbf{t}_i = \mathbf{K}_i ({}^h\mathbf{c}_i - {}^o\mathbf{c}_i) + \mathbf{B}_i ({}^h\dot{\mathbf{c}}_i - {}^o\dot{\mathbf{c}}_i), \tag{1}$$

where vectors ${}^h\mathbf{c}_i$ and ${}^o\mathbf{c}_i$ describe the postures of two contact frames, the first one on the manipulator and the second one on the object, where the i–th contact spring and damper are anchored. Matrices $\mathbf{K}_i$ and $\mathbf{B}_i$ are symmetric and positive definite (p.d.) and the dimensions of vectors involved in (1) depend on the particular model used to describe the contact interaction (cf. [10]). The computation and control of the stiffness matrix have been considered in depth by Cutkosky and Kao [11].

To simplify the notation, contact forces $\mathbf{t}_i$'s, contact points ${}^h\mathbf{c}_i$'s and ${}^o\mathbf{c}_i$'s are grouped into vectors $\mathbf{t}$, ${}^h\mathbf{c}$ and ${}^o\mathbf{c}$. Similarly, $\mathbf{K}_i$'s and $\mathbf{B}_i$'s are grouped to build the block diagonal grasp stiffness and damping symmetric and p.d. matrices $\mathbf{K}$ and $\mathbf{B}$.

The *Jacobian* $\mathbf{J}$ and *grasp* matrix $\mathbf{G}$ of the manipulation system, cf. [1], are defined by the linear maps relating the velocities of vectors ${}^h\mathbf{c}$ and ${}^o\mathbf{c}$ with the joint and object velocities $\dot{\mathbf{q}}$ and $\dot{\mathbf{u}}$, respectively:

$$\begin{aligned} {}^h\dot{\mathbf{c}} &= \mathbf{J}\dot{\mathbf{q}}; \\ {}^o\dot{\mathbf{c}} &= \mathbf{G}^T\dot{\mathbf{u}}. \end{aligned}$$

Note that, dually, $\mathbf{J}^T\mathbf{t}$ and $\mathbf{G}\mathbf{t}$ represent the effects of contact forces $\mathbf{t}$ on the manipulation and object dynamics whose full non-linear models are:

$$\begin{aligned} \mathbf{M}_h\ddot{\mathbf{q}} + \mathbf{Q}_h &= -\mathbf{J}^T\mathbf{t} + \tau; \\ \mathbf{M}_o\ddot{\mathbf{u}} + \mathbf{Q}_o &= \mathbf{G}\mathbf{t} + \mathbf{w}. \end{aligned} \tag{3}$$

Here, $\mathbf{M}_h$ and $\mathbf{M}_o$ are inertia symmetric and p.d. matrices, while $\mathbf{Q}_h$ and $\mathbf{Q}_o$ are terms including velocity-dependent and gravity forces of the manipulator and of the object, respectively.

To proceed with the analysis of the linearized model of the full manipulation system, consider a reference equilibrium configuration

$$\mathbf{q} = \mathbf{q}_o, \quad \mathbf{u} = \mathbf{u}_o, \quad \dot{\mathbf{q}} = \dot{\mathbf{u}} = 0, \quad \tau = \tau_o, \quad \mathbf{w} = \mathbf{w}_o \quad \mathbf{t}=\mathbf{t}_o,$$

such that

$$\tau_o = \mathbf{J}^T\mathbf{t}_o \quad \text{and} \quad \mathbf{w}_o = -\mathbf{G}\mathbf{t}_o.$$

The linear approximation of the manipulation system in the neighbourhood of such equilibrium is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau\delta\tau + \mathbf{B}_w\delta\mathbf{w}, \tag{4}$$

where state and input vectors are defined as the departures from the reference equilibrium configuration:

$$\begin{aligned} \mathbf{x} &= \left[\delta\mathbf{q}^T, \delta\mathbf{u}^T, \delta\dot{\mathbf{q}}^T, \delta\dot{\mathbf{u}}^T\right]^T = \left[(\mathbf{q}-\mathbf{q}_o)^T (\mathbf{u}-\mathbf{u}_o)^T \dot{\mathbf{q}}^T \dot{\mathbf{u}}^T\right]^T, \\ \delta\tau &= \tau - \mathbf{J}^T\mathbf{t}_o, \\ \delta\mathbf{w} &= \mathbf{w} + \mathbf{G}\mathbf{t}_o, \end{aligned} \tag{5}$$

and the dynamic, input and disturbance matrices are

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \mathbf{B}_\tau = \begin{bmatrix} 0 \\ 0 \\ \mathbf{M}_h^{-1} \\ 0 \end{bmatrix}; \mathbf{B}_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{M}_o^{-1} \end{bmatrix}. \tag{6}$$

To simplify the notation, the symbol δ henceforth will be omitted.

According to [9], by neglecting gravity, assuming a locally isotropic model of visco–elastic phenomena (stiffness matrix $\mathbf{K}$ is proportional to damping matrix $\mathbf{B}$), and assuming that local variations of the Jacobian and grasp matrices are small, to ensures that blocks $\mathbf{L}_k$ and $\mathbf{L}_b$ in $\mathbf{A}$ are simply obtained as

$$\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k \qquad \mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b, \tag{7}$$

where

$$\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o), \mathbf{P}_k = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{K} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix}, \mathbf{P}_b = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{B} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix}.$$

2.1 A Grasp and its Geometric Property

The following grasp properties have a relevant influence on the dynamic behaviour of the manipulation system, cf. [9,1]. These properties are based on the existence of the null spaces of the grasp matrices $\mathbf{G}$ and of their transpose of matrices.

Definition 1. A grasp is said to be "indeterminate" if $\ker(\mathbf{G}^T) \neq 0$.

If the grasp is indeterminate, there exist motions of the objects under which no variations of contact force occur (2). In other words, indeterminacy implies that the object is not firmly grasped.

Definition 2. A manipulation system is said to be "graspable" if $\ker(\mathbf{G}) \neq 0$.

If the system is graspable it is possible to exert contact forces with zero resultant forces on the object. Usually in the literature the forces belonging to the null space of $\mathbf{G}$ are referred to as "internal forces". Such forces play a fundamental role in controlling the manipulation task. It is intuitive that, without internal forces squeezing the object, a manipulator only accommodates the object, rather than grasping it. Whenever the effect of a disturbance action on the object is in the tangential direction of a manipulator contact, the system cannot reject such a disturbance by simply opposing a contact force. It must generate an additional internal force to keep the total contact force in the friction cone and to maintain the contact. As far as stabilizability of the linearized dynamics is concerned, the following proposition stated in [1] is reported.

Proposition 1. If the system is not indeterminate, i.e. $\ker(\mathbf{G}^T) = 0$, then the minimal **A**–invariant subspace containing the $\mathrm{im}(\mathbf{B}_\tau)$, $\min I(\mathbf{A}, \mathbf{B}_\tau)$, is externally stable.

From now on, non indeterminacy is assumed, $\ker(\mathbf{G}^T) = 0$. Such an assumption is needed to assume that the linearized manipulation system (4) is stabilizable.

3. INTERNAL FORCES

The main goal of manipulation tasks consists of controlling the motion of the manipulated object. The stimulating aspect of manipulation control is that the manipulated object is not anchored to the robotic device, but this one acts on the object through passive (not directly actuated) "joints" consisting of a mechanical unilateral contact. Unilateral contacts are phenomena occurring between different parts of the system and are usually modelled as inequality constraints on the direction of forces and kinematic constraints on rolling and sliding motions.

Since contact constraints ensure both the object grasp and motion control, their non–violation is of paramount importance. Assuming that a general task specification is given in terms of the object motion, the remaining degrees of freedom by which contact phenomena can be controlled correspond to the "internal forces". These forces belong to the null space of the grasp matrix **G** and, as already pointed out, they are called "internal" as their resultant action on the object dynamics is zero.

3.1 Reachable Internal Contact Forces

Contact forces **t** are exerted on the object by the manipulating system in order to maintain the grasp, to reject disturbance wrenches **w** and control the object motion. Therefore, the control of contact forces is a fundamental part of the manipulation control problem, as the better the control of the forces is, the finer the manipulation.

In [9] the reachable subspace of contact forces as outputs of the dynamic system (4) was studied. The main result is reported in the next proposition.

Let us define $\delta\mathbf{t}$ as the departures of contact force vector **t** from the reference equilibrium $\mathbf{t}_o$ (5). Its first order approximation can be easily evaluated by substituting differential kinematics (2) in **t**, the grouped vector of $\mathbf{t}_i$'s (1). Hence

$$\mathbf{t} = \mathbf{C}_t \mathbf{x} \text{ where}$$
$$\mathbf{C}_t = \begin{bmatrix} \mathbf{KJ} & -\mathbf{KG}^T & \mathbf{BJ} & -\mathbf{BG}^T \end{bmatrix}. \tag{8}$$

Proposition 2. According to Definition (2), the reachable subspace of contact forces $\mathbf{t}$, under the hypothesis **K** is proportional to **B**, is

$$\mathcal{R}_{t,\tau} = \mathbf{C}_t \min\mathcal{I}(\mathbf{A}, \mathbf{B}_\tau) = \min\mathcal{I}(\mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{KJ}).$$

Control of the contact forces belonging to the null space of the grasp matrix **G** is normally an area of great interest of the research in this field. Obviously, in general, the null space of **G** is not completely reachable.

The importance of the reachability of internal forces in grasping was clarified in [12], where the principle of virtual work was used to describe the subspace of *active* internal forces, and in [1] where the *asymptotically reachable* internal forces were studied as a steady state behaviour of a suitable transfer function.

In this work the *reachable internal forces subspace* as an intersection of subspaces are characterized.

Definition 3. The reachable internal forces subspace $\mathcal{R}_{ti,\tau}$ is

$$\mathcal{R}_{ti,\tau} = \mathcal{R}_{t,\tau} \cap \ker(\mathbf{G}).$$

The following theorem provides an explicit formula for the reachable internal forces subspace:

Theorem 1. Under the hypothesis **K** is proportional to **B** then

$$\mathcal{R}_{ti,\tau} = \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{C}_t) = \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}).$$

Proof. The theorem statement is equivalent to

$$\mathcal{R}_{ti,\tau} \supseteq \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}) \tag{9}$$

$$\mathcal{R}_{ti,\tau} \subseteq \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}). \tag{10}$$

From Definition 3 and Proposition 2, the inclusion (9) turns into

$$\left(\min\mathcal{I}(\mathbf{K}\mathbf{G}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{K}\mathbf{J}) \cap \ker(\mathbf{G})\right) \supseteq \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}). \tag{11}$$

It is known that

$$\ker(\mathbf{G}) \supseteq \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}), \tag{12}$$

because the matrix $(\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J})$ is a projection onto the null space of $\mathbf{G}$. Moreover,

$$\min\mathcal{I}(\mathbf{K}\mathbf{G}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{K}\mathbf{J}) \supseteq \mathrm{im}\begin{bmatrix}\mathbf{K}\mathbf{J} & \mathbf{K}\mathbf{G}^T\mathbf{M}_o^{-1}\mathbf{G}\mathbf{K}\mathbf{J}\end{bmatrix} \supseteq \mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}), \tag{13}$$

because $\mathbf{M}_o^{-1}$ and $(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}$ are nonsingular matrices. Hence, (11) follows from (12) and (13).

Now, instead of proving the inclusion (10), its orthogonal complement is considered

$$\mathcal{R}_{ti,\tau}^{\perp} \supseteq (\mathrm{im}((\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}))^{\perp}. \tag{14}$$

Again from Definition 3, the previous relationship is equivalent to

$$\mathcal{R}_{ti,\tau}^{\perp} = \mathrm{im}(\mathbf{G}^T) + \mathcal{R}_{t,\tau}^{\perp} \supseteq \ker(\mathbf{J}^T\mathbf{K}(\mathbf{I} - \mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G}\mathbf{K}))$$

and being im ($\mathbf{G}^T$) the null space of the projection matrix $(\mathbf{I} - \mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G}\mathbf{K})$ the following relationship is obtained

$$\mathrm{im}(\mathbf{G}^T) + \mathcal{R}_{t,\tau}^{\perp} \supseteq \mathrm{im}(\mathbf{G}^T) + \mathrm{im}(\mathbf{I} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GK}) \cap \ker(\mathbf{J}^T\mathbf{K}).$$

Now, to prove (14) and end the theorem's proof, it will suffice to show that

$$\mathcal{R}_{t,\tau}^{\perp} \supseteq \mathrm{im}(\mathbf{I} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GK}) \cap \ker(\mathbf{J}^T\mathbf{K})$$

and this is trivial by considering the orthogonal

$$\mathcal{R}_{t,\tau} = \min I(\mathbf{KG}^T\mathbf{M}_o^{-1}\mathbf{G}, \mathbf{KJ}) \subseteq \ker(\mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G} - \mathbf{I}) + \mathrm{im}(\mathbf{KJ}).$$

According to this result, the subspace of reachable internal forces is obtained by projector $\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G}$ acting on the column space of $\mathbf{C}_t$. Notice that Theorem 1 states the equality of $R_{ti,\tau}$ with the active internal forces in [12] and with the asymptotically reachable internal forces in [1].

In order to specify consistent control outputs, the suggestion of Theorem 1 is followed. In fact, it is possible to choose as regulated force output $\mathbf{e}_{ti}$ the projection of the contact force vector $\mathbf{t}$ on the null space of $\mathbf{G}$. Then the output matrix is defined as follows

$$\mathbf{e}_{ti} = \mathbf{E}_{ti}\mathbf{x}; \quad \text{with} \quad \mathbf{E}_{ti} = (\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{C}_t = \begin{bmatrix} \mathbf{Q}_k & 0 & \mathbf{Q}_\beta & 0 \end{bmatrix}, \tag{15}$$

where

$$\mathbf{Q}_k = (\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ} \tag{16}$$

and

$$\mathbf{Q}_\beta = (\mathbf{I} - \mathbf{BG}^T(\mathbf{GBG}^T)^{-1}\mathbf{G})\mathbf{BJ}. \tag{17}$$

It should be noted that im $(\mathbf{Q}_k) = \mathrm{im}(\mathbf{Q}_\beta)$ under the hypothesis $\mathrm{im}(\mathbf{K}) = \mathrm{im}(\mathbf{B})$.

4. CONCLUSIONS

The geometric approach is used throughout the paper. The main result provides an explicit formula by using matrices for the reachable internal forces subspace. This work is in progress on the analysis of the forces structure of manipulation systems.

REFERENCES

[1] D. Prattichizzo and A. Bicchi. Consistent task specification for manipulation systems with general kinematics. *ASME Journal of Dynamics Systems Measurements and Control*, Vol. **119**, pp. 760–767, 1997.

[2] R.M. Murray, Z. Li, and S.S. Sastry. *A mathematical introduction to robotic manipulation*. CRC, Boca Raton, Florida, 1994.

[3] P. Mercorelli and D. Prattichizzo. A geometric procedure for robust decoupling control of contact forces in robotic manipulation. *Kybernetika*, Vol. **39(4)**, pp. 433–445, 2003.

[4] D. Prattichizzo and P. Mercorelli. On some geometric control properties of active suspension systems. *Kybernetika*, Vol. **36(5)**, pp. 549–570, 2000.

[5] G. Basile and G. Marro. A state space approach to non-interacting controls. *Ricerche di Automatica*, **1(1)**: 68–77, 1970.

[6] G. Basile and G. Marro. *Controlled and conditioned invariants in linear system theory*. Prentice Hall, New Jersey, 1992.

[7] W.M. Wonham and A.S. Morse. Decoupling and pole assignment in linear multivariable systems: a geometric approach. *SIAM J. Control*, Vol. **8(1)**, pp. 1–18, 1970.

[8] A.S. Morse and W.M. Wonham. Decoupling and pole assignment by dynamic compensation. *SIAM J. Control*, **(1)**: 317–337, 1970.

[9] D. Prattichizzo and A. Bicchi. Dynamic analysis of mobility and graspability of general manipulation systems. *Transaction on Robotic and Automation*, Vol. **14(2)**, pp. 251–218, 1998.

[10] J.K. Salisbury and B. Roth. Kinematic and force analysis of articulated mechanical hands. *J. Mech. Transm. Automat. in Des.*, 105, 1983.

[11] M.R. Cutkosky and I. Kao. Computing and controlling the compliance of a robotic hand. *IEEE Trans. Robotics Automat.*, Vol. **5(2)**, pp. 151–165, 1989.

[12] A. Bicchi. Force distribution in multiple whole–limb manipulation. In *ICRA*, 1993.