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COMPARATIVE STUDY OF FLOW AND HEAT TRANSFER BEHAVIOR OF NEWTONIAN AND NON-NEWTONIAN FLUIDS OVER A PERMEABLE STRETCHING SURFACE

AMIT PARMAR AND SHALINI JAIN

Abstract: In this article, we have investigated a mathematical model for flow and heat transfer behavior of Newtonian fluid and non-Newtonian fluids including Maxwell fluid, Casson fluid, Williamson fluid past a stretching surface in the presence ofthermal radiative effect with transverse magnetic field and suction/injection boundary condition. By using suitable transformation, the governing equations are converted into non-linear coupled ODEs and solved by bvp4c solver with MATLAB. Velocity and temperature profile is discussed and depicted graphically.

1. Introduction

The flow and thermal energy transfer of Newtonian and non-Newtonian fluids play importance role in industry, research and engineering field. Newton's law of motion describes that the shearing stress and rates of strain are proportional to each other. A fluid that obeys Newton's law of motion is called Newtonian fluid. Non-Newtonian fluids are divided in different categories such as Maxwell fluid, Casson fluid, Williamson fluid, viscoelastic fluid, couple stress fluid, micro polar fluid, power-law flow, Jeffrey fluid etc.

Casson fluid modal constitutes a plastic fluid modal. It exhibits shear thinning characteristics, high shear viscosity and yield stress. Din et. al., [1] investigated the moving and stationary boundary problems for Casson fluid with effects of viscous dissipation over a convective boundary conditions. Ramzan et. al., [2] studied viscous dissipation of Casson-nano fluidover thermal boundary conditions. Mahanta and Shaw [3] studied the mixed convection Casson stagnation point flow past convective boundary conditions. Bortteir et. al., [4] analyzed MHD Casson fluid over a vertical permeable surface with chemical reaction. Megahed [5-6] examined the unsteady Casson fluid toward a permeable and non-permeable stretching sheet with first and second-order slip flow and thermal slip. They examined the influence of the internal heat generation/absorption and thermal radiation on unsteady Casson fluid modal. Reddy et. al., [7] discussed the phenomena of thermal and concentration transfer in MHD Casson fluid past suction/injection exponentially stretching surface. Casson fluid flow with viscous dissipation and thermal energy transfer past a porous shrinking sheet investigated by Qasima et. al., [8]. Patel et. al., [9] discussed the MHD Casson fluid flow modal with radiation and chemical reaction.

Keyword. Radiation; Suction/injection; Maxwell fluid; Casson fluid; Williamson fluid; Newtonian fluid bvp4c solver.

Williamson fluid model describes the flow of shear thinning non-Newtonian fluids. Williamson fluid is viscoinelastic fluid. The viscoinelastic fluids contain both viscous and elastic properties. The Navier Stokes equations are explaining the rheological properties of fluids. A real fluid has both minimum viscosity and maximum viscosity depending upon the molecular shape of the fluid. Williamson [10] examined the flow of pseudo-plastic materials, pseudo-plastic fluids. Nadeem and Hussain [11-13] presented the concept of heat transfer of Williamson fluid. They have investigated various surface condition such as heated surface, stretching surface and exponentially stretching surface. Gorla et. al., [14] discussed the influence of MHD on non-Newtonian Williamson-nano fluid in porous medium with chemical reaction over a melting heat transfer surface condition. Dapra and Scarpi [15] presented the pulsatile Williamson fluid flow in a rock fracture. They solved the problem by perturbation method. Williamson fluid flow with thermal radiative and Ohmic dissipation over hydro magnetic boundary layer flow analyzed by Alsaedi et. al., [16].

Maxwell fluid model is mainly useful for polymers of low molecular weight. Maxwell model is used to show the stress relaxation. Ramesh et. al., [17] studied of Nano-Maxwell stagnation point flow fluid towards a permeable surface. Ibrahim [18] analyzed the heat transfer of UCM-Nano fluid on MHD stagnation point flow and past a convective stretching sheet. Kameswaran et. al., [19] discussed the unsteady MHD UCM fluid flow with non-linear chemical reaction toward a stretching surface. Zhang et. al., [20] studied the UCM-Nano fluid flow, thermal energy and mass transfer with Cattaneoe Christov double-diffusion toward a stretching sheet with slip velocity.

In this study, we discussed the impact of transverse magnetic field and thermal radiation on momentum and heat transfer behavior for Casson, Maxwell, Williamson fluid and Newtonian fluid past a suction/injection surface.

2. Mathematical Formulation

Consider two-dimensional steady fluid flow and thermal energy transfer an incompressible Newtonian and non-Newtonian fluids past a permeable stretching surface. Let surface is stretching along x axis and moves with velocity ax. Where $a > 0$ is stretching parameter. Assuming that U_w, and T_w are respectively the fluid velocity, and temperature near the surface.

The continuity, momentum and energy equations of fluid flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \\ &\quad - k_0\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) - \frac{\sigma \mathrm{B}_0^2 u}{\rho} \end{aligned} \tag{2}$$

$$u\frac{\partial \mathrm{T}}{\partial x} + v\frac{\partial \mathrm{T}}{\partial y} = \frac{k}{\rho \mathrm{C}_p}\frac{\partial^2 \mathrm{T}}{\partial y^2} - \frac{1}{\rho \mathrm{C}_p}\frac{\partial q_r}{\partial y} + \frac{\sigma \mathrm{B}_0^2}{\rho \mathrm{C}_p}u^2 \tag{3}$$

where, $u(x, y)$ and $v(x, y)$ are the horizontal and vertical velocity components, ρ, ν and ρC_p are respectively fluid density, kinematic viscosity and heat capacities of particles. T: temperature and T_∞: ambient fluid temperature. Subject to boundary conditions

$$\begin{aligned} &\text{at} \quad y = 0; \quad u = U_w, \quad v = -V_w, \quad T = T_w \\ &\qquad y \to \infty; \quad u \to 0, \quad T \to T_\infty \end{aligned} \tag{4}$$

and the surface is stretching with velocity $U_w = bx$ and V_w suction/injection velocity.

Following Rosseland approximation q_r, the radiation heat flux is given $q_r = -\left(\frac{4\sigma}{3k^*}\right)\frac{\partial T^4}{\partial y}$, expanding T^4, in a Taylor series about T_∞, on neglecting higher order term, we get

$$T^4 \approx T_\infty^4 + 4T_\infty^3 T - 4T_\infty^3 T_\infty$$

$$\frac{\partial q_r}{\partial y} = \frac{\partial}{\partial y}\left(\frac{-4\sigma}{3k^*}\frac{\partial T^4}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{-4\sigma}{3k^*}\frac{\partial(T_\infty^4 + 4T_\infty^3 T - 4T_\infty^3 T_\infty)}{\partial y}\right) = \frac{-16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}$$

Equation (3) can be rewritten as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\rho C_p k^*}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho C_p}u^2 \tag{5}$$

Solution:

We now introduce the following relations for u, v as

$$u = bxf'(\eta), \quad v = -\sqrt{bv}f(\eta), \quad \eta = y\sqrt{\frac{b}{v}} \quad \text{and} \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Equation (2) and (5) thus reduces to the following non-dimensional form

$$\left(1 + \frac{1}{\beta}\right)f''' - \lambda(f^2 f''' - 2ff'f'') - f'^2 + f''f - Mf' + \mathrm{We}\, f''f''' = 0 \tag{7}$$

$$\theta''\left(1 + \frac{4}{3}R\right) + \Pr f\theta' + \Pr M\,\mathrm{Ec}\, f'^2 = 0 \tag{8}$$

Boundary conditions (6) reduces as:

$$\begin{aligned} \eta = 0: &\quad f'(\eta) = 1, \quad f(\eta) = S, \quad \theta(\eta) = 1 \\ \eta \to \infty: &\quad f'(\eta) \to 0, \quad \theta(\eta) \to 0 \end{aligned} \tag{9}$$

where, $\mathrm{We} = \Gamma x\sqrt{\frac{2b^3}{\nu}}$; non-Newtonian Williamson parameter, $\Pr = \frac{k}{\mu C_p}$; Prandtl number, $R = \frac{4\sigma T_\infty^3}{kk^*}$; Radiation parameter, k^*; thermal radiation parameter, $\mathrm{Ec} = U^2/C_p(T_w - T_\infty)$; Eckert number, $M = \frac{\sigma B_0^2}{\rho b}$; Magnetic field parameter, β; Casson fluid parameter and $\lambda = k_0 b$; Maxwell fluid parameter.

The problem is analyzed the following cases:

- If We = 0, $\lambda = 0$ and $\beta = \infty$ then the problem represents Newtonian model.
- If We = 0, $\beta = \infty$ and $\lambda \neq 0$ then the problem represents Maxwell fluid model.
- If $\lambda = 0$, $\beta = \infty$ and We $\neq 0$ then the problem represents Williamson fluid model.
- If We = 0, $\lambda = 0$ and $\beta \neq \infty$ then the problem represents Casson fluid modal.

The governing equations (7) and (8) subject to the boundary condition equation (9) has been solve using bvp4c solver.

3. Result Discussion

The fix value of physically parameters S = 0.5, R = 1, Pr = 2, M = 1, $\lambda = 0.5$, We = 0, $\beta = 2$, Ec = 1 and excluding the varied value of particular graph. Several sets of numerical solution have been carried out for different combinations of pertinent parameters namely, Williamson fluid parameter (We), Maxwell fluid parameter (λ), Casson fluid parameter (β), radiation parameter R, Magnetic field parameter (M), Eckert number (Ec) and suction/injection parameter (S). Figures (1) and (2) shows the influence of magnetic field parameter on velocity and temperature profile with Newtonian fluid, Casson fluid, Maxwell fluid and Williamson fluid. Increases the magnetic field parameter (M) reduces the momentum boundary layer and rises in thermal boundary layer thickness. It is interesting to mention the velocity profile for Casson fluid is extremely effected by magnetic field parameter (M) compared with other fluids and in temperature profile, Maxwell fluid is highly effected by magnetic field parameter (M) compared with other fluids. Physically, magnetic field perpendicular to an electrically-conducting fluid has the tendency to produce a drag-like force known as the Lorentz force which acts in the against flow, causing a flow deceleration effect. It reduces the momentum boundary layer thickness and rises the thermal boundary layer thickness. Figures (3) and (4) show the influence of suction/injection parameter on velocity profile and temperature profile with Newtonian fluid and non-Newtonian fluids. Rise the value of suction/injection parameter reduces the momentum boundary layer and reduces the thermal boundary layer thickness. It is observed from velocity profile of Casson fluid is highly influenced of suction/injection parameter condition compared with other fluids and in temperature profile, Maxwell fluid is highly influenced of suction/injection parameter condition compared with other fluids. Physically, increasing suction parameter is to eliminate the low-momentum fluid around the warm wall and delays both transition and separation since the fluid near the heated wall ($\eta = 0$) is pushed past the surface where the buoyancy forces can do the fluid due to high impact of the viscosity. Figure (5) shows the influence of radiation parameter on temperature profile with Newtonian and non-Newtonian fluids. Increases radiation parameter rises thermal boundary layer thickness. In temperature profile, Maxwell fluid is more influenced of radiative parameter liken with other fluids. Generally, increasing values of R which is due to the reason that as thermal radiation parameter increases, the mean absorption coefficient decreases, which results in rise to the divergence of radiative heat flux. Hence, the rate of radiative heat transferred to the fluid shoot up, so that the fluid temperature increases. High values of radiation parameter cause increased dominance of conduction thereby decreasing the temperature profile. Figure (6) shows the effect of Prandtl number on temperature profile with various fluid condition. Increases the value of Prandtl number reduces the thermal boundary layer thickness. It temperature profile, Maxwell fluid is more influenced of Prandtl number liken with other Newtonian and non-Newtonain fluids. Prandtl number signifies the ratio between momentum diffusivity to thermal energy diffusivity. Fluids with lower value of Prandtl number will possess higher thermal energy conductivities so that heat can diffuse from the sheet faster than for higher Pr fluids (thinner boundary layers).

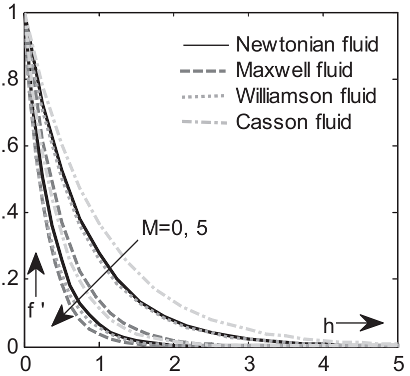

Figure 1. Influence of Magnetic field parameter with various fluids condition on velocity profile

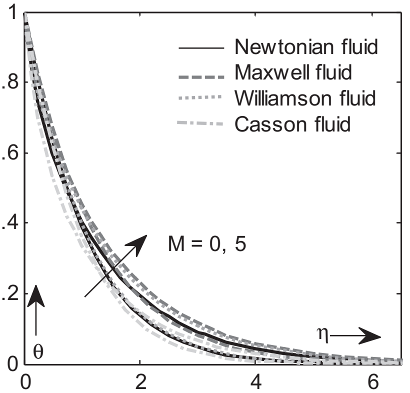

Figure 2. Influence of Magnetic field parameter with various fluids condition on temperature profile

Table 1 and Table 2 are the comparison table of $-\theta'(0)$ and $-f''(0)$

Table 1

Pr	*Comparison of $-\theta'(0)$ for different values Pr in the absence of the parameters $S = R = We = Ec = M = \lambda = 0$ and $\beta \to \infty$*					
	Nadeem et. al., [22]	*Khan et. al., [23]*	*Golra et. al., [24]*	*Wang [25]*	*Narayana et. al., [19]*	*Present study*
0.7	0.454	0.454	0.454	0.454	0.4539	0.454049257
2.0	0.911	0.911	0.911	0.911	0.9114	0.911360664

Prandtl number can be used to rise the cooling rate in conducting flows. At high Prandtl number the fluid is very viscous and the velocity is reduced. Figure (7) shows the effect of Eckert number on temperature profile with Newtonian and non-Newtonian fluids condition. Increases the value of Eckert number rises the thermal energy boundary layer thickness. In temperature profile, Maxwell fluid is highly influenced of Eckert number compared with other fluids. Physically, rising the values of the Eckert number than generates energy in the fluid due to frictional heating. Thus, increasing the values of Ec, rises the temperature within the fluid flow.

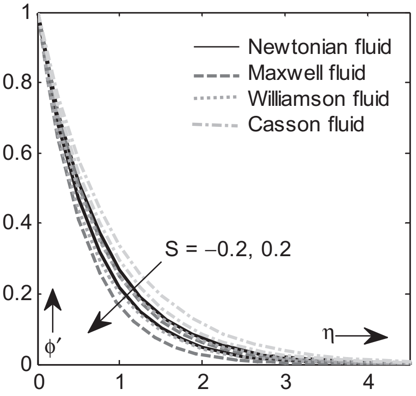

Figure 3. Influnece of suction/injuction parameter with various fluids condition on velocity profile

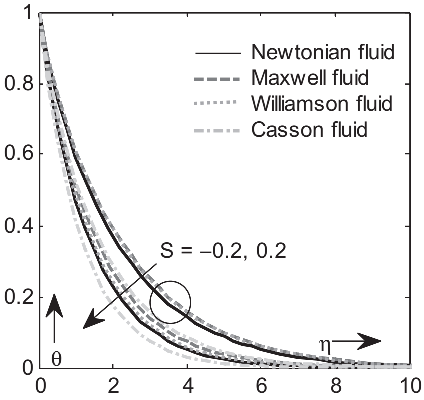

Figure 4. Influnece of suction/injuction parameter with various fluids condition on temperature profile

Table 1 and 2 shows the comparison of the present results with the existed results of Nadeem et. al., [22], Khan et. al., [23], Golra et. al., [24], Wang [25], Narayana et. al., [19], Anderson et. al., [26], Prasad et. al., [27], Mukhopadhyay et. al., [28] and Palani et. al., [29]. Under some special conditions, present results have an excellent agreement with the existed results. This shows the validity of the present results along with the accuracy of the numerical technique we used in this study.

Table 2

Comparison of $-f''(0)$ for different values M in the absence of the parameters $S = R = We = \lambda = 0$ and $\beta \to \infty$

M	*Anderson et. al., [26]*	*Prasad et. al., [27]*	*Mukhopadhyay et. al., [28]*	*Palani et. al., [29]*	*Present study*
0.0	1.000000	1.000174	1.000173	1.00000	1.000001171
0.5	1.224900	1.224753	1.224753	1.224745	1.224744873
1	1.414000	1.414449	1.414450	1.414214	1.414213562
1.5	1.581000	1.581139	1.581140	1.581139	1.581138830
2	1.732000	1.732203	1.732203	1.732051	1.732050808

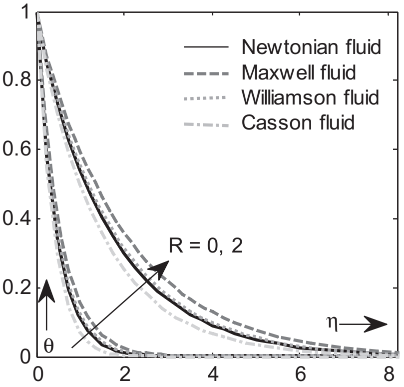

Figure 5. Influnece of radiation parameter with various fluids condition on temperature profile

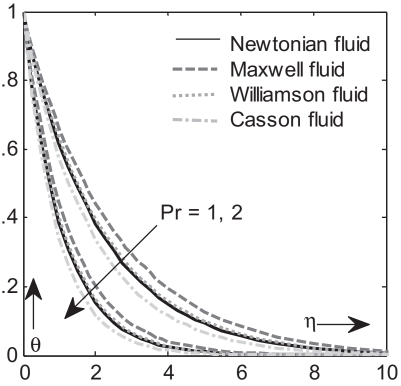

Figure 6. Influnece of Prandtl number with various fluids condition on temperature profile

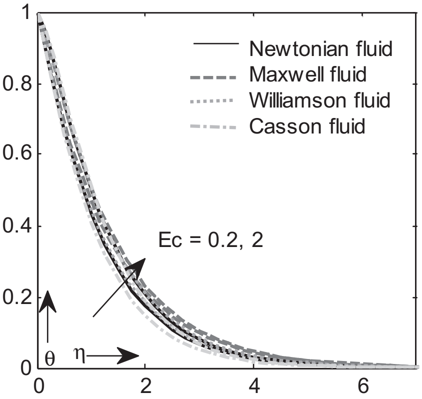

Figure 7. Influnece of Eckert number with various fluids condition on temperature profile

4. Conclusion

This study presents a numerical solution to analyze the momentum and thermal heat transfer behavior of Newtonian and non-Newtonian fluid such as Casson, Maxwell, Williamson fluids past a stretching surface in the presence of thermal radiation, transverse magnetic field and suction/injection effects. The governing PDEs corresponding to the momentum and thermal energy equations are converted into non-linear coupled ODEs and solved by bvp4c solver. The conclusions of the present study are shown as follows:

- Radiation parameter (R) and Eckert number (Ec) have propensity to rises the thermal heat transfer rate.
- Maxwell fluid has better thermal heat transfer performance than the Casson fluid, Williamson fluid and Newtonian fluid.
- Increases the value of magnetic field parameter reduces the momentum boundary layer whereas rises the thermal boundary layer thickness.

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Department of Mathematics and Statistics, Manipal University Jaipur, Rajasthan, India
E-mail address: **`shalini.jain@jaipur.manipal.edu, amit.198631@gmail.com`**