1. INTRODUCTION

The goal of this study is to contribute to the remodeling simulation of bone tissue. This can be achieved after understanding the mechanosensation of bone cells. The presented work is focused on the micro length scale but, the homogenised results might be of great importance to the specification of bone’s growth and remodeling. The better understanding of problems like implantation of hip-joint-prosthesis (Ebbecke and Nackenhorst (2003)) or osteoporosis (Mullender et al. (1996)) and the cooperation of the engineers with medical professionals will lead to the improvement in diagnostics and the decrease in the often painful healing of bone fractures.

Nowadays, numerous simulations based on computational methods and clinical observations are carried out (Ebbecke and Nackenhorst (2003)). However, these kind of calculations cannot take into consideration all processes which occur at the microscale. Especially the process of detecting the external forces by bone tissue namely mechanosensation. It is certain that the network of osteocytes is responsible for detecting the external loading, however, sending signals to other cells in order to remodel the bone is not yet known (You et al. (2001)). The latest results (Lenz and Nackenhorst (2003)) describe these relations between the mechanical loading and strain detection in bone tissue and give a better view over most length scales (nano-micro-meso).

The hierarchical structure is of great importance in this work. It helps to identify on which of the length scales need to be carried out the modeling process. The most important bone components for the mechanical simulation are briefly introduced. Further on the biomechan of skeletal adaptation is presented along with several theories concerning this process. Next, the continuum mechanics concepts (kinematics, tensor description) as well as the constitutive relations (linear isotropy and transversal orthotropy) are presented.

Finally the simulation of the living tissues is presented. Formulation of the model requires wide knowledge in both mechanical and biological fields. In order to obtain the geometry which would describe the biological reality in the proper way, it is necessary to use all available medical methods including e.g. microtomographic pictures. The exactness of the biological geometry was carefully studied in this work and significantly improved in comparison to previous research. Knowing the geometry of the objects under consideration along with the bone tissue behavior under mechanical loading it is possible to attempt the modeling process.

The main goal of this work is to create a three dimensional model of a single osteon and numerical simulation of the selected boundary value problems that reflect the micro strain distribution responsible for the remodeling of bone tissue on the micro level. The complexity of the task as in three dimensional geometrical modeling and numerical computations, requires strong computational environment of the most advanced preprocessors and solvers.

The mechanical approach is carried out in a typical engineering way. The model is prepared and computed with the use of standard engineering software. The
complexity of the modeled structure requires Finite Element Model analysis. The detailed description of the modeling approach is given. Several models with different material properties and shapes of bone cells responsible for the strain detection are presented. Obtained results (strain, stress and displacement visualization) are shown and compared with existing theories ((Frost (1992), You et al. (2001)), the fluid-flow theory (Cowan (2002), Turner and Pavalko (1998)), streaming potentials theory (Korenstein et al. (1984)) and damage-repair theory (Doblare and Garcia (2002)). A brief summary of the models and results along with the concluding remarks is given and future perspectives are suggested.

2. BIOLOGICAL BACKGROUND

Human’s skeleton is like a very complex building. It determines the shape of the body, protects the organs and works closely together with the muscular system to allow us to move. There are 206 bones which build up a skeleton. It means that all these components must function and cooperate properly in order to create a structure which will handle all external, sometimes extreme, loading conditions. In order to provide such conditions it is crucial to understand the behavior and structure of a single skeleton’s component. In the following section a multiscale bone structure is presented and an overview of the bone and its tissue is given.

Bone is a two-phase porous natural composite material comprised primarily of collagen and mineral, which together provide its mechanical properties. The organic component, mainly collagen, gives bone its form and contributes to its ability to resist tension, while mineral component, inorganic, primarily resists compression. Bones are biological structural materials made of dynamically adaptable tissue. Its remodeling is described by Wolf’s law (Wolff (1986)).

Although bones are similar in structure and development, they vary considerably in size and shape. The architecture of bones is composed of very complex material with extraordinary properties that optimally adjust to external loads or self-replacing mechanisms. In order to understand these properties it is important to understand the mechanical properties of all bone’s component phases and the structural relationships among them at the various levels of hierarchical structural organization (shown in Figure 1 from et al. (1998)).

Although all computations were carried on a microscale it was necessary to use results from other length scales (e.g. osteonal parameters from nanoscale, Lenz and Nackenhorst (2003)) in order to understand or describe mechanical and material properties. Figure 1 gives a good introduction into the hierarchical structure of a long bone.
3. MECHANOSENSITIVITY

The fact that bone tissue adapts its material properties, and the whole bone shape, in response to altered mechanical loading was proved and confirmed long time ago. However, the physiological mechanism by which the mechanical loading applied to the bone is sensed and the mechanism by which the sensed signal is transmitted to the cells has not been finally identified yet.

In order to make the theoretical description entirely complete, one more 'ingredient' ought to be presented – the mechanosensation process. Although the capacity of bone tissue to adapt to changing mechanical demands is well known, the reaction of bone cells is not fully understood. In the last decades significant progress has been made in understanding the bone cells reaction on mechanical signals, derived from bone loading. Lately, several theories concerning this problem have been developed (Cowin (1999), V. D. Meulen and Prendergast (2000), Turner and Pavalko (1998), or books like Cowin (2001), Martin et al. (1998)).

The mechanosensation process is responsible for detection of the mechanical loading on the bone. It is one of two main subprocesses of the mechanotransduction (see below).

**Mechanotransduction in Bone**

- bone loading
- \(\rightarrow\)
- matrix strain
- \(\rightarrow\)
- mechanosensation by bone cells
- \(\rightarrow\)
- intercellular signaling:
  - mobilization of BMU’s
  - (basic multicellular units)
- \(\rightarrow\)
- bone remodeling

Mechanotransduction is the process which converts biophysical forces into cellular response and includes the cells reaction to the external loads.

The current theoretical and experimental evidence suggests that osteocytes (Cowin (2002)) are the principal mechanosensory cells of bone, which are activated by shear stress from fluid flowing through the canaliculi. The electrically coupled three-dimensional network of osteocytes and lining cells is the communication system for the control of bone structural strain detection. The described mechanosensory mechanisms in bone consist of:

1. The cell system that is stimulated by external mechanical loading applied to the bone,
2. The system that transduces that mechanical loading to a communicable signal,
3. The system that transmits that signal to the effector cells (osteoblasts, osteoclasts) for the maintenance of bone homeostasis and strain adaptation of the bone structure.

However, the way strains are detected in bones is still under research. After taking a closer look at bone, it becomes obvious that the network consisting of the above mentioned cells situated directly inside the bone tissue would have the capability to fulfill the demands of mechanosensation. At this point, it is necessary to address the problem of strain detection in bone cells. Figure 3 shows the relationship between bone activity and strain detection. As presented there are two characteristic points \(MES_c\) and \(MES_s\) which are boundary values, according to which the remodeling processes of bone (resorption, creation) are carried out. These two values point out the so called dead zone, in which no change in bone occurs.

However, the problem is much more complicated than shown in Figure 3. Quantitative studying of the strain in bones of performing animals (galloping horses, fast-flying birds, even a running human volunteer) found a maximal strain not higher than 0.2% to 0.3% (Rubin (1984), Burr et al. (1996). This poses the problem in interpreting the results of in vitro studies of strained bone cells, where much higher deformations, 1% to 10%, are needed to obtain a cellular response (Murray and Rushton (1990), Burger and Veldhuijzen (1993)). According to Neidlinger-Wilke et al. (1995) 1% of unidirectional cell stretching is required to receive the response in cultures of human bone cells. However, in an animal study using rats, 0.15% bending strain of the intact tibia was already sufficient to activate cell proliferation and bone formation in vivo (Turner et al. (1994), Forwood (1996)). Assuming the strain to be somehow involved in bone cell mechanosensing means that bone tissue must possess a lever system whereby the small matrix strains are transduced into a larger signal that is sensed by bone cells.

![Figure 3: Relationship between the strain detection on the osteon and bone reaction](image-url)
4. PURE STRAIN CONCEPT

The core of this concept is a reaction of osteocytes to pure strains or bone matrix deformations. The external forces act on the whole bone, and the strains caused on the macro length scale are the same as on the micro length scale of the osteocytes. But this problem is much more complex than described above. Several biological experiments were carried out and it was found that the strain occurring in osteocytes must be significantly above (about two orders magnitude higher) the strain expected in the bone (You et al. (2000), Burger and Veldhuijzen (1993)). According to other experiments strain values of about 1000 microstrain is required in order to get a response from osteoblasts (Kaspar et al. (2000)) while Frost (1992) described that minimum as 1500 microstrain. With the strains of this magnitude bone will undergo remodeling and increase its density in order to reduce local strains. This action will last as long as the local strains are small enough. However, higher local strains will not cause any action and too high strains will cause damage in the bone tissue. There are two interesting models suggested. One of them is trying to explain how the osteocytes could amplify the strain to a level which would be detected by cellular mechanisms (You et al. (2001)) while the second attempts to clarify the problem of the dead zone (Rubin et al. (2002)). In this work the second model (see Figure 4) will be presented where osteocytes undergo even higher strains.

Figure 4: Osteocyte between two lamella with additional shear deformation

As shown in Figure 4 the model of two lamellae with different fiber orientation is presented with inbetween located osteocyte. The axial pressure, loading top surfaces of lamellae, causes additionally stretching strain, which is clearly higher in osteocyte localization than inside the lamellae. There is no evidence that these shear strains will be able to amplify the cell strains significantly enough. Fulfilling the models mentioned above and causing strain detection by osteocytes as a final effect seems however, quite probable.

It is generally accepted that continuum bone matrix strains are found to be in the order of 0.3 and 2 microstrains. Therefore, the minimum effective strains for numerical models presented in this work will be determined after the values given by Frost (1992).

5. FLUID FLOW HYPOTHESIS

The fluid flow is induced by dynamical external loads which cause a local hydrostatic pressure gradient. A hypothesis concerning the mechanism by which the osteocytes placed in the lacunae of mechanically loaded bone sense the load applied to the bone by the detection of strains was suggested by several researchers (Duncan and Turner (1995), Burger and Klein-Nulend (1999a), Burger and Klein-Nulend (1999b)). It was proposed that the osteocytes are stimulated by relatively small fluid shear stresses acting on the membranes of their osteocytic processes. The fluid flow is induced by dynamical external loads which cause a local hydrostatic pressure (see Figure 5).

Figure 5: Creation of fluid flow through the hydrostatic pressure gradients

As presented on Figure 5 the bending forces not only deform the osteocytes situated inside the bone, but further generate a pressure gradient from compression to tension regions (Duncan and Turner (1995)). This causes the bone fluid to flow through the canalicular spaces, and in addition to causing shear stresses on cell membranes it also provides the cell with nutrition supplies. Additionally, a very characteristic electric potential is created which is called streaming potential and was introduced by
Korenstein et al. (1984). It is known that extracellular bone matrix is negatively charged due to its proteins. When the fluid moves, the excess positive charge is convected developing streaming currents. The amount of electrical current created is dependent on the pore sizes through which the fluid is pressed. The anatomical site in bone tissue that is the source of the experimentally observed strain generated potentials is still under discussion (Pollack and Petrov (1984)).

Although numerous experiments were carried out by Burger and Klein-Nulend (1999b), Owan et al. (1997), Wang et al. (2003) it is not finally proven if the mechanosensation process can be explained with the fluid flow theory.

6. CONTINUUM MECHANICS

The specific objects represent different outlook when seen with the naked eye and under a microscope. Material which appears to be continuum is not the same in a finer length scale. However, in continuum theory the fact that matter is made of atoms and that has some sort of heterogeneous microstructure is ignored in the simplifying approximation and physical quantities, such as energy and momentum, can be handled in the infinitesimal limit. Differential equations can thus be employed in solving problems in continuum mechanics.

Some of these differential equations are specific to the materials being investigated, while others capture fundamental physical laws, such as conservation of mass or conservation of momentum (Zienkiewicz and Taylor (1994), Altenbach and Altenbach (1994), Betten (1993), and Wriggers (2001)).

The kinematic basics of continuum mechanics contain the description of solid body deformations, distortions and time derivation of the kinematic variables.

Formally, a solid body $B$ can be described as an assembly of points in an Euclidian space $E$. The configuration of $B$ is the explicit transformation of $\varphi : B \rightarrow E$ where the particle of $B$ is included in $E$. Therefore, the location of particle $X$ of body $B$ in configuration $\varphi(B)$ (coupling operator) is defined as $x = \varphi(X)$. Figure 6 (based on illustration in Wriggers (2001)) shows configurations: on the left undeformed called also reference (B) and on the right deformed or actual $\varphi(B)$. As stated earlier, both configurations are coupled by the operator $\varphi$.

According to Figure 6 a particle $A$ is situated in both configurations: reference $B$ and actual $\varphi(B)$ but at different locations, respectively $X$ and $x$. The location of particle $A$ in time space $t \in R$ is described as follows:

$$x = \varphi(X, t)$$

(1)

The deformation of body $B$ causes the displacement of particle $A$ which is given by Equation 2:

$$u(X, t) = x(X, t) - X$$

(2)

To describe deformation processes it is crucial to introduce the deformation gradient $F$ which combines the deformation of the line element $dx$ in reference configuration $B$ with the deformation in actual configuration. This relation is given as:

$$dx = FdX$$

(3)

Using partial derivatives the deformation gradient can be expressed as:

$$F = \frac{\partial \varphi}{\partial X} = \frac{\partial X}{\partial x} + \frac{\partial u}{\partial X} = 1 + \text{Grad} u = 1 + H$$

(4)

where $\text{Grad} u = H$ and is defined as the displacement gradient tensor.

If the deformation gradient operator is linear, the transformation given by Equation 3 is also linear. In order to provide the continuum structure of $B$ during the deformation, Equation 3 must be unique.

$$J = det F \neq 0$$

(5)

where $J$ is the Jacobian determinant. When the deformation gradient $F$ is not singular there exists the inverse $F^{-1}$ and Equation 2 can be written as follows

$$dX = F^{-1} \cdot dx$$

(6)

The Jacobian determinant is of a great importance in continuum mechanics and the corresponding numerical formulation. It relates infinitesimal small volume elements such that

$$dv = JdV$$

(7)

Deformation gradients can be decomposed into stretch and rotation tensors using the polar decomposition theorem as follows:

$$F = RU = VR$$

(8)
where respectively $\mathbf{R}$ is the orthogonal rotation tensor, $\mathbf{U}$ is the right stretch tensor and $\mathbf{V}$ is the left stretch tensor with

$$
\mathbf{R}^T \mathbf{R} = 1, \quad \mathbf{U}^2 = \mathbf{F}^T \mathbf{F} = \mathbf{C}, \quad \mathbf{V}^2 = \mathbf{F} = \mathbf{b}
$$

where $\mathbf{C}$ is described as the right and $\mathbf{b}$ as the left Cauchy-Green tensor.

After describing the deformation gradient tensor $\mathbf{F}$ the Green-Lagrange strain tensor can be introduced as follows:

$$
\mathbf{E} := \frac{1}{2} (\mathbf{F}^T \mathbf{F} - 1) = \frac{1}{2} (\mathbf{C} - 1)
$$

The deformation gradient contains all information about displacement and rotation of the body. It is convenient to describe the Green-Lagrange tensor with use of the displacement gradient $\mathbf{H}$ that gives

$$
\mathbf{E} = \frac{1}{2} (\mathbf{H} + \mathbf{H}^T + \mathbf{H}^T \mathbf{H})
$$

where the linear and non linear parts can be distinguished. In this work (except for one example) the linear theory will be used. The non linear part can be omitted which results in the linear strain tensor

$$
\varepsilon = \frac{1}{2} (\mathbf{H} + \mathbf{H}^T).
$$

The constitutive relation between the applied stresses and strains for linear materials at small strains is given by:

$$
\sigma_{ij} = C_{ijkl} \varepsilon_{kl}
$$

where $C_{ijkl}$ represents the fourth rank material tensor (81 components). However on the basis of symmetry of strain (Equation 18) and stress (Equation 19) tensor it is possible to reduce the number of components to 36.

$$
\sigma^T = \begin{bmatrix}
\sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{23} & \sigma_{31}
\end{bmatrix}
$$

In this work the used materials are described below. Hydroxyapatite crystal, collagen matrix and osteocyte are isotropic linear elastic materials. With the use of only two independent constants, Poisson ratio $\nu$ and Young modulus $E$, typical for engineering problems, it is possible to build the whole material matrix $\mathbf{C}$ as shown in Equation (20):
strain field of the matrix is approximated through the average strain field in a sufficient distance. The load on each inclusion depends on the existence of other inclusions and the average matrix strain and stress fields created by them. The interaction does not include the fluctuation of the fields. Finally, the effective material $C^*$ is calculated by:

$$C^* = C_m + C_i \cdot (C_i - C_m) \cdot L_{MT}$$

where $C_m$ and $C_i$ are the fourth order material tensors of matrix and inclusion, respectively. The parameter $c_i$ is the concentration of inclusion and $L_{MT}$ is the localization tensor based on the Mori & Tanaka assumption

$$L_{MT} = [I + c_m S_m \cdot (C_i^{-1} \cdot C_i - I)]^{-1}.$$  

The localization tensor represents the solution of a linear elastic boundary value problem of the following form

$$e_i^*(x) = L_{ij}^u(x) \cdot e_i^o_j \text{ for } u_i = e_i^o j \text{ on } \partial \Omega,$$

which means that if the strain $e^o$ in the matrix material acting on the boundary of the inclusion $\partial \Omega$ is known (assumed to be constant in this case) then the calculation of the strain field $e^*$ inside the inclusion becomes possible.

The localization tensor itself depends on both material tensors, the matrix concentration and the fourth order Eshelby tensor $S$ (Eshelby (1957)). The Eshelby solution denotes that inside ellipsoidal inclusions, if the strain field $e^*$ is constant, the resulting total strain field $e^*$ is also constant. These two strain fields are coupled by the fourth rank Eshelby tensor $S$:

$$e_i^* = S_{ijkl} \cdot e_k^o \text{ in } \Omega \text{ for } v_i = e_i^o x_j \text{ on } \partial \Omega.$$  

In the case of simple inclusion geometries, analytical solutions for the Eshelby tensor are known and may be denoted explicitly. In other cases, numerical approximations of this tensor have to be computed.

8. MATERIAL MODEL

The mechanical behavior of osteons depends on their elastic properties which are essential and must be determined to carry out the finite element analysis. Thereby many factors play a role like the fiber pattern of the lamella, the degree of mineralization as well as the elastic properties of the main constituents of bone tissue: HA-crystals and collagen molecules. Taking into consideration the scale in which this research was done the dimensions of osteon and the dimensions of HA-crystals (2x4x20nm), homogenization methods become necessary. The homogenization allows the creation of a material which has the same elastic properties as a composite of HA-crystals and collagen as shown in Figure 8:

$$\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix} \begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
e_{12} \\
e_{23} \\
e_{31}
\end{bmatrix} \quad (21)$$

where:

$$c_{ij} = \frac{1 - \nu_{21}^2 E_i}{\Delta} \quad e_{12} = \frac{\nu_{21} + \nu_{31}^2 E_i}{\Delta} \quad e_{13} = \frac{(1 + \nu_{21}) v_{31}}{\Delta}$$

$$c_{31} = \frac{(1 - \nu_{31}) E_i}{1 - \nu_{21} - 2 \nu_{31} v_{31}} \quad c_{44} = \mu_{11} \quad c_{66} = \mu_{12} = \frac{E_i}{2(1 + \nu_{21})} \quad (22)$$

with

$$\Delta = \frac{1 + \nu_{21}}{E_i} \left[ 1 - \nu_{21} - 2 \nu_{31}^2 \frac{E_i}{E_1} \right]. \quad (23)$$

7. HOMOGENIZATION

The constitutive material relations shown in this work have been derived under the assumption that the material parameters are homogenous in the entire body B. For heterogeneous materials, e.g. composite materials, the constitutive equations are applicable only to homogeneous volume fractions. In these cases the homogenization procedures are used to create effective, homogenous materials out of heterogeneous composites. The effective material properties strongly depend on the elastic properties, the volume fraction, the orientation and dimension of the constituents on the micro level. In case of biological materials of small size, experiments to derive the material properties are very difficult to carry out. Analytical or numerical estimations of homogenized material parameters are sometimes useful alternatives, but they remain approximations. Having knowledge about the mechanical properties on the nano scale of the material constituents, effective material parameters can be derived for micro scale calculations. Therefore several homogenization procedures are described in the literature. They all have to fulfill certain requirements concerning RVE’s (representative volume element). It has to be small enough on the computation scale, but large enough on the scale, where its properties are known. It can than be considered statistically representative.

One of the most widely used classical homogenization methods is the Mori & Tanaka (1973) method. It was used by Lenz and Nackenhorst (2003) to obtain nano parameters of the bone on the micro scale.

This method is an extended approach to homogenization considering the interaction between the inclusions and their finite volume fraction. Therefore, the
Another important fact of the osteon material belongs to its anisotropy. This anisotropy arises basically from the approximately ellipsoidal (also cuboidal according to other researchers) HA-crystal structure. Despite of the isotropy of collagen and hydroxyapatite, the homogenization yields an orthotropic lamella material. Thus, three principle axis for the stiffness of the lamella material are obtained where the highest stiffness belongs to the semi-major axis of the ellipsoid.

For homogenization the classical Mori-Tanaka method is used and the following elastic properties of the components are applied as shown in Table 1:

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus</th>
<th>Poisson’s ratio</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>collagen</td>
<td>1.2GPa</td>
<td>0.35</td>
<td>isotropic</td>
</tr>
<tr>
<td>hydroxyapatite</td>
<td>144GPa</td>
<td>0.27</td>
<td>isotropic</td>
</tr>
<tr>
<td>composite</td>
<td>5-18GPa</td>
<td>0.3</td>
<td>orthotropic</td>
</tr>
</tbody>
</table>

The big dispersion of the composite material stiffness is strongly connected with the concentration of ellipsoidal HA (mineralization) in the RVE. The problem is addressed in the following section below.

For the process of homogenization, the HA-crystals are called inclusions (particles) and the collagen is referred to as matrix. With the use of the Mori-Tanaka method, it is possible to calculate the material matrix $C^*$ of the composite material (lamella material).

Finally, the lamella material $C^*$ has to be rotated in the directions in which they appear in the osteon. Here the example of lamellae ±45° (see also Figure 8) is introduced:

$$C^*_{ijkl} = C^*_m D_p (45°) D_m (45°) D_m (45°) D_m (45°).$$  \hspace{1cm} (28)

After the execution of all steps, including the rotation procedure, the orthogonal material tensor can be obtained.

1. **Geometry of the Model**

To solve the mechanical problem of an ordinary osteon under physiological boundary conditions a three dimension finite element model is developed. The geometry of such a model is idealized to a simple hollow cylinder, and further, with the use of periodical boundary conditions, simplified to 60° section (see Figure 9 on the right and Figure 14).

The modeling process in this work is carried out according to the theory presented earlier. All properties are defined in a cylindrical coordinate system (Figure 9) in order to obtain axial symmetry. The small osteon models presented in this work contain only two lamellae. Each of them has a radial width of 5 µm and a height of 50 µm. The radius of the inner hole is 40 µm.

There are several theories describing the geometry of osteocytes. According to Cowin (2001) they are spherically shaped with diameters of 4–14µm. Martin...
et al. (1998) suggested even smaller dimensions while Fritton et al. (2004) accept more specific shapes of a triaxial ellipsoid with axes lengths of $17.6 \pm 0.3 \mu m$, $6.1 \pm 0.3 \mu m$ and $4.0 \pm 0.2 \mu m$. According to these data the osteocyte geometry was chosen and is presented below.

The geometry of the models prepared in previous research (e.g. Lenz and Nacken-Horst (2003)) poorly approximated the shape of the osteocyte which did not reflect the real biological geometry. Here, this geometry is substantially improved. It was decided to create two kinds of models one with spherical osteocytes with diameter of $5\mu m$ and the second one with ellipsoidal osteocytes with axial lengths of $2.5\mu m$, $2.5\mu m$ and height of $5\mu m$. That gives the opportunity to compare the differences arising during the computation process and present the geometrical development of the model. Models with both kind of inclusions are presented in Figure 10. The location of osteons (red on Figure 10) is random.

![Figure 10: The lamella with spherical (left) and ellipsoidal (right) osteocytes](image1)

To make the geometrical description complete the number of osteocytes in the lamellar tissue must be discussed. Here arises another disaccord occurring in several sources. According to Cowin (2001) the number of lacunae per $mm^3$ is about 15000. Martin et al. (1998) supports this number, suggesting 13000-17000 lacunae per $mm^3$, but Fritton et al. (2004), who also suggested the largest axial sizes of the osteocytes, proposes much larger value of 80600 lacunae per $mm^3$. These relatively large differences confirm the lack of consensus in research. In order to hold one source geometrical data the number of osteocytes for the final model as well as the shape and size are those proposed by Cowin (2001) and Martin et al. (1998). However, for the small two lamellae models which should only describe the modeling approach these values are not applied. Here four osteocytes are placed between lamellar layers. At this point the model development is shown. The exactness of the biology representation is discussed later.

![Figure 11: Discretization of lamella inclusion; spherical (left) and ellipsoidal (right)](image2)

2. Material Properties

The created models in this research use great variety of the material properties (described earlier) but all of them are generated for the mineralization degree of 65%. All orthogonal material tensors used for the modeling in this research and describing the properties of the lamella with specific fiber orientation are expressed in VOIGT notation. The orthotropic tensors for the fiber orientation $0^\circ$ and $90^\circ$ look identical to transversal isotropic material matrices. The other material tensors look very similar. The differences lie in the rotation routine which causes the material axes not to coincide with the cartesian coordinate system axes. However, the obtained homogenized material behaves in transversal isotropic way which means that exchanging the x and z-axes would not affect the material properties.

According to the biological studies the osteocytes are modeled with much softer material than lamellae, as fully elasto–isotropic material, described with two material parameters Young’s moduli $E = 0.2 GPa$ and Poisson ratio $\nu = 0.3$.

3. Model Discretization

For the discretization process the 10-node tetrahedral elements with quadratic shape functions are used to describe the geometry. The discretization is shown in Figure 11.

The discretization is carried out in a very specific way. Except of big size differences between modeled elements (lamella layer and osteocyte) which is discussed later, the use of MPC (multi point constraint) boundary condition requires characteristic properties. Therefore the right side of the lamellae is discretized first with the two dimensional triangle elements. Next, these elements are copied to the opposite surfaces on the left, hence corresponding nodes are created. Thereafter, the entire net was created based on the already existing one. The obtained discretization fulfills the boundary conditions
requirements and additionally provides very good visualization of the results.

The sizes of osteocytes in comparison to the lamellar layers are relatively small. Although the discretization of the osteocytes must be fine to obtain satisfactory results, the size of the elements composing the lamellae layers cannot be the same. That would cause enormous number of elements and makes the computation difficult to carry. In Figure 12 the single discretization of the osteocytes placed in the lamella is shown.

![Figure 12: Illustration of the discretized osteocytes with surrounding lamella; spherical (left) and ellipsoidal (right)](image)

The number of elements for the spherical osteocytes is smaller than the number of ellipsoidal osteocytes (as shown in Figure 12). This comes off the size differences—the smaller ellipsoidal inclusion requires finer discretization than the larger spherical ones.

The discretization has to provide the union between the lamellae layers and osteocytes placed between them. This could be proved by the continuities of the displacements and strains occurring in the osteocytes. By using the nodal equivalence function which eliminates double nodes in the model, the expected results are obtained in computation.

4. Boundary Conditions

There are two main boundary conditions applied to the generated model. The first one fixes the bottom of the model in the z-direction. The second, makes possible the reduction of the entire cylindrical osteon. In this work, only the section of 60° is modeled and computed (see Figure 13).

This reduction is possible due to the periodical boundary conditions which are applied at the side boundaries. This is realized by Multi Point Constraint (MPC), by which the same deformations in the section are obtained as if the whole osteons are analyzed. Periodical boundary conditions imply that the deformation of two, axial symmetric corresponding nodes (same radial distance to the center and the same z-dimensional hight at both sides) is equal. In order to obtain such a method the side surfaces (Figure 14) had to be discretized, one after another, with triangle surface elements that results in creation of the corresponding nodes.

The corresponding nodes are constrained with MPC. This tactic tremendously reduces the time and required memory of the computations. In Figure ??0 the MPC for a 2-lamellae osteon model is presented. The red circles show the node locations and the red lines connect two corresponding nodes.

5. Loading

On the top of the model a uniform physiological pressure of 6.25 MPa is applied. This value was estimated by Lenz (2005) according to the experimental and literature review – COWIN (2001), Martin et al. (1998) and others. The illustration of the loading application is shown on the right in Figure 13.

6. Results without MPC Boundary Conditions

In Figure 15 the displacements of the model without MPC boundary conditions is shown. The displacement distribution is similar for both cases. A very interesting behavior of the displacement can be noticed which relates...
to the material parameters $C$ and the fibriform structure with opposite fiber direction ($\pm 45^\circ$) for each lamella.

In Figure 16 the cross sectional view of the Huber von Mises strains are presented. This model is calculated with the use of all modeling methods described in this work excluding the MPC boundary condition. The HMH (Huber, Mises, Hencky) strains are calculated according to the equation

$$\varepsilon_{\text{red}} = \sqrt{2\varepsilon_{ij} : \varepsilon_{ij}}. $$

The strains in osteocyte are clearly higher than those in the lamellae, which, according to the material parameters, is expected.

![Figure 15: Total displacements of both models, left with sphere and right with elliptic osteocytes](image1)

![Figure 16: Cross-sectional view of Mises strains, left with sphere and right with elliptic osteocytes](image2)

The presentation of the displacement and strains showed that the model is created properly. The displacement of separately discretized lamellae are united and the strains occurring in softer osteocytes are higher than those obtained in the lamellar tissue.

7. Results with MPC Boundary Conditions

With the use of MPC boundary conditions, the displacements become symmetric as expected. They are presented in Figure 17. The cross-sectional strains are very similar to those presented in the previous section, and will not be shown again.

At this point one more very characteristic displacement property will be presented which is illustrated in Figure 18. As stated earlier, the periodical boundary conditions guarantee the same behavior of a section of $60^\circ$ as if the whole cylindrical osteon would have been analyzed. It means that there should not be any possibility of displacement of the section’s sides in the $\varphi$ direction. With the use of a greater scale factor $(0.1)$ and proper view it is possible to notice the displacement differences between models with and without MPC. They are compared in Figure 18.

![Figure 17: The displacement of osteon models with periodical boundary conditions, model with spherically shaped osteocytes on the left and with elliptically on the right](image3)

![Figure 18: The model with MPC on the left, and without, on the right](image4)

The whole modeling procedure is presented and explained in this work. All the modeling requirements are fulfilled. With such an experience, creating the entire model is much easier and takes less effort. However, the computation time is longer because of the model size. In the following section detailed description of the entire models with both kinds of osteocytes (spherical and ellipsoidal) will be given and obtained results will be compared with the theory developed earlier.

9. NUMERICAL EXAMPLES

The finite element analysis results of the osteon model with ten lamellae layers is presented in these examples. The aim is to study the deformations between the osteonal lamellae where the lacunae are situated. Each lacuna accommodates an osteocyte, and the strains at these locations are responsible for the initiation of the remodeling processes.

The full model description is given in Table 2.
presented earlier. Table 3 presents the calculation of the number of osteocytes for the model.

The location of the osteocytes is random and is chosen to be between 3rd and 4th, and 7th and 8th lamella. Both models with spherical and ellipsoidal osteocytes are shown in Figure 20.

Table 2
Parameters of the model (properties, boundary conditions and loading)

<table>
<thead>
<tr>
<th>Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of entire ostenal cylinder</td>
</tr>
<tr>
<td>Diameter of inner whole (Haversian canal)</td>
</tr>
<tr>
<td>Height of the entire model</td>
</tr>
<tr>
<td>Number of lamellae</td>
</tr>
<tr>
<td>Radial width of single lamellae</td>
</tr>
<tr>
<td>Height of single lamellae</td>
</tr>
<tr>
<td>Diameter of the spherical osteocytes</td>
</tr>
<tr>
<td>Axial dimensions of the ellipsoidal</td>
</tr>
<tr>
<td>Bottom boundary conditions</td>
</tr>
<tr>
<td>Side boundary conditions</td>
</tr>
<tr>
<td>Loading top surface pressure</td>
</tr>
</tbody>
</table>

Figure 19: Geometry and loading pressure of the entire osteon model

The geometry, boundary condition and loading are now determined and the model is almost ready for the simulation. The computation of both models, as presented in Table 5, is carried out with different lamellae material parameters. Six different models are created, three with spherical and three with elliptical osteocytes. For all simulations the hydroxyapatite concentration (degree of mineralization) was set to 65%, which describes fully mineralized osteons.

The orientation of collagen fibers between lamellae may change up to 90° in adjacent lamellae. Except of orthogonal models, where the collagen fibrils only assume one of two directions which are out of phase 90° with each other, there is also the possibility of a twisted model. In this case parallel collagen fibrils continuously rotate by a constant angle from plane to plane in a concentric structure. In this work, three most common lamellae fibril orientations are simulated. The first group of two models with spherical and ellipsoidal inclusions, has the angles of fiber inclination for a cross ply lamellae pattern of ±45°, in the second, the angles are respectively 0° and 90° and the third group is an example of the twisted model, where the angle of fiber inclination is rising from –45° to 90° with 15° increase between each lamellae. The summary of the all models with different material properties is given in Table 4.

The geometry of both models with different shapes of osteocytes was presented before. Discretization and setting the boundary condition with use of MPC was carried out in the same way as it was presented for the small model in the previous sections.

Table 3
Number of osteocytes in a model
Calculation of number of osteocytes

<table>
<thead>
<tr>
<th></th>
<th>Calculation of number of osteocytes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of all cylindrical model including Haversian canal</td>
<td>0.000769mm³</td>
</tr>
<tr>
<td>Volume of Haversian canal</td>
<td>0.0000628mm³</td>
</tr>
<tr>
<td>Volume of all cylindrical model without Haversian canal</td>
<td>0.000706mm³</td>
</tr>
<tr>
<td>Volume of 60° section</td>
<td>0.000117mm³</td>
</tr>
<tr>
<td>Number of lacunae (Cowin (2001), Martin et al. (1998))</td>
<td>15000-17000/mm³</td>
</tr>
<tr>
<td>Number of lacunae for the final model (60° section)</td>
<td>1.7 – 2.01 = 2</td>
</tr>
</tbody>
</table>

Figure 20: The entire model geometry with the location of spherical (left) and ellipsoidal (right) osteocytes

However, between the two models presented in Figure 20 there are some discretization differences. This arises due to the fact that the geometry of the inside inclusions is not the same. Smaller ellipsoidal osteocytes require finer tetrahedral elements than the larger spherical ones. The differences of the discretization data are shown in Table 5.
The FEM models with both kinds of osteocytes look identical. Though their surfaces are discretized in the exactly same way to obtain the corresponding nodes and MPC so that there is no visual differences in the external view. The entire discretized model is presented in Figure 21.

The finite element model, presented in Figure 21 is generated in MSC.PATRAN Simulating Reality software. After creating the model, an input file is produced. ABAQUS is used to solve the linear elastic mechanical problem. There is also one nonlinear computation carried out for the model with spherical inclusions and ±45° fibers orientation. It is presented and compared with linear calculations. According to different model parameters, which are the shape of inclusions and material parameters, the displacements at nodal points are calculated. Next, strains and stresses are obtained. These results are presented in the following Sections.

1. Model Including Lamellae with ±45° Fiber Orientation

In Figure 22 the displacements of models with different inclusions are shown in millimeters. On the left, the model with spherical osteocytes and, on the right, with the ellipsoidal osteocytes. The displacement distribution for both models is very similar and does not differ significantly from each other. Due to the periodical boundary conditions the problem is symmetrical. The largest displacements are on the top for both models, and they get smaller in the lower portion. These values vary between 4.31×10⁻² and 0.884×10⁻² micrometers for model with spherical inclusions, and 4.29×10⁻² and 0.935×10⁻² micrometers for model with ellipsoidal inclusions. The displacements on the top of the models are nearly constant while in the lower region they get larger with increasing radius. It is visible in Figure 22 that the shape of the osteocytes do not have significant influence on the displacement distribution and values.

Additionally, the computations of the the model with spherical inclusions is carried out with the nonlinear theory to compare the plausible differences to the linear elastic computation. The displacement field is shown in Figure 23. The non linear computation did not show any differences in displacement field distribution as well as in displacement values in comparison to the linear computations of the same model. Both results are quasi identical (Figure 22 on the left for linear and Figure 23 for nonlinear computation).

The strains presented in Figures 24 and 25 are essential for this work, especially the values at the lacuna locations where the osteocyte elements are placed. The strain in the osteonal corpus of the models are relatively constant therefore in Figures 24 and 25 only the two characteristic lamellar layers with the inclusions-4th and 8th lamella are presented.

Table 4
Material parameters (65% mineralization) for six osteonal models computed in this work

<table>
<thead>
<tr>
<th>Kind of inclusion</th>
<th>Fiber orientation</th>
<th>Description of appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model with spheres</td>
<td>±45°</td>
<td>lamella with smallest radius –45°</td>
</tr>
<tr>
<td>Model with spheres</td>
<td>0;90°</td>
<td>lamella with smallest radius 0°</td>
</tr>
<tr>
<td>Model with spheres</td>
<td>–45°/(+15°)/ + 90°</td>
<td>lamella with smallest radius –45°</td>
</tr>
<tr>
<td>Model with ellipses</td>
<td>±45°</td>
<td>lamella with smallest radius –45°</td>
</tr>
<tr>
<td>Model with ellipses</td>
<td>0;90°</td>
<td>lamella with smallest radius 0°</td>
</tr>
<tr>
<td>Model with ellipses</td>
<td>–45°/ +14°/ + 90°</td>
<td>lamella with smallest radius –45°</td>
</tr>
</tbody>
</table>

Table 5
FEM data of the models with different inclusions

| Number of nodes | 160274 | 201862 |
| Number of elements | 109905 | 142806 |
| Number of equations | 480822 | 605586 |
| Required memory for the calculation | 3.04G Bytes | 4.87G Bytes |
In Figure 24 both presented models contain spherical osteocytes. They are computed according to two different theories, the model on the left is calculated with linear analysis while the model on the right is calculated with nonlinear analysis. It is clearly visible that the results of these two different computational methods are almost identical. The reason lies in the resulting small strains, which makes the nonlinear analysis obsolete. Both models present exactly the same strain values and distribution. The strains in the lacunae are between \(9.67 \times 10^{-4}\) and \(1.65 \times 10^{-3}\) (red, yellow and pink colors) while the strains in the osteon corpus are much more constant from \(5.79 \times 10^{-4}\) to \(6.76 \times 10^{-4}\) (red and blue colors). For the entire osteon corpus the strains can be approximated with about 0.7 millistrains while at the lacunae the maximum value of 1.65 millistrains is located. These comparatively higher strains at the lacuna result from the lower stiffness of the osteocytes elements. The main contribution to the total strain derives from the longitudinal \(\varepsilon_z\) strain component which has nearly the same value as the total strain. The influence of all the other strain components is much smaller and remains lower than 0.5 millistrains.

In Figure 25 the obtained strains are presented; on the left the strains in the model with ellipsoidal inclusion and on the right the magnification of the strains in the osteocyte. As in the previous models, the osteocyte material is much softer than the osteonal corpus therefore the strains in the lacunae are three to four times higher. Their values vary between \(1.15 \times 10^{-3}\) and \(2.66 \times 10^{-3}\) (red, pink and green color) while the strains in the osteonal corpus present even more stable values (than for the spherical inclusions 24) of \(6.44 - 8.12 \times 10^{-4}\) (constant blue color). The approximation of the strains in the model would result in 0.7 millistrains for the osteonal corpus. The maximal strains in the osteocytes is computed to be 2.66 millistrains. The main contribution to the total strain derives, as for the previous models, from the longitudinal \(\varepsilon_z\) strain component, while the other components remain much smaller (less than 0.4 millistrains).

An additional characteristic property is now presented – the shear stress \(\tau_z\). The lamellae in both models (see Figure 26) changes the value in a very specific, alternating way (colors green and pink). For both examples the green color presents the shear stress \(\tau_z\) in lamellae as negative while the pink color presents the positive values. The reason for these results is the orientation pattern of the fibers in the lamellae which correlates to the main stiffness axes. These axes of the pink lamellae lie in the positive while the green lamellae lie in the negative circumferential direction.

The comparison of the presented strains for spherical and ellipsoidal inclusions shows that the strains in the osteonal corpus under specific pressure remains the same, 0.7 millistrains for both kind of models. Comparing these values with the simulation made by Lenz and Nackenhorst (2003) (their model was programmed in Matlab programming language) it turned out that the results in the osteonal corpus are very similar.

The strains obtained in the osteocytes vary significantly. The maximal value for the spherical inclusions is estimated to be 1.65 millistrains while for the ellipsoidal 2.66 millistrains. In comparison with the results presented by Lenz and Nackenhorst (2003), about 1.16 millistrains, both results obtained in this work are up to almost 1.5 times higher. The model by Lenz and Nackenhorst (2003) had the same material parameters, but much simpler geometry for the osteocytes (two hexagonal elements). These relatively high difference in strain detection, caused by the different geometry of the osteocytes, shows how important the modeling approach is, and might suggest and explain the most proper shape for the osteocytes.

The other conclusion concerns the nonlinear calculation, which takes much more time and requires, in comparison to the linear theory, much more memory to save the huge results data. There are no differences between the results obtained in linear elastic and nonlinear analysis. Therefore, the following computations will be carried out only according to the linear elastic theory which turns out satisfactory for this example.
direction, so their stiffness in this direction is the highest. Analogically the stiffness of fibers with orientation of 90° for such an axial loading is the smallest. It causes the entire lamella stiffness with fibers orientation of 0° to be much higher than the other lamellae. This explains the characteristic displacement, but also the strain distribution in the top region. In the middle part of both models the displacement fields are nearly constant, very similar to the previous example and in the lowest part they get larger with increasing radius however, not as much as those in the previous example. The displacement values in the top region are slightly higher than in the model with fibers orientation of ±45° but in the lower part they appear to be much smaller. For both models the displacement values are very similar and vary between 3.55·10⁻² and 4.07·10⁻⁶ micrometers for the model with spherical inclusions and between 3.50·10⁻² and 6.51·10⁻⁶ for the ellipsoidal inclusions.

In Figure 28 the strains in the models with both kinds of inclusions are presented.

The distribution, as well as strain values for the osteonal corpus are qualitatively similar for both models. The above explained stiffness of lamellae with different fiber orientations (see displacement description) is the reason of the characteristic strain distribution in the top region of the models. The softer material under specific loading results in higher strains than the stiffer one. It is clearly visible especially in the 1st and last lamella of both models. The strain values in the softer lamellae, the one with the fiber orientation of 90°, vary between 6.51·10⁻⁴ and 9.10·10⁻⁴ for the model with spherical inclusions and between 5.11·10⁻⁴ and 9.71·10⁻⁴ for the model with ellipsoidal inclusions. For the lamellae with higher stiffness (fiber orientation 0°) these values vary between 1.33·10⁻² and 2.63·10⁻⁴, and 1.66·10⁻⁴ and 2.81·10⁻⁴ respectively. The strain values presented at the top of both models are very alike and do not differ very much. In the lower parts, the strains in both kinds of lamellae stabilize, become constant and equal respectively 2.63–3.92·10⁻⁴ for the model with spherical and 2.81–3.96·10⁻⁴ for the model with ellipsoidal inclusions. They can be approximated to 0.3-0.35 millistrains and is much smaller than in the model presented earlier.
In Figure 29, the cross-sectional strain distribution in the osteocytes are presented. As explained, in the top region higher strains can be noticed which are the results of the material parameters. Although the strains of osteonal corpus are much lower than those obtained for the model presented earlier, the strains inside the osteocytes still remain relatively high. For the spherical inclusion maximal obtained strains are equal to 1.50 \times 10^{-3} while for the ellipsoidal they are even higher and equal to 1.78 \times 10^{-3}. These strains can be approximated to 1.5 and 1.78 millistrains, respectively. Here the obtained values are slightly smaller than those in the previous example but it must be also pointed out that the strains detected in osteonal corpus are about twice smaller than those in the previous example. Assuming the osteonal corpus strains are clearly smaller than in the previous example, the strains in the osteocyte stays on the very high level and still remains much higher than those obtained in Lenz and Nackenhorst (2003). Again, the strain detection in ellipsoidal osteocytes resulted in highest values and turned out to be the most efficient.

In Figure 30, the characteristic behavior can be noticed. As already explained previously, the more the fiber orientation angle differs from 0° location, the more the displacements of the lamella are larger. Accordingly the smallest displacement would be the one with fiber angle of 0° and the softest one, with largest displacement respectively, would be the one with angle of 90°. This property is clearly visible on Figure 30. The 4th lamella described by the material tensor $C(0°)$ has the smallest displacements while the last, 10th lamella, with the material tensor $C(90°)$ has the highest displacements. With increase of the fiber angle, no matter if in positive or negative direction, the lamellae displacements are getting larger. In the lower half of the model the displacement’s distribution is more stabilized and grows with increasing radius. Both displacement distributions and the values presented in Figure 30 are quite similar. For the model with spherical inclusions these values vary between 5.52 \times 10^{-2} and 2.01 \times 10^{-3} micrometers, while for the model with ellipsoidal inclusions between 5.49 \times 10^{-2} and 1.33 \times 10^{-3}, respectively. In this last example it is again shown, that the inclusion shape does not affect the global displacements in a significant way.

According to the above description a characteristic strain distribution field could be expected. Taking into consideration the material properties, their stiffness and the type of example, the strain distributions in the top region of the model should behave in a similar way as the displacements. The softer material under the loading undergoes higher strains than the stiffer one. All these issues are confirmed in Figure 31, which presents the strain distribution for both models, on the left with spherical and on the right with ellipsoidal inclusions.

The strains presented in Figure 31 shows the expected properties. The strain values in the stiffest lamella (4th lamella with fiber orientation 0°) are the smallest while for the softest ones (the last 10th lamella with fiber orientation 90°) are the highest. The characteristic of the distribution of the strain field is congenial to the displacement field. According to the material parameters like the lamellae stiffness, the strains in osteonal corpus vary between 2.60 \times 10^{-4} and 8.66 \times 10^{-4}.
for the models with spherical inclusions, and between $2.27 \times 10^{-4}$ and $8.80 \times 10^{-4}$ for the model with ellipsoidal inclusions (see also Figure 31). These values can be approximated to $0.3-0.9$ and $0.25-0.9$ millistrains, respectively. The higher strains (0.9 millistrains) appear in the softer lamellae while the strains in the stiffer ones stay on a lower level (0.3 millistrains).

The last Figure 32 presents the cross-sectional view of the strains in both kinds of osteocytes. Although the strains in osteonal corpus vary because of the different material parameters of each lamella, the strains in the osteocytes still remain much higher. The presented Figure 32: The strains in 4th and 8th lamella with reference to the values in spherical (left) and ellipsoidal (right) osteocytes

![Figure 32: The strains in 4th and 8th lamella with reference to the values in spherical (left) and ellipsoidal (right) osteocytes](image)

Table 6: Strains obtained from the analysis

<table>
<thead>
<tr>
<th>Fiber orientation</th>
<th>Shape of the Osteocytes</th>
<th>Strains in Osteonal Corpus</th>
<th>Max Strains in Osteocytes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 45^\circ$</td>
<td>Sphere</td>
<td>$5.79-6.76 \times 10^{-4}$</td>
<td>$1.65 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pm 45^\circ$</td>
<td>Ellipse</td>
<td>$6.44-8.12 \times 10^{-4}$</td>
<td>$2.66 \times 10^{-3}$</td>
</tr>
<tr>
<td>0; 90$^\circ$</td>
<td>Sphere</td>
<td>$2.63-3.92 \times 10^{-4}$</td>
<td>$1.50 \times 10^{-3}$</td>
</tr>
<tr>
<td>0; 90$^\circ$</td>
<td>Ellipse</td>
<td>$2.81-3.96 \times 10^{-4}$</td>
<td>$1.78 \times 10^{-3}$</td>
</tr>
<tr>
<td>$-45^\circ / (+15^\circ) / + 90^\circ$</td>
<td>Sphere</td>
<td>$2.60-8.66 \times 10^{-4}$</td>
<td>$1.96 \times 10^{-3}$</td>
</tr>
<tr>
<td>$-45^\circ / (+15^\circ) / + 90^\circ$</td>
<td>Ellipse</td>
<td>$2.27-8.80 \times 10^{-4}$</td>
<td>$2.51 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

For all of the models the main contribution to the total strain derives from the longitudinal ($c_z$) strain component (as was suggested by Cowin (2001) or Lenz (2005)) which always had nearly the same value as the total strain. The influence of the other strain components is much smaller.

According to the previous investigation (Lenz and Nackenhorst (2003)) the strain detection obtained by the osteocytes improved. The two hexagonal elements approximating the osteocyte in Lenz and Nackenhorst (2003) detected 1.15 millistrains (with the same loading, boundary condition etc. and for the $\pm 45^\circ$ fiber orientation), while in this work the spherical osteocytes detected 1.65 millistrains and ellipsoidal even 2.66 millistrains. The results of this research satisfy the pure strain theory by [*?*], according to which the results are interpreted. It is shown in Table 6 that the values obtained in the osteocytes are much higher than the values in osteonal corpus, and fulfill the required minimum (Frost (1992)) of 1.5 millistrains to send the remodeling signal to the BMU groups.

The total osteon stiffness can be approximated to 9.2GPa which gives good correlation with the measured values by Ascenzi and Bonucci (1968) (7.3 – 9.5 GPa).

10. SUMMARY AND CONCLUSIONS

The mechanosensation process describes the bone physiological reactions. They are coupled with the biomechanical signals produced by the osteocytes under mechanical deformation. The detailed knowledge of all processes occurring in micro scale gives the opportunity to describe more exactly the bone remodeling algorithms which might be of a great importance for healing the fractures and osteoporosis.

The mechanical simulation of a single osteon which denotes the properties of the compact bone is presented...
in the beginning of this paper. The analysis is done on
the micro length scale. The aim of the possibly close
approximation of a real biological geometry is obtained.
The three dimensional model of a single osteon is created
and computed with the linear elastic theory. The geometry
of the osteocytes is generated in two different approaches,
according to the data obtained in the literature, as
spherical and ellipsoidal inclusions. In comparison with
previous research in the mechanosensation and
biomechanical field, the presented geometry of the model
better describes the biological reality. Six different models
are created and computed.

The deformations, strains and stresses inside of a
single osteon are calculated with the use of the finite
element method and according to the linear elastic
analysis. It is shown, that nonlinear computations give
the same results as the linear analysis. The model is
discretized with relatively a fine mesh. The mesh
differences resulting from different shapes of the
osteocytes are shown and discussed. The discretisation
process is carried out using the tetrahedral elements.
These elements are described using the quadratic shape
functions.

The material parameters on the nano scale are found
in the literature. Mori-Tanaka homogenization method
is applied in this work. This transformation, which
combines nano and micro scales, results in the material
parameters on the micro scale which are presented as an
orthogonal material tensor. These calculations are carried
out and taken from Lenz and Nackenhorst (2003). All
the necessary steps to understand and follow the process,
including theoretical background, are presented. The
resulting orthotropic material is the function of the
mineralization level of the lamellae. These homogenization
methods allow one to describe the model
parameters which are very close to the realistic biological
properties.

The shape of the osteocyte inclusions has a very
important role for the strain detection in the bone tissue.
In all computations the detected strains are always
the highest for the ellipsoidal inclusions. Comparing all the
obtained results in this and in other research, where the
osteocytes are described with two hexagonal elements,
it can be suggested that the ellipsoidal shape is the proper
way to describe the biological geometry.

The improvement of the geometry describing the
osteocytes provided the required minimum of the pure
strain theory presented by Frost (1992). Both, spherical
and ellipsoidal osteocytes reached at least the value of
1.5 millistrains in this mechanosensation process. It
means that under such a specific loading the osteocytes
would send the remodeling signal to the other bone cells
in the bone tissue. The previous research with values of
1.15 millistrains, where the osteocytes did not reflect the
real biological geometry (Lenz and Nackenhorst (2003)),
did not meet the required minimum. This additionally
points the importance of preciseness in the geometry
modeling approach.

Detailed comparisons with data from the literature
and previously carried experiments is given in this work.
The main contribution to the total strain derives from the
osteon longitudinal (ε) strain component which always
has nearly the same value as the total strain. The influence
of all the other strain components is much smaller.

The micro-length scale simulations show the
importance of the modeling approach. It is shown that
more satisfactory results are obtained only with more
precise description of the biological geometry. The results
of this work are of a big importance for further
investigation. Especially bridging the micro and meso
length scales or attempting the possible exact description
of the bone properties on meso scale can be carried with
use of results presented here. This work provides some
contribution to the progress in the development of the
biomechanical field of study.

NOTE

1. Strains are reported in units of microstrain, with 10000
microstrain = 1% change in length.

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