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EXTENSIVE SUBCLASSES OF MULTIVALENT ANALYTIC FUNCTIONS INTERPRETED BY GENERALIZED $q-$ VARIANCE DIFFERINTEGRAL OPERATOR

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ABSTRACT. In present paper, we defined the generalized $q-$ variance differintegral operator inspired by theory of $q-$calculus. We developed two subclasses of star like functions and determined some conditions such as coefficient estimation, deformation axiom, operations of $q-$fractional calculus, Fekete-Szego inequality and integral representation.

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1. Introduction

Let A_p denote the class of all functions of the type

$h(z)=z^p+\sum_{k=1}^{\infty}a_{p+k}z^{p+k}, \quad k>p;\ k,p\in\mathbb{N}$ (1-1) we consider the open unit disk represented by $D=\{z:|z|<1\}$ and having all the functions of the class A_p and let $H(p,k)$ be the subclass of A_p and a function $h(z)\in H(p,k)$ is defined by

$$h(z)=z^p-\sum_{k=1}^{\infty}A_{p+k}z^{p+k}, k>p;\quad k,p\in\mathbb{N} \tag{1-2}$$

For the functions $h(z)$ and $g(z)$ belong to D, the function $h(z)$ is subordinate to $g(z)$, written as $h(z)\prec g(z)$, if there exists a function $w(z)$, which is analytic in D with $w(0)=0$ and $|w(z)|<1, (z\in D)$ such that $h(z)=g(w(z)),\ (z\in D)$. Moreover, if the function $g(z)$ is univalent in D, then the following relation satisfies

$$h(z)\prec g(z) \iff h(0)=g(0)\ \& \ h(D)\subset g(D)$$

For detail see [16], [17]

Earlier, many researchers proposed and examined about an extended fractional differintegral operator. For detail see the work of Patel & Mishra [19], also see [1], [14], [20], 26].

Two renowned authors introduced in [19] about an extended fractional differintegral operator $\Omega_z^{(\lambda,p)} h : A_p \to A_p$ and for a function h of the type (1-1) we have:

$$\Omega_z^{(\lambda,p)} h = z^p + \sum_{k=1}^{\infty} C_{p,k}^{\lambda} a_{p+k} z^{p+k}; \lambda \in (-\infty, p+1) \subset \Re; p \in \mathbb{N} \quad (1\text{-}3)$$

Where, $$C_{p,k}^{\lambda} = \frac{\Gamma(p+1-\lambda)\Gamma(p+k+1)}{\Gamma(p+1)\Gamma(p+k-\lambda+1)} \quad (1\text{-}4)$$

We define a function $T(p,k;\lambda)$ the subclass of A_p, which contains the functions of the type:

$$\Omega_z^{(\lambda,p)} h(z) = z^p - \sum_{k=1}^{\infty} C_{p,k}^{\lambda} a_{p+k} z^{p+k}, z \in D \quad (1\text{-}5)$$

Within a few years, many researchers have outlined their concerns about the concept of q-calculus theory in geometric function theory because of its extensive advanced use in mathematical and physical sciences. Jackson [9-10] pioneered the theory of q-calculus. Ismail et al. [8] introduced the concept of derivative in q-calculus. Its intensive operations in the field of geometric function theory were given by Srivastava in a book chapter (see, for more details, [21] (pp. 347 et seq.)). In the same chapter Agrawal and Sahoo [2], given the use of q-hyper geometric functions. The contemporary contribution of this subject is given by Srivastava et al. [22-23]. Kanas and Raducanu [11] also worked and studied further in [3-15]. There are many researchers who have introduced some special classes of multivalent functions using q-calculus. For detailed contributions see [4-24]. Recently MirajUl-Haq et al. have been working on the theory of $q-$calculus in [18].

The $q-$difference operator, which was introduced by Jackson [9], and Heine [7], is defined by

$$\partial_q h(z) = \begin{cases} \dfrac{h(qz)-h(z)}{(q-1)z} & z \neq 0 \\ h'(0) & z = 0 \end{cases} \quad (1\text{-}6)$$

A.H. El-Qadeem et al. explained and generalized the q-difference operator in [5].

Now,

$$\partial_q^0 h(z) = h(z), \quad \partial_q^1 h(z) = \partial_q h(z), \quad \partial_q^2 h(z) = \partial_q(\partial_q h(z))$$

$$\partial_q^m h(z) = \partial_q(\partial_q^{m-1} h(z)), \quad m \in \mathbb{N} \tag{1-7}$$

If $h(z) \in H(p,k)$ and given by (1-2), we have

$$\partial_q^1 h(z) = \partial_q h(z) = h(z) = [p]_q z^{p-1} - \sum_{k=1}^{\infty} [p+k]_q a_{p+k} z^{p+k}, z \neq 0$$

$$[k]_q = \frac{1-q^k}{1-q} \tag{1-8}$$

$$\partial_q^m h(z) = \left\{ \prod_{i=1}^{m} [p-i+1]_q \right\} z^{p-m}$$

$$-\sum_{k=1}^{\infty} \left\{ \prod_{i=1}^{m} [k-i+1]_q \right\} a_{p+k} z^{p+k-m} \tag{1-9}$$

$(m \in \mathbb{N}; z \neq 0)$

Following the idea of A.H. El-Qadeem and M.A. Mamon [5], we introduced generalized q-variance differintegral operator for any integer λ given by

$$\partial_q \Omega_z^{(\lambda,p)}; \ \lambda \in \mathbb{Z}; \ \lambda < p+1$$

For $h \in T(p,k;\lambda)$, we have

$$\partial_q \Omega_z^{(\lambda,p)} h = [p]_q z^p - \sum_{k=1}^{\infty} [p+k]_q C_{p,k}^{\lambda} a_{p+k} z^{p+k}, z \in D; \lambda \in \mathbb{Z} \tag{1-10}$$

Definition 1.1 Let $h(z) \in T(p,k;\lambda)$ and also $h(z) \in T_q(\Omega_z, \lambda, p, \alpha; \psi)$

$$\Leftrightarrow \quad \frac{1}{p-\alpha} \left\{ \frac{z\partial_q \Omega_z^{(\lambda,p)} h}{[p]_q \Omega_z^{(\lambda,p)} h} - \alpha \right\} \prec \psi(z) \tag{1-11}$$

$(0 \le \alpha < p; z \in D); p \in \mathbb{N}; \lambda \in \mathbb{Z}$

Definition1.2 The function $T_q(\Omega_z, \lambda, p, \alpha; A, B)$ is said to be the subclass of functions $h(z) \in T(p,k;\lambda)$ and satisfies the condition

$$\frac{z\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} \prec [p]_q (p-\alpha) \frac{(1+Az)}{(1+Bz)} + \alpha [p]_q \tag{1-12}$$

$$-1 \le A < B \le 1 \quad \& \quad 0 \le \alpha < [p]_q$$

It is equivalent

$$T_q(\Omega_z, \lambda, p, \alpha; A, B) =$$

$$\left\{ h(z) \in T(p,k;\lambda) : \left| \frac{\dfrac{z\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} - p[p]_q}{B \dfrac{z\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} + \left\{ Ap + (A-B)\alpha \right\} [p]_q} \right| < 1, z \in D \right\}$$

where $\alpha \in [0,[p]_q], q \in [0,1); A, B \in [-1,1]; a < b; k, p \in \mathbb{N}$

In q —calculus the q —integral operator $\Im_{t,p}^q$ is expressed as

$$\Im_{t,p}^q h(z) = \frac{[t+p]_q}{z^t} \int_0^z u^{t-1} h(u) d_q u \tag{1-13}$$

For the function $h(z)$ defined by (1-2) we express

$$\Im_{t,p}^q h(z) = z^p - \sum_{k=n}^{\infty} \frac{[t+p]_q}{[t+k]_q} a_{p+k} z^{p+k}, (t < -p; p \in \mathbb{N}; 0 < q < 1) \tag{1-14}$$

Gasper and Rahman in [6] introduced the " q-Gamma function" for $z \in \mathbb{C}, z \ne -n, n \in \mathbb{N}_0$

$$\Gamma_q(z) = \frac{(q;q)_\infty}{(q^z;q)_\infty} (1-q)^{1-z}, q \in (0,1) \tag{1-15}$$

Where $(\alpha;q)_\infty = \prod_{m=0}^{\infty} (1-\alpha q^m), q \in (0,1)$

Koepf in [13] introduced by using q-gamma function the q-analogue of the factorial for $k \in \mathbb{N}$

$$[k]_q! = faq(k,q) = \frac{(q;q)_k}{(1-q)^k} = \Gamma_q(k+1) \tag{1-16}$$

The fractional $q-$integral operator is defined by Wongsaijai and Sukantamala in [25] for $h \in H(p,k)$.

$$D_{q,z}^{-\varepsilon}h(z) = \frac{1}{\Gamma_q(\varepsilon)}\int_0^z (z-qu)_{1-\varepsilon}h(u)d_q u\,;\ \varepsilon > 0 \tag{1-17}$$

Where, $(z-qu)_{1-\varepsilon}$ is called as $q-$binomial and defined by

$$\begin{aligned}(z-qu)_{1-\varepsilon} &= z^{\varepsilon-1}\phi_0[q^{-\varepsilon+1};-;q;\frac{uq^{\varepsilon}}{z}] \\ &= z^{\varepsilon-1}\prod_{m=0}^{\infty}\left(\frac{1-(\frac{uq}{z})q^m}{1-(\frac{uq}{z})q^{\varepsilon+m-1}}\right)\end{aligned} \tag{1-18}$$

The series $\phi_0[\varepsilon;-;q;z]$ is a singular when $|\arg z| < \pi, |z| < 1$ so that $(z-qu)_{1-\varepsilon}$ is singular if $\left|\arg\left(\frac{-uq^{\varepsilon}}{z}\right)\right| < \pi, |uq^{\varepsilon}| < 1$ and $|z| < 1$

Thus, if $h(z)$ defined by (1-2) then we have,

$$D_{q,z}^{-\varepsilon}h(z) = \frac{[p]_q\,!}{[p+\varepsilon]_q\,!}z^{p+\varepsilon} - \sum_{k=1}^{\infty}\frac{[k]_q\,!}{[k+\varepsilon]_q\,!}a_{p+k}z^{p+k+\varepsilon} \tag{1-19}$$

L. Shi and Q. Khan in [24] defined an operator named $q-$derivative operator of order ε $(0 \le \varepsilon < 1)$ represented by $\Delta_{q,z}^{\varepsilon}$ of $h \in H(p,k)$

$$\Delta_{q,z}^{\varepsilon}h(z) = \frac{1}{\Gamma_q(1-\varepsilon)}\partial_q\int_0^z (z-qu)_{-\varepsilon}h(u)d_q u \tag{1-20}$$

$$\Delta_{q,z}^{\varepsilon}h(z) = \frac{[p]_q\,!}{[p-\varepsilon]_q\,!}z^{p-\varepsilon} - \sum_{k=1}^{\infty}\frac{[k]_q\,!}{[k-\varepsilon]_q\,!}a_{p+k}z^{p+k-\varepsilon} \tag{1-21}$$

2. Coefficient Estimation

Theorem 2.1 If any function h is expressed by (1-5) satisfying the conditions, $\alpha \in [0,[p]_q); q \in [0,1); A, B \in [-1,1]; b > a; k, p \in \mathbb{N}$ then

$$h \in T_q(\Omega_z, \lambda, p, \alpha; a, b) \quad \Leftrightarrow$$

$$\sum_{k=1}^{\infty} C_{p,k}^{\lambda}\left[\left\{(1+A)p-(A-B)\alpha\right\}[p]_q-(1-B)[p+k]_q\right]a_{p+k}$$
$$\leq\left\{A-(A-B)\alpha+(1+A)p-1\right\}[p]_q \tag{2-1}$$

Proof. It is supposed that the result (2-1) holds true, we concluded from (1-2), (1-12) and (2-1) that

$$\left|z\partial_q\Omega_z^{(\lambda,p)}h-p[p]_q\Omega_z^{(\lambda,p)}h\right|-$$
$$\left|Bz\partial_q\Omega_z^{(\lambda,p)}h-\left\{(A-B)\alpha-Ap\right\}[p]_q\Omega_z^{(\lambda,p)}h\right|$$
$$=\left|[p]_q(1-p)z^p+\sum_{k=1}^{\infty}C_{p,k}^{\lambda}a_{p+k}\left\{p[p]_q-[p+k]_q\right\}z^{p+k}\right|-$$
$$\left|[p]_q\left\{A-\vartheta\alpha+Ap\right\}z^p+\sum_{k=1}^{\infty}C_{p,k}^{\lambda}a_{p+k}\left[\left\{\vartheta\alpha-Ap\right\}[p]_q-[p+k]_q\right]z^{p+k}\right|,$$

$A-B=\vartheta$

$$\leq[p]_q\left\{(1-p)+B-\vartheta\alpha+Ap\right\}|z|^p+$$
$$\sum_{k=1}^{\infty}C_{p,k}^{\lambda}a_{p+k}\left[\left\{\vartheta\alpha-Ap+p\right\}[p]_q-(1+B)[p+k]_q\right]|z|^{p+k}$$

Therefore, we obtained

$$\left|z\partial_q\Omega_z^{(\lambda,p)}h-p[p]_q\Omega_z^{(\lambda,p)}h\right|-$$
$$\left|Bz\partial_q\Omega_z^{(\lambda,p)}h-\left\{(\vartheta\alpha-Ap\right\}[p]_q\Omega_z^{(\lambda,p)}h\right|$$
$$<-[p]_q\left\{\vartheta\alpha+(1-A)p-1\right\}|z|^p+$$
$$\sum_{k=1}^{\infty}C_{p,k}^{\lambda}a_{p+k}\left[\left\{\vartheta\alpha+(1-A)p\right\}[p]_q-(1+B)[p+k]_q\right]|z|^{p+k}\leq 0$$

Now we apply the principle of maximum modulus, we obtained that

$$\left|\frac{\dfrac{z\partial_q\Omega_z^{(\lambda,p)}h}{\Omega_z^{(\lambda,p)}h}-p[p]_q}{B\dfrac{z\partial_q\Omega_z^{(\lambda,p)}h}{\Omega_z^{(\lambda,p)}h}+\left\{Ap+\vartheta\alpha\right\}[p]_q}\right|<1$$

Therefore, $h\in T_q(\Omega_z,\lambda,p,\alpha;A,B)$

Contrariwise, let us suppose $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$ be expressed by (1-5), then from (1-5) and (1-10), we obtained that

$$\left| \frac{\frac{z\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} - p[p]_q}{B \frac{z\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} + \left\{ Ap + \vartheta\alpha \right\} [p]_q} \right| =$$

$$\left| \frac{(1-p)[p]_q z^p + \sum_{k=1}^{\infty} C_{p,k}^{\lambda} \left\{ p[p]_q - [p+k]_q \right\} a_{p+k} z^{p+k}}{\left\{ B - \vartheta\alpha + Ap \right\} [p]_q z^p - \sum_{k=1}^{\infty} C_{p,k}^{\lambda} \left[B[p+k]_q - \left\{ \vartheta\alpha - Ap \right\} [p]_q a_{p+k} z^{p+k} \right]} \right|$$
$$< 1$$

Since we know that, $\mathbb{R}e(z) \leq |z|, \ \forall z$, then we conclude

$$\mathbb{R}e \left\{ \frac{(1-p)[p]_q z^p + \sum_{k=1}^{\infty} C_{p,k}^{\lambda} \left\{ p[p]_q - [p+k]_q \right\} a_{p+k} z^{p+k}}{\left\{ B - \vartheta\alpha + Ap \right\} [p]_q z^p - \sum_{k=1}^{\infty} C_{p,k}^{\lambda} \left[B[p+k]_q - \left\{ \vartheta\alpha - Ap \right\} [p]_q a_{p+k} z^{p+k} \right]} \right\} < 1 \qquad (2\text{-}2)$$

Now, if z is taken on the real axis, so that the expression $\frac{z\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h}$ becomes real. After simplifying equation (2-2) and assuming $z \to 1^-$ for the real z, we obtained

$$\sum_{k=1}^{\infty} C_{p,k}^{\lambda} \left\{ p[p]_q - [p+k]_q \right\} a_{p+k}$$
$$\leq \left\{ B - \vartheta\alpha + (1+A)p - 1 \right\} [p]_q - \sum_{k=1}^{\infty} C_{p,k}^{\lambda} \left[B[p+k]_q - \left\{ \vartheta\alpha - Ap \right\} [p]_q \right] a_{p+k}$$

Thus the required condition is obtained.

Corollary 1 If $h(z)$ is expressed by (1-5) and

$h(z) \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$, then

$$a_{p+k} \le \frac{\{B - \vartheta\alpha + (1+A)p - 1\}[p]_q}{C^{\lambda}_{p,k}\left[\{(1+A)p - \vartheta\alpha\}[p]_q - (1-B)[p+k]_q\right]}, \quad A - B = \vartheta \tag{2-3}$$

This obtained conclusion is sharpened for h defined by

$$h = z^p - \frac{\{B - \vartheta\alpha + (1+A)p - 1\}[p]_q}{C^{\lambda}_{p,k}\left[\{(1+A)p - \vartheta\alpha\}[p]_q - (1-B)[p+k]_q\right]} z^k \tag{2-4}$$

3. Deformation Axiom

In this section, we define deformation axiom for the class $T_q(\Omega_z, \lambda, p, \alpha; A, B)$.

Theorem 3.1 If h is expressed by (1-5) and $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$ then for a function given by (1-8) and $m \in \mathbb{N}$, we obtained

$$\left\{ \prod_{i=1}^{m}[p-i+1] - \frac{\prod_{i=1}^{m}[k-i+1]_q \{A - \vartheta\alpha + (1+A)p - 1\}[p]_q}{[(1+A)p - \vartheta\alpha][p]_q - (1-B)[p+n]_q} |z|^n \right\} |z|^{p-m},$$

$$A - B = \vartheta$$

$$\le \left|\partial_q \Omega_z^{(\lambda,p)} h\right|$$

$$\le \left\{ \prod_{i=1}^{m}[p-i+1] + \frac{\prod_{i=1}^{m}[k-i+1]_q \{B - \vartheta\alpha + (1+A)p - 1\}[p]_q}{[(1+A)p - \vartheta\alpha][p]_q - (1-B)[p+n]_q} |z|^n \right\} |z|^{p-m} \tag{3-1}$$

The above obtained result is sharpened for the function h defined by

$$h = z^p - \frac{\{B - \vartheta\alpha + (1+A)p - 1\}[p]_q}{C^{\lambda}_{p,n}\left[\{(1+A)p - \vartheta\alpha\}[p]_q - (1-B)[p+n]_q\right]} |z|^{p+n} \tag{3-2}$$

Proof Assuming the inequality (2-1) holds true, we conclude that

$$\frac{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}\sum_{k=1}^{\infty}a_{p+k},\quad A-B=\vartheta$$

$$\leq\sum_{k=1}^{\infty}\frac{C_{p,k}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+k]_q\right]}{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}a_{p+k}\leq 1$$

Now we obtained,

$$\sum_{k=1}^{\infty}a_{p+k}\leq 1\frac{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}\tag{3-3}$$

Then, theorem 3.1 would follow from (1-10) and (3-3).

4. Operations of theory of q-fractional

This segment contains the examination of deformation axiom for functions of $T_q(\Omega_z,\lambda,p,\alpha;A,B)$ by using the operators $\Im_{t;p}^{q}$, $\Delta_{q,z}^{-\varepsilon}$, and $\Delta_{q,z}^{\varepsilon}$,provided that $(0<\varepsilon\in\mathbb{Z},t>-p,p\in\mathbb{N})$.

If $h(z)$ is given by (1-5), so that

$$\begin{gathered}\Delta_{q,z}^{-\varepsilon}\left(\left(\Im_{t;p}^{q}h\right)(z)\right)=\lambda_1 z^{p+\varepsilon}-\sum_{k=1}^{\infty}\lambda_2 a_{p+k}z^{p+k+\varepsilon},\\ \lambda_1=\frac{[p]_q\,!}{[p+\varepsilon]_q\,!},\ \lambda_2=\frac{[t+p]_q}{[t+k]_q}\frac{[k]_q\,!}{[k+\varepsilon]!}\end{gathered}\tag{4-1}$$

$$\begin{gathered}\Delta_{q,z}^{\varepsilon}\left(\left(\Im_{t;p}^{q}h\right)(z)\right)=\lambda_3 z^{p-\varepsilon}-\sum_{k=1}^{\infty}\lambda_4 a_{p+k}z^{p+k-\varepsilon},\\ \lambda_3=\frac{[p]_q\,!}{[p-\varepsilon]_q\,!},\lambda_4=\frac{[t+p]_q}{[t+k]_q}\frac{[k]_q\,!}{[k-\varepsilon]!}\end{gathered}\tag{4-2}$$

$$\begin{gathered}\Im_{t;p}^{q}\left(\Delta_{q,z}^{-\varepsilon}(h(z))\right)=\frac{[t+p]_q[p]_q\,!}{[t+p+\varepsilon]_q[p+\varepsilon]_q\,!}z^{p+\varepsilon}\\ -\sum_{k=1}^{\infty}\frac{[t+p]_q}{[t+k+\varepsilon]_q}\frac{[k]_q\,!}{[k+\varepsilon]_q\,!}a_{p+k}z^{p+k+\varepsilon}\end{gathered}\tag{4-3}$$

and

$$\Im_{t;p}^{q}\left(\Delta_{q,z}^{\varepsilon}(h(z))\right)=\frac{[t+p]_q[p]_q\,!}{[t+p-\varepsilon]_q[p-\varepsilon]_q\,!}z^{p-\varepsilon}$$
$$-\sum_{k=1}^{\infty}\frac{[t+p]_q}{[t+k-\varepsilon]_q}\frac{[k]_q\,!}{[k-\varepsilon]_q\,!}a_{p+k}z^{p+k-\varepsilon} \tag{4-4}$$

Theorem 4.1 If h is expressed by (1-5) and $h\in T_q(\Omega_z,\lambda,p,\alpha;A,B)$, then

$$\left|\Delta_{q,z}^{-\varepsilon}\left(\left(\Im_{t;p}^{q}h\right)(z)\right)\right|\geq$$
$$\left\{\lambda_1-\lambda_2\frac{\{b-\vartheta\alpha+(1+a)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+a)p-\vartheta\alpha\}[p]_q-(1-b)[p+n]_q\right]}|z|^n\right\}|z|^{p+\varepsilon} \tag{4-5}$$

and,

$$\left|\Delta_{q,z}^{-\varepsilon}\left(\left(\Im_{t;p}^{q}h\right)(z)\right)\right|\leq$$
$$\left\{\lambda_1+\lambda_2\frac{\{B-\vartheta\alpha+(1+A)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+A)p-\vartheta\alpha\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\right\}|z|^{p+\varepsilon} \tag{4-6}$$

Proof Let us suppose the function of the type

$$\varphi(z)=\frac{[p+\varepsilon]_q\,!}{[p]_q\,!}z^{-\varepsilon}\Delta_{q,z}^{-\varepsilon}((\Im_{t;p}^{q}h)(z)$$

$$\varphi(z)=z^p-\sum_{k=1}^{\infty}\frac{[t+p]_q}{[t+k]_q}\frac{[k]_q\,![p+\varepsilon]_q\,!}{[k+\varepsilon]_q\,![p]_q\,!}a_{p+k}z^{p+k}=z^p-\sum_{k=1}^{\infty}\rho(k)a_{p+k}z^{p+k}$$

$$\rho(k)=\sum_{k=1}^{\infty}\frac{[t+p]_q}{[t+k]_q}\frac{[k]_q\,![p+\varepsilon]_q\,!}{[k+\varepsilon]_q\,![p]_q\,!} \tag{4-7}$$

Since $\rho(k)$ decreases according as $k(n\leq k)$, then for $0<\varepsilon\in\mathbb{Z}$, we get

$$0<\rho(k)\leq\rho(n)=\frac{[t+p]_q}{[t+n]_q}\frac{[n]_q\,![p+\varepsilon]_q\,!}{[n+\varepsilon]_q\,![p]_q} \tag{4-8}$$

Therefore, from (3-3) and (4-8), we obtained that

$$|\varphi(z)|\geq|z|^p-\rho(n)|z|^n\sum_{k=n}^{\infty}a_{p+k}\geq$$

$$|z|^p - \frac{[t+p]_q}{[t+n]_q}\frac{[n]_q![p+\varepsilon]_q!}{[n+\varepsilon]_q![p]_q} \frac{\{A-\vartheta\alpha+(1+A)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+A)p-\vartheta\alpha\}[p]_q-(1-B)[p+n]_q\right]}|z|^n \tag{4-9}$$

and,

$$|\varphi(z)| \le |z|^p + \rho(n)|z|^n \sum_{k=n}^{\infty} a_{p+k} \ge$$

$$|z|^p + \frac{[t+p]_q}{[t+n]_q}\frac{[n]_q![p+\varepsilon]_q!}{[n+\varepsilon]_q![p]_q} \frac{\{A-\vartheta\alpha+(1+A)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+A)p-\vartheta\alpha\}[p]_q-(1-B)[p+n]_q\right]}|z|^n \tag{4-10}$$

Thus we obtained the inequalities (4-5) and (4-6) of theorem 4.1

The inequalities in (4-5) and (4-6) are also obtained for the function h given in (3-2)

$$\Delta_{q,z}^{-\varepsilon}\left(\left(\Im_{t;p}^{q}h\right)(z)\right) = \left\{\frac{[p]_q!}{[p+\varepsilon]_q!} - \frac{[t+p]_q}{[t+n]_q}\frac{[n]_q!}{[n+\varepsilon]_q!} \frac{\{B-\vartheta\alpha+(1+A)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+A)p-\vartheta\alpha\}[p]_q-(1-B)[p+n]_q\right]}z^n\right\} z^{p+\varepsilon} \tag{4-11}$$

or

$$(\Im_{t;p}^{q}h)(z) = z^p - \frac{[t+p]_q}{[t+n]_q} \frac{\{B-\vartheta\alpha+(1+A)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+A)p-\vartheta\alpha\}[p]_q-(1-B)[p+n]_q\right]}|z|^n \tag{4-12}$$

Thus, the required proof of theorem 4.1 is obtained.

Theorem 4.2 If h is expressed by (1-5) and $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$ then

$$\left|\Delta_{q,z}^{\varepsilon}\left\{\left(\Im_{t,p}^{q}h\right)(z)\right\}\right| \geq \left\{\lambda_3 - \lambda_4 \frac{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\right\}|z|^{p-\varepsilon} \tag{4-13}$$

and,

$$\left|\Delta_{q,z}^{\varepsilon}\left(\left(\Im_{t;p}^{q}h\right)(z)\right)\right| \leq \left\{\lambda_3 + \lambda_4 \frac{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\right\}|z|^{p-\varepsilon} \tag{4-14}$$

the results obtained in (4-13) and (4-14) are sharpened.

Theorem 4.3 If h is expressed by (1-5) and $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$, then

$$\left|\Im_{t;p}^{q}\left(\Delta_{q,z}^{-\varepsilon}\left(h(z)\right)\right)\right| \geq \left\{\begin{array}{c}\frac{[t+p]_q[p]_q\,!}{[t+p+\varepsilon]_q[p+\varepsilon]_q\,!} - \frac{[t+p]_q}{[t+n+\varepsilon]_q}\frac{[n]_q\,!}{[n+\varepsilon]_q\,!} \\ \frac{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\end{array}\right\}|z|^{p+\varepsilon}$$

and

$$\left|\Im_{t;p}^{q}\left(\Delta_{q,z}^{-\varepsilon}\left(h(z)\right)\right)\right| \leq \left\{\begin{array}{c}\frac{[t+p]_q[p]_q\,!}{[t+p+\varepsilon]_q[p+\varepsilon]_q\,!} + \frac{[t+p]_q}{[t+n+\varepsilon]_q}\frac{[n]_q\,!}{[n+\varepsilon]_q\,!} \\ \frac{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\end{array}\right\}|z|^{p+\varepsilon}$$

Theorem 4.4 If h is expressed by (1-5) and $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$, then

$$\left|\Im_{t;p}^{q}\left(\Delta_{q,z}^{\varepsilon}\left(h(z)\right)\right)\right| \geq \left\{\begin{array}{c}\frac{[t+p]_q[p]_q\,!}{[t+p-\varepsilon]_q[p-\varepsilon]_q\,!} - \frac{[t+p]_q}{[t+n-\varepsilon]_q}\frac{[n]_q\,!}{[n-\varepsilon]_q\,!} \\ \frac{\left\{B-\vartheta\alpha+(1+A)p-1\right\}[p]_q}{C_{p,n}^{\lambda}\left[\left\{(1+A)p-\vartheta\alpha\right\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\end{array}\right\}|z|^{p+\varepsilon} \tag{4-15}$$

$$\left|\Im_{t;p}^{q}\left(D_{q,z}^{\varepsilon}\left(h(z)\right)\right)\right| \le$$

and
$$\left\{\begin{array}{l}\dfrac{[t+p]_q[p]_q\,!}{[t+p-\varepsilon]_q[p-\varepsilon]_q\,!}+\dfrac{[t+p]_q}{[t+n-\varepsilon]_q}\dfrac{[n]_q\,!}{[n-\varepsilon]_q\,!}\\ \dfrac{\{B-\vartheta\alpha+(1+A)p-1\}[p]_q}{C_{p,n}^{\lambda}\left[\{(1+A)p-\vartheta\alpha\}[p]_q-(1-B)[p+n]_q\right]}|z|^n\end{array}\right\}|z|^{p+\varepsilon} \tag{4-16}$$

5. Fekete-Szegö Inequality for the class $T_q(\Omega_z,\lambda,p,\alpha;A,B)$

This segment contains Fekete-Szegö inequality, Integral representation formula for the subclasses $T_q(\Omega_z,\lambda,p,\alpha;\psi)$ and $T_q(\Omega_z,\lambda,p,\alpha;A,B)$

Theorem 5.1 If $h\in T(p,k;\lambda)$ and is given by (1-5), with $\alpha\in[0,p)$, $A,B\in[-1,1];\ A<B$, if $h\in T_q(\Omega_z,\lambda,p,\alpha;A,B)$, then

$$\left|A_{p+2}-\chi A_{p+1}^2\right|\le\tau_2\max\left\{1,|\eta|\right\} \tag{5-1}$$

where $\eta=\dfrac{\tau_1-\chi\tau_3}{\tau_2}$, $\tau_1=\dfrac{B\left\{[p+1]_q-\alpha[p]_q\right\}-A[p]_q(p-\alpha)}{C_{p,2}^{\lambda}\left(p[p]_q-[p+1]_q\right)\left(p[p]_q-[p+2]_q\right)}$

$$\tau_2=\frac{\{A(p-\alpha)-B(1-\alpha)\}[p]_q}{C_{p,2}^{\lambda}\left(p[p]_q-[p+2]_q\right)}$$

$$\tau_3=\left[\frac{\{A(p-\alpha)-B(1-\alpha)\}[p]_q}{C_{p,1}^{\lambda}\left(p[p]_q-[p+1]_q\right)}\right]^2 \tag{5-2}$$

the result obtained in (5-1) is sharpened.

Proof Since $h\in T_q(\Omega_z,\lambda,p,\alpha;A,B)$, we have

$$\frac{1}{p-\alpha}\left[\frac{z\partial_q\Omega_z^{(\lambda,p)}h}{[p]_q\,\Omega_z^{(\lambda,p)}h}-\alpha\right]=\frac{1+Aw}{1+Bw} \tag{5-3}$$

where $w(z)=\sum_{k=1}^{\infty}w_kz^k$ is a bounded analytic function and satisfying the condition $w(0)=0$ and $|z|<1$, $\forall z\in D$, or

$$\begin{aligned}&\left[Bz\partial_q\Omega_z^{(\lambda,p)}h(z)-\left\{Ap-\alpha\vartheta\right\}[p]_q\Omega_z^{(\lambda,p)}h(z)\right]w(z)\\&=p[p]_q\Omega_z^{(\lambda,p)}h(z)-z\partial_q\Omega_z^{(\lambda,p)}h(z)\end{aligned}\tag{5-4}$$

Writing corresponding series expansion in (5-4), we get

$$\begin{bmatrix}-\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q z^p-\\ \sum_{k=1}^{\infty}C_{p,k}^{\lambda}\begin{Bmatrix}B\left([p+k]_q-\alpha[p]_q\right)-\\A[p]_q(p-\alpha)\end{Bmatrix}a_{p+k}z^{p+k}\end{bmatrix}(w_1z+w_2z^2+........)$$

$$=(p-1)[p]_q z^p-\sum_{k=1}^{\infty}C_{p,k}^{\lambda}\left(p[p]_q-[p+k]_q\right)z^{p+k},\quad A-B=\vartheta$$

Equating the coefficient of z^{p+1} and z^{p+2}

$$-\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q w_1=-C_{p,1}^{\lambda}\left(p[p]_q-[p+1]_q\right)a_{p+1}$$

$$a_{p+1}=\frac{\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q w_1}{C_{p,1}^{\lambda}\left(p[p]_q-[p+1]_q\right)}\tag{5-5}$$

and,

$$\begin{aligned}&-\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q w_2-\\&C_{p,1}^{\lambda}\left[B\left\{[p+1]_q-\alpha[p]_q\right\}-A[p]_q(p-\alpha)\right]a_{p+1}w_1\\&=-C_{p,2}^{\lambda}\left(p[p]_q-[p+2]_q\right)a_{p+2}\end{aligned}$$

$$\begin{aligned}&\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q w_2-\\&C_{p,1}^{\lambda}\left[B\left\{[p+1]_q-\alpha[p]_q\right\}-A[p]_q(p-\alpha)\right]\\&\frac{\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q}{C_{p,1}^{\lambda}\left(p[p]_q-[p+1]_q\right)}w_1^2\\&=-C_{p,2}^{\lambda}\left(p[p]_q-[p+2]_q\right)a_{p+2}\end{aligned}$$

$$a_{p+2} = \frac{-\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q}{-C_{p,2}^{\lambda}\left(p[p]_q-[p+2]_q\right)} \left[\frac{B\left\{[p+1]_q-\alpha[p]_q\right\}-A[p]_q(p-\alpha)}{\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q\left(p[p]_q-[p+1]_q\right)}w_1^2-w_2\right] \tag{5-6}$$

Let χ be a complex quantity then we have

$$\left|a_{p+2}-\chi a_{p+1}^2\right| =$$

$$\left|\frac{\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q}{C_{p,2}^{\lambda}\left(p[p]_q-[p+2]_q\right)}\left[\frac{B\left\{[p+1]_q-\alpha[p]_q\right\}-A[p]_q(p-\alpha)}{\left(p[p]_q-[p+1]_q\right)\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q}w_1^2-w_2\right] -\chi\left[\frac{\left\{A(p-\alpha)-B(1-\alpha)\right\}[p]_q w_1}{C_{p,1}^{\lambda}\left(p[p]_q-[p+1]_q\right)}\right]^2\right|$$

$$=\left|\tau_1 w_1^2-\tau_2 w_2-\chi\tau_3 w_1^2\right|$$

$$=\tau_2\left|w_2-\eta w_1^2\right| \tag{5-7}$$

$$\text{Where, }\eta=\frac{\tau_1-\chi\tau_3}{\tau_2} \tag{5-8}$$

From the result of Keogh and Mcrker [12], If η be any complex number, it is given

$$\left|w_2-\eta w_1^2\right|\le \max\left\{1,\left|\eta\right|\right\},$$

This result is sharpened for the functions

$h_0(z)=z^p$ and $h_1(z)=z^{p+1}$ for $\left|\eta\right|\ge 1$ & $\left|\eta\right|<1$ respectively

From (5-7), it follows that

$$\left|a_{p+2}-\chi a_{p+1}^2\right|\le \tau_2 \max\left\{1,\left|\eta\right|\right\}$$

Where, η is given by (5-7).

6. Integral Representations for the Classes $h \in T_q(\Omega_z, \lambda, p, \alpha; \psi)$ and $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$

Theorem 6.1 Let $h(z) \in T(p,k;\lambda)$ of the form (1-5) be in the class $h \in T_q(\Omega_z, \lambda, p, \alpha; \psi)$ if and only if there exist a Schwarz function $w(z)$ such that

$$\Omega_z^{(\lambda,p)} = z^{\alpha[p]_q} \exp\int_0^z \frac{[p]_q(p-\alpha)\psi(w(z))}{t} d_q t \tag{6-1}$$

In particular, if $h \in T_q(\Omega_z, \lambda, p, \alpha; A, B)$

$$\Omega_z^{(\lambda,p)} h = \exp\left(\alpha[p]_q \int_0^z \frac{\left[\frac{p}{\alpha} - \left\{ \vartheta - \frac{Ap}{\alpha} \right\} L(t) \right]}{t(1-BQ(t))} d_q t \right) \tag{6-2}$$

Where $|L_z| < 1$ and

$$\Omega_z^{(\lambda,p)} h = \left(\frac{z}{1-yzB} \right)^{p[p]_q} \exp\int_0^z \log(1-yzB)^{\frac{\left\{ \vartheta - \frac{Ap}{\alpha} \right\} y\alpha[p]_q}{B}} d_q\mu(y)$$

Where $\mu(y)$ be the probability measure on the set $Y = \{ y : |y| = 1 \}$.

Proof Since $h \in A_p$ is supposed to be in the subclass $T_q(\Omega_z, \lambda, p, \alpha; \psi)$

$$\Leftrightarrow \quad \frac{1}{p-\alpha}\left[\frac{z\partial_q \Omega_z^{(\lambda,p)} h}{[p]_q \Omega_z^{(\lambda,p)} h} - \alpha \right] \prec \psi(z) \tag{6-3}$$

$$\frac{\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} - \frac{\alpha[p]_q}{z} = \frac{[p]_q(p-\alpha)\psi(w(z))}{z} \tag{6-4}$$

After integrating we obtained

$$\Omega_z^{(\lambda,p)} h = z^{\alpha[p]_q} \exp\int_0^z \frac{[p]_q(p-\alpha)\psi(w(z))}{t} d_q t \tag{6-5}$$

Again, from the condition of the subclass $T_q(\Omega_z, \lambda, p, \alpha; A, B)$

$$\left| \frac{w - \frac{p}{\alpha}}{Bw - \left\{ \vartheta - \frac{Ap}{\alpha} \right\}} \right| < 1, \text{ where } w = \frac{z\partial_q \Omega_z^{(\lambda,p)} h}{\alpha [p]_q \Omega_z^{(\lambda,p)} h}$$

$$\frac{w - \frac{p}{\alpha}}{Aw - \left\{ \vartheta - \frac{Ap}{\alpha} \right\}} = L_z \text{ Then, } |L_z| < 1$$

Finally we have

$$\frac{z\partial_q \Omega_z^{(\lambda,p)} h}{\alpha [p]_q \Omega_z^{(\lambda,p)} h} = \frac{\frac{p}{\alpha} - \left\{ \vartheta - \frac{Ap}{\alpha} \right\} L_z}{1 - BL_z} \tag{6-6}$$

$$\frac{\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} = \frac{\alpha [p]_q \left[\frac{p}{\alpha} - \left\{ \vartheta - \frac{Ap}{\alpha} \right\} L_z \right]}{z(1 - BL_z)}$$

On Integrating we obtained

$$\log \Omega_z^{(\lambda,p)} h = \alpha [p]_q \int_0^z \frac{\left[\frac{p}{\alpha} - \left\{ \vartheta - \frac{Ap}{\alpha} \right\} L_t \right]}{t(1 - BL_t)} d_q t \tag{6-7}$$

Therefore we get (6-1).

For obtaining the third representation let $Y = \{ y : |y| = 1 \}$ then, we have

$$\frac{w - \frac{p}{\alpha}}{Bw - \left\{ \vartheta - \frac{Ap}{\alpha} \right\}} = yz, \, y \in Y, \, z \in D$$

and then we conclude that

$$\frac{\partial_q \Omega_z^{(\lambda,p)} h}{\Omega_z^{(\lambda,p)} h} = p[p]_q \left\{ \frac{1}{z} + \frac{yb}{1 - yzb} \right\} - \frac{\left\{ \vartheta - \frac{Ap}{\alpha} \right\} y\alpha [p]_q}{(1 - yzB)} \tag{6-8}$$

On Integrating, we get

$$\log \Omega_z^{(\lambda,p)} h = p[p]_q \log\left(\frac{z}{1-yzB}\right) + \frac{\left\{\vartheta - \frac{Ap}{\alpha}\right\} y\alpha[p]_q}{B} \log(1-yzB)$$

Or

$$\Omega_z^{(\lambda,p)} h = \left(\frac{z}{1-yzB}\right)^{p[p]_q} \exp\int_0^z \log(1-yzB)^{\frac{\left\{\vartheta - \frac{Ap}{\alpha}\right\} y\alpha[p]_q}{B}} d_q\mu(y) \qquad (6\text{-}9)$$

The function $\mu(y)$ is defined by the probability measure on $Y = \{y : |y| = 1\}$.

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