**Global and Stochastic Analysis**
***Special Issue:*** *25th International Conference of Forum for Interdisciplinary Mathematics*

MUK PUBLICATIONS
Open Access Publisher

# EVALUATION OF PRE-ASSESSMENT METHOD ON IMPROVING STUDENTS PERFORMANCE IN LINEAR ALGEBRA COURSE

N. LOHGHESWARY, Z.M. NOPIAH, A.A. AZIZ AND E. ZAKARIA

Abstract: Engineering Mathematics has been the fundamental and vital courses in any engineering curriculum. Lack of basic knowledge of mathematics will reflect badly on the student when they are unable to grasp higher level of mathematics subjects or other engineering subjects. Thus the objective of this study is to cross examine the students' performance in Linear Algebra subject in Mechanical, Chemical, Civil and Electrical Engineering departments in Univeriti Kebangsaan Malaysia (UKM). Students were given pre-final test with the combination of different questions related to the course outcome of the course. The results of the pre-final test were then analysed using the Rasch measurement. Preliminary findings indicated that majority of the students have problems with understanding vector space and power series. With this early identification, the existing teaching method needs to be re-evaluated. Some suggestions were given for future improvement in the teaching and learning of the Linear Algebra course.

## 1. Introduction

Engineering Mathematics is one of the pillar or foundation subjects for all engineering courses. Therefore engineering students should have good grasp of fundamental mathematics knowledge in their earlier years of engineering course. This foundational knowledge later will be applied in other engineering subjects as well as in solving real world problem.

Linear Algebra is one of the compulsory subjects across the departments of chemical, civil, mechanical and electrical engineering in the Universiti Kebangsaan Malaysia, UKM. Linear Algebra course requires students to be able to understand, analyse and apply the concepts of matrices, vector space and series in solving practical engineering problem. However, previous results for other batch of students revealed that students fail to understand the fundamental concept of Linear Algebra.

Many researchers have conducted research on examining the Programme Outcome and Course Outcome. Ayob *et. al.*, (2011) used student exit strategy to assess students' Programme Outcomes. An exit strategy consists of the exit survey, exit test and exit interview. All twelve Programme Outcomes are achieved with this exit strategy. In this study, only the exit survey and exit test are used to assess the Programme Outcomes. Both results from the exit survey and exit test agree with each other by representing the students' achievement of all Programme Outcomes.

Osman *et. al.*, (2011) compared the students' achievement of the Programme Outcomes (POs) for the Reinforced Concrete Design Course for civil engineering

*Keyword.* Pre-final test, engineering mathematics, linear algebra, Rasch measurement, teaching and learning.

students. The project which was given to the students was related to the real structural design project. POs achievement is measured based on the final examination, mid semester examination, tutorials and group project. There was an increment of 9-39% on all POs on students' achievement between the two sessions. Moreover all POs have achieved more than 50% marks in the later session which made the study successful.

Osman *et. al.*, (2012a) conducted a study to evaluate the Course Outcomes for the Civil Engineering Design II Course. The Course Outcomes from the peer assessment contributes to the highest percentage in this study. The study concludes that students still have some difficulties in understanding design projects.

Saifuddin *et. al.*, (2010) commented that the Rasch Measurement Model is able to associate the pattern between students and the performance level of each Course Outcome whereby this cannot be achieved using the standard measurement method. Osman *et. al.*, (2012b) developed an assessment model based on students' mark register with the Rasch Measurement Model. This model measured students' performances in terms of the Course Outcomes for the Civil Engineering Design II Course. Rasch results were compared with the conventional distribution marks. Rasch results have similar patterns with the conventional method.

Ibrahim *et. al.*, (2012) claimed students' achievement which based on test, quizzes and assignment provides an overall indicator of students' cognitive ability. Yet it is lacking in measuring achievement of individual. Thus stochastic Rasch model gives an insight view of individual performance. A study was conducted to analyse the performance of matriculation students in Physics examinations. The measure of questions gives a good fit measurement where else the measure of persons gives a fair fit.

Said (2014) evaluated students' performance in Computing II subject using Rasch measurement model. Every final examination question is mapped to the number of students who answered correctly. Based on the findings of 75 students from the Foundation of Engineering, students can answer fairly question from various level of difficulty of final examination questions.

## 2. Methodology

All the engineering students in Universiti Kebangsaan Malaysia are subject to take Vector Calculus, Linear Algebra and Differential Equations subjects in their first second and third semester. All the subjects are 4 credit hour subjects.

A pre-final for Linear Algebra was conducted in semester II 2015/2016. A total of 282 students from Chemical, Mechanical, Civil and Electrical Engineering department participated in the test. Questions were constructed based on Course Outcome and Programme Outcome and was validated by two experts. The duration of the test was 2 hours and 5 subjective questions were prepared. In constructing the pre-final questions, Bloom Taxanomy (Knowledge, Comprehension, Application, Analysis, Evaluation and Creation) were considered.

Table 1 lists the Course Outcome for the Linear Algebra subject. Table 2 shows the Programme Outcome for Linear Algebra Subject. Table 3 illustrates the distribution of pre-final questions together with the marks. The details entry of each pre-final question is given in Table 4.

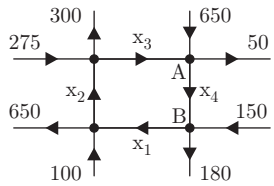
**Table 1. Course Outcome for Linear Algebra subject**

| *CO* | *Description* |
|---|---|
| 1. | Understand the fundamental concepts on the matrix and its basic operations and applications. |
| 2. | Able to use the concepts of vector space, linear independent in the space dimension and matrix transformation. |
| 3. | Able to apply the eigenvector and eigenvalue in engineering problems. |
| 4. | Able to use the diagonalization and quadratic forms in the matrix solution for engineering problems. |
| 5. | Able to understand the concepts of Power series. |

**Table 2. Programme Outcome for Linear Algebra subject**

| *PO* | *Description* |
|---|---|
| 1 | Engineering knowledge |
| 2 | Problem analysis |
| 3 | Design/development of solutions |
| 4 | Investigation |
| 5 | Modern tool usage |
| 6 | The engineer and society |
| 7 | Environment and sustainability |
| 8 | Ethics |
| 9 | Communication |
| 10 | Individual and team work |
| 11 | Lifelong learning |
| 12 | Project management and finance |

**Table 3. Pre-final questions**

| *Question* | *Description* | *Marks* |
|---|---|---|
| 1 | The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.   | |
| | (i) Setup a linear system which solution provides the unknown flow rates | 4 |
| | (ii) Determine whether or not the system has a solution. Justify your answer. | 6 |
| | (iii) Explain how you might use the least square methods to estimate the flow rates on each street. Show all your work. | 10 |

| *Question* | *Description* | *Marks* |
|---|---|---|
| 2 | Determine whether or not the set of vectors under addition and scalar multiplication defined by $< a_1, 0, a_3 > + < b_1, 0, b_3 > = < a_1 + b_1, 0, a_3 + b_3 > k < a_1, 0, a_3 > = < ka_1, 0, a_3 >$ is a vector space. | 15 |
| 3 | Suppose that the temperature at a point $(x, y)$ on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2)$. A ladybug walks on the plate along a circle of radius 5 centered at the origin. | |
| | (i) Use the Quadratic Forms to determine the highest and lowest temperature encountered by the ladybug and state the point where it attains the highest and the lowest temperature. | 13 |
| | (ii) Give the location and classification of the critical points of $T(x, y)$. | 7 |
| 4 | Given the following matrix: $A = \begin{bmatrix} -13 & -60 & -60 \\ 10 & 42 & 40 \\ -5 & -20 & -18 \end{bmatrix}$ | |
| | (i) Show that the matrix is A diagonalizable. | 5 |
| | (ii) Find a matrix P that diagonalizes A. | 5 |
| 5 | Use the geometric series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$ to find the series representation of $\ln \lvert 1 + x \rvert$. | 5 |

**Table 4. Entry number for pre-final questions**

| *Question* | *Course Outcome* | *Programme Outcome* | *Bloom Taxonomy* | *Description* |
|---|---|---|---|---|
| 1(i) | 1 | 1 | 1 | Knowledge |
| 1(ii) | 1 | 1 | 2 | Comprehension |
| 1(iii) | 1 | 1 | 3 | Application |
| 2 | 2 | 1 | 2 | Comprehension |
| 3(i) | 3 | 2 | 3 | Application |
| 3(ii) | 3 | 2 | 2 | Comprehension |
| 4(i) | 4 | 2 | 3 | Application |
| 4(ii) | 4 | 2 | 2 | Comprehension |
| 5 | 5 | 1 | 2 | Comprehension |

## 3. Results and Discussion

All the marks are entered in the Excel **prn* format. This file was transferred using Bond & Box Steps (Bond & Fox, 2006) which is a customized WINSTEPS. The WINSTEPS provides detail description on Summary Statistics and Person-Item Distribution Map.

Summary Statistics gives the detail description of the items (questions) and the persons (Students). Figure 1 illustrates the Summary Statistics for person. Early

analysis on the summary statistics reveals that internal consistency of the raw score is at average with Cronbach-alpha value at 0.49. Other than that Rasch provides indication of sufficiency of spread of both the item and person. This is reflected in the person reliability value which is 0.22 indicating that there is poor spread of person ability within the sample used. According to Linacre (1994), higher value of person reliability increases the possibility of the person having similar likelihood of ability when given similar test.

The spread of person ability is between the maximum location of the person on the logit ruler, which is +0.98 logit, and the minimum location of the person which is at −1.84 logit. The summary statistics reveals also that the mean person value is at −0.70 logit, which means that generally, this students' cohort didn't achieved the objective of the test.

SUMMARY OF 282 MEASURED (EXTREME AND NON-EXTREME) Person

| | TOTAL SCORE | COUNT | MEASURE | MODEL ERROR | INFIT MNSQ | INFIT ZSTD | OUTFIT MNSQ | OUTFIT ZSTD |
|---|---|---|---|---|---|---|---|---|
| MEAN | 16.0 | 9.0 | -.70 | .41 | | | | |
| S.D. | 5.0 | .0 | .57 | .26 | | | | |
| MAX. | 34.0 | 9.0 | .98 | 1.13 | | | | |
| MIN. | 9.0 | 9.0 | -1.84 | .27 | .12 | -2.1 | .09 | -.8 |
| REAL RMSE | .50 | TRUE SD | .27 | SEPARATION | .53 | Person RELIABILITY | | .22 |
| MODEL RMSE | .48 | TRUE SD | .30 | SEPARATION | .62 | Person RELIABILITY | | .28 |
| S.E. OF Person MEAN = .03 | | | | | | | | |

Person RAW SCORE-TO-MEASURE CORRELATION = .96
CRONBACH ALPHA (KR-20) Person RAW SCORE "TEST" RELIABILITY = .49

**Figure 1. Summary Statistics for Persons**

Figure 2 shows the summary statistics for items. The summary statistics reveals an excellent spread of item difficulty with item reliability of 0.98. This means among this pre-final question there are some which is difficult and some which is easy. The spread of the items are from minimum logit of -0.76 to maximum logit at 0.86. Item mean (measure) is 0. The item separation indicates number of the groups the questions can be divided. The summary statistics shows that the item separation is 7.84. There are 2 extreme questions in the test.

SUMMARY OF 7 MEASURED (NON-EXTREME) Item

| | TOTAL SCORE | COUNT | MEASURE | MODEL ERROR | INFIT MNSQ | INFIT ZSTD | OUTFIT MNSQ | OUTFIT ZSTD |
|---|---|---|---|---|---|---|---|---|
| MEAN | 562.0 | 282.0 | .00 | .06 | 1.05 | .2 | 1.14 | .3 |
| S.D. | 237.6 | .0 | .59 | .03 | .18 | 1.5 | .59 | 2.0 |
| MAX. | 901.0 | 282.0 | .86 | .12 | 1.42 | 1.8 | 2.30 | 4.1 |
| MIN. | 313.0 | 282.0 | -.76 | .04 | .80 | -3.0 | .42 | -2.5 |
| REAL RMSE | .07 | TRUE SD | .58 | SEPARATION | 7.84 | Item RELIABILITY | | .98 |
| MODEL RMSE | .07 | TRUE SD | .58 | SEPARATION | 8.53 | Item RELIABILITY | | .99 |
| S.E. OF Item MEAN = .24 | | | | | | | | |

MAXIMUM EXTREME SCORE: 2 Item

**Figure 2. Summary Statistics for Items**

Figure 3 is the Item Distribution Map where Rasch provides the location of all person on the vertical logit ruler, indicated by the dashed vertical line. The higher the person located on the map it indicate the most competent the student is. On the contrast, the item is like the hurdle to the students. The higher the item is on the ruler the more difficult the question is. The Summary Statistics of person indicated that the students cannot be divided into any group. Therefore the ability of every student is the same. Nevertheless, the Person Distribution Map shows the Chemical, Mechanical, Civil and Electrical Engineering students separately. From Figure 3, Chemical students perform better than the other departments. This can be shown from Figure 3 whereby Chemical students rank on the top followed by Mechanical, Civil and Electrical students. In term of performance, there are 54 Chemical students above the mean line 0 for person from Figure 3. This is followed by 34 students for Civil, 27 for Mechanical and 26 students from Electrical Engineering department. This indicates that Chemical students put some effort to answer the difficult questions in this pre-final.

Figure 4 refers to the Item Distribution Map. The discussion aims at the performance of the item all of the 9 questions spread on the logit scale. On the right hand side of the dashed line, the questions are aligned from easy to difficult, starting from the bottom. The distribution of students is on the left side of the vertical dashed line in increasing the order of ability. Letter "M" describes the student and item mean, "S" is one standard deviation distance from the mean and "T" marks two standard deviation distances from the mean. Since the number of students who participated in the pre-test was 282, all their metric numbers are not shown in Figure 4. Instead, that "#" represents 5 students. Below than 5 students, it will be represented by ".".

From Figure 4, the questions can be divided into 3 groups. In order to establish the cut-off point of the 3 groups on the logit ruler, it starts from the mean item 0.00 logit. Aziz (2009) claimed that at mean 0.00, Rasch established a virtual zero of the logit ruler. At this point, given a task, a person has 50:50 chance of success in performing the task, before he decides based on his experience that he may or may not be able to complete the task.

Questions above the mean line are defined as being in the 'mediocre' and 'very difficult' category. From a total of 9 questions, 2 questions fall into the 'very difficult' and 4 questions fall in 'mediocre' category. The questions in the 'very difficult' category are question 2 and question 5. Question 3(i), question 4(i), question 4(ii) and question 1(ii) belongs to 'mediocre' group. On the other hand, questions below the mean line are easy for students to answer during the pre-test. Question 1(i), question 1(iii) and question 3(ii) are the ones deemed easy to solve.

In terms of the Course Outcomes, Course Outcome 1 and Course Outcome 3 fall into the easy category. In Course Outcome 1, students perform well in questions related to knowledge and application. In addition, Course Outcome 3 related to comprehension is also easy for students. Therefore, the fundamental concepts on the matrix and its basic operation, eigenvector and eigenvalue are considered as easy chapters for the Linear Algebra subject. Other Course Outcomes for instance Course Outcome 2 and Course Outcome 5 are extremely difficult for students to

solve. Students are not able to answer of the comprehension questions for both these Course Outcomes.

Course Outcome 4 is also considered mediocre for students. This means the questions are not difficult or easy to be answer. In particular, for Course Outcome 4, students are average in the comprehension and application stages. It is noted that students are average problem solver in the concepts of vector space, diagonalization and quadratic forms and power series.

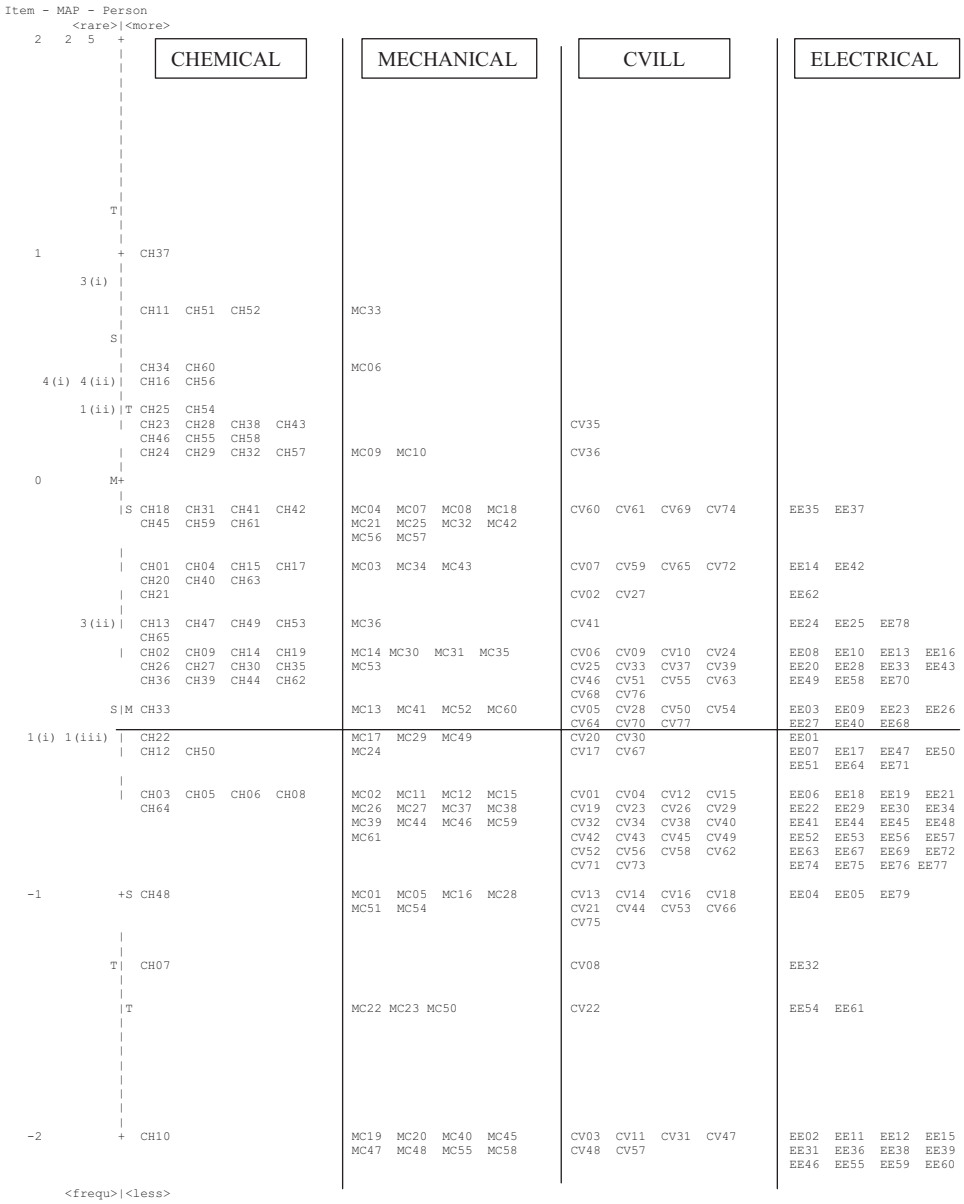



**Figure 3. Person Distribution Map**

There exists a big gap between questions in the difficult category. This gap is between question 3(i) and question 2 and question 5. This gap is indicated by the line with both side arrows in Figure 4. The difficulty level of both questions 2 and 5 is extremely high for the engineering students. Although both questions are comprehension level from Bloom Taxonomy students do badly in answering these questions which are related to Course Outcome 2 and Course Outcome 5.

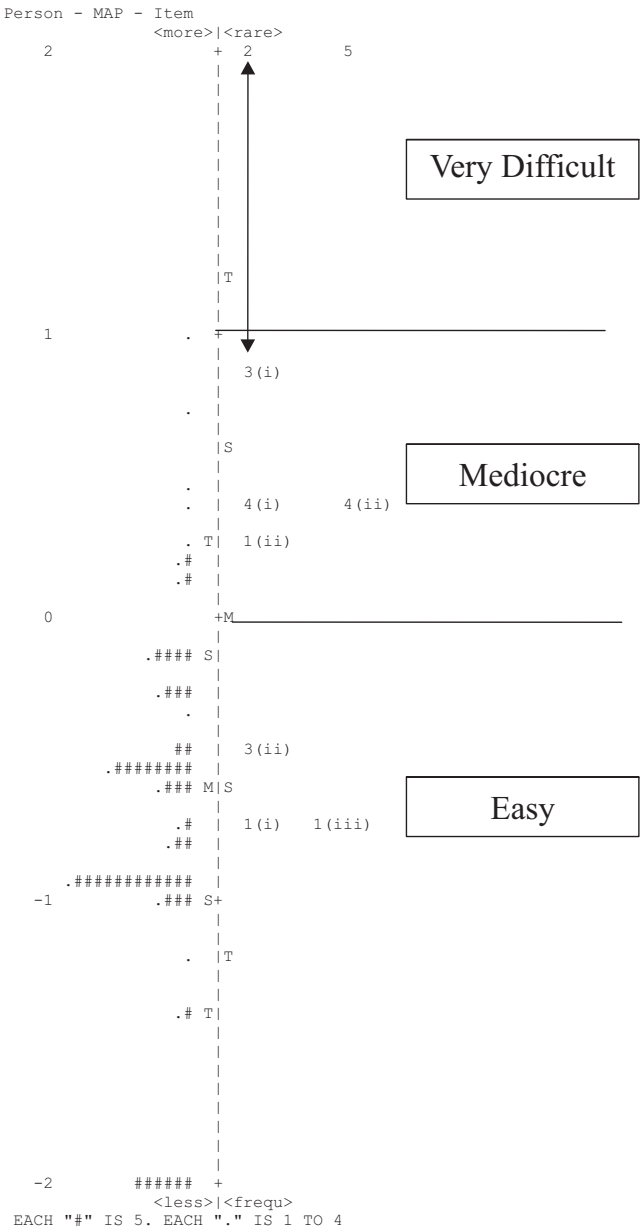



**Figure 4. Item Distribution Map**

## 4. Conclusion

Performance students in Linear Algebra course was assess using Rasch Measurement Model. Rasch measurement is capable of measuring the reliability and validity of the intended questions and to group the students according to their level of understanding on the course. The correlation level between the performance of students from each department and the pre-final questions was identified.

Summary statistics shows that person separation is less than 1. This supports the Person Distribution Map where students cannot be separated into any groups. The Item Distribution Map shows that pre-test questions can be classified into 3 groups. They are very difficult, mediocre and easy. The pre-test questions need to be revised to reduce the gap between the difficult questions in the Item Distribution Map.

**Acknowledgements**

The authors wish to express their utmost gratitude towards Universiti Kebangsaan Malaysia (UKM) and SEGi University for supporting the research.

## References

1. Ayob, A.; Wahab, H.F.A.; Arshad, N.; Bakar, A.A.A. And Husain, H. Assessment of Student Program Outcomes through a Comprehensive Exit Strategy, *Procedia Social and Behavioral Sciences*, 18, (2011) 33-38.
2. Aziz, A.A. Rasch Model Fundamentals: Scale Construct and Measurement Structure, Malaysia, (2009).
3. Bond, T.G. and Fox, C.M. *Bonds & Fox Steps* (customized version of WINSTEPS), Computer Software, (2006).
4. Ibrahim, F. M., Aziz, A.A., Zubairi, Y.Z. and Zaharim, *A.* Using The Rasach Model To Assess Examination Beyond Students' Scores. *Global Engineering Education Conference (EDUCON)*, (2012).
5. Linacre, J.M. Sample Size and Item Calibration Stability, *Rasch Measurent Transactions*, 7, (1994), 328.
6. Osman, S.A.; Badaruzzaman, W.H.W.; Hamid, R.; Taib, K.A. and Rashid, A.K. (2011). The Program Outomes Achievement for Basic Reinforced Concrete Design Course between Two Different Sessions, *Procedia – Social and Behavioral Sciences*, 18, 260-265.
7. Osman, S.A.; Jaafar, O. Badaruzzaman, W.H.W. and Rahmat, R.A.A.O.K. The Course Outcomes (COs) Evaluation for Civil Engineering Design II Course, *Procedia – Social and Behavioral Sciences*, 60, (2012a), 103-111.
8. Osman, S.A.; Naam, S.I.; Jaafar, O.; Badaruzzaman, W.H.W. and Rahmat, R.A.A.O.K. Application of Rasch Model in Measuring Students' Performance In Civil Engineering Design II Course, *Procedia – Social and Behavioral Sciences*, 56, (2012b), 59-66.
9. Said, R.F.M. Application of Rasch Measurement Model in Evaluating Student Performance for Foundation of Computing II. *7th International Conference on University Learning and Teaching (InCULT 2014) Proceedings*, (2014), 251-259.
10. Saifuddin, M.; Azrilah, A.A.; Rodzo'an, N.A.; Omar, M.Z.; Zaharim, A. and Basri, H. Easier Learning Outcomes Analysis Using Rasch model in Engineering Education Research. *EDUCATION'10 Proceedings of the 7th WSEAS International Conference on Engineering Education*, (2010), 442-447.

Corresponding Author: Faculty of Engineering and Built Environment, SEGi University, Malaysia
*E-mail address:* **lohgheswarynagarethinam@gmail.com**

Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, Malaysia

Faculty of Computing and IT, King Abdulaziz University, Saudi Arabia

Faculty of Education, Universiti Kebangsaan Malaysia, Malaysia