Global and Stochastic Analysis
Special Issue: *25th International Conference of Forum for Interdisciplinary Mathematics*

MUK PUBLICATIONS
Open Access Publisher

NUMERICAL STUDY OF UNSTEADY INCLINED MHD CASSON FLUID FLOW AND HEAT TRANSFER THROUGH POROUS MEDIUM ALONG A VERTICAL SHEET

PREETI GUPTA AND SHALINI JAIN

ABSTRACT: In present paper, we have investigated unsteady two-dimensional fluid flow and heat transfer of Casson fluid through a porous medium along with porous, vertical, non-conducting sheet. This phenomenon modeled in the form of partial differential equation with boundary condition. Using suitable dimensionless quantities, these governing equations transformed into non-dimensional partial differential equations. The corresponding non-dimensional partial differential equations are solved numerically with Crank-Nicolson implicit finite difference scheme. The pertinent findings are analyzed and displayed through figures.

1. Introduction

The study of non-Newtonian fluids have been increases due to its wide variety of industrial and engineering applications. Non-Newtonian fluid model for Casson fluid was presented in1995. Crane [1] has first initiated the study of boundary layer flow over stretching plate, later on Sakiadis [2] and Tsou et. al., [3] has extended his work. Nadeem et. al., [4] has investigated Casson fluid flow of three dimensional MHD porous stretching sheet. Shaw et. al., [5] obtained results for pulsatile Casson fluid flow.

The study of MHD has been initiated by Swedish engineer Hannas Alfven [6]. He has investigated steady motion of fluid under transverse magnetic field Singh [7] analyzed steady MHD fluid flow between two parallel plates. Several researchers Mahmud and Fraser [8], Chauhan and Kumar [9], Raptis and Kafoussias [10], Chamkha [11], Jain and Choudhary [12], nazar et. al., [13], Ishak et. al., [14] has investigated flow and heat transfer characteristic in the presence of porous medium and magnetic field.

Based on literature review mentioned above, there have been reported no studies on unsteady flow and heat transfer of Casson fluid through porous medium over a vertical sheet. We have investigated unsteady two-dimensional fluid flow and heat transfer of Casson fluid through a porous medium, vertical, non-conducting platsheet. This phenomenon modeled in the form of partial differential equation with boundary condition.

2. Formulation of the Problem

The unsteady two-dimensional flow of Casson fluid through a porous medium bounded by a vertical, infinite, porous, non-conducting sheet in the presence of

Keyword. Casson fluid, heat transfer, porous medium,vertical plate, inclined vertical sheet.

inclined magnetic field be taken along the sheet in upward direction is taken normal to it. The rheological equation of state, for an isotropic and incompressible flow of a Casson fluid as follows:

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c \\ 2(\mu_B + p_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c \end{cases}$$

where, τ_{ij} is the (i, j)-th component of the stress tensor, π is the product of the component of the deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian casson fluid, and p_y is the yield stress of the fluid. When stress is greater than the yield stress fluid starts to move under the force going assumption, the governing equation of such type of flow are:

The governing equations of flow and heat transfer through porous medium are as follows:

$$\frac{\partial v}{\partial y} = 0, \; v = -V_0 \text{ (Constant)}$$

$$\rho \frac{\partial u^*}{\partial t^*} - \rho V_0 \frac{\partial u^*}{\partial y^*} = \mu_B \left\{1 + \frac{1}{\gamma}\right) \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\mu}{K'} u^* - \sigma B_0^2 \sin^2 \alpha u^* \tag{1}$$

$$\rho c_p \left(\frac{\partial T^*}{\partial t^*} - V_0 \frac{\partial T^*}{\partial y^*} \right) = k \frac{\partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*} \right)^2 \tag{2}$$

Subject to the boundary conditions

For $t^* \leq 0$; $u^* = 0$, $T^* = T_\infty$, for all y^*

For $t^* > 0$; $u^* = U_0, \frac{\partial T^*}{\partial y^*} = -\frac{q^*}{\kappa}$, at $y^* = 0$

$$u^* \to 0, \; T^* \to T_\infty, \text{ as } y^* \to \infty \tag{3}$$

where, u^*, t^*, T^*, μ_B, γ, ρ, c_p, k, σ, and K are the velocity of the fluid in x-direction, time, temperature, plastic dynamic viscosity, Casson parameter, constant density, specific heat at constant pressure and thermal conductivity, electric conductivity of the fluid and permeability of the fluid respectively.

Introducing the following dimensionless quantities:

$$u = \frac{u^*}{U_0} \quad y^* = \frac{\upsilon y}{V_0} \quad t^* = \frac{\upsilon t}{V_0^2} \quad k' = \frac{\upsilon^2 K}{V_0^2} \quad M^2 = \frac{\sigma B_0^2 \upsilon}{\rho V_0^2}$$

$$Ec = \frac{U_0^2}{T_r C_p} \quad Pr = \frac{\mu C_p}{\kappa} \quad T^* = T_\infty + (q^* \upsilon / k V_0)\theta \quad T_r = \frac{q^* \upsilon}{k V_0} \tag{4}$$

into the equations from (1) to (4), we get

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{K} u - M^2 \sin^2 \alpha u \tag{5}$$

$$P_r\left(\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial y}\right)=\frac{\partial^2\theta}{\partial y^2}+P_rE_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{6}$$

The corresponding initial and boundary conditions in dimensionless form are

For $t \leq 0$; $u = 0$, $\theta = 0$, for all y^*

For $t > 0$; $u = 1$, $\frac{\partial\theta}{\partial y} = -1$ at $y = 0$

$$u \to 0,\ \theta \to 0, \quad \text{as} \quad y \to \infty \tag{7}$$

Numerical Solution. The governing equations (5) and (6) subject to the boundary conditions (7) are solved by Crank-Nicolson implicit finite difference scheme. By substituting the finite difference approximations to derivatives in equations (5) and (6), the governing equations are reduced to the following algebraic equations:

$$u_{i+1,j}\left(\lambda\Delta y+\lambda\left(1+\frac{1}{\gamma}\right)\right)-u_{i,j}\left(K^{-1}\Delta t+\lambda\Delta y-2+2\lambda\left(1+\frac{1}{\gamma}\right)+M^2\Delta t\sin^2\alpha\right)+u_{i-1,j}\left(\lambda\left(1+\frac{1}{\gamma}\right)\right)$$
$$=-u_{i+1,j+1}\left(\lambda\Delta y+\lambda\left(1+\frac{1}{\gamma}\right)\right)+u_{i,j+1}\left(K^{-1}\Delta t+\lambda\Delta y+2+2\lambda\left(1+\frac{1}{\gamma}\right)+M^2\Delta t\sin^2\alpha\right)-u_{i-1,j+1}\left(\lambda\left(1+\frac{1}{\gamma}\right)\right) \tag{8}$$

$$\theta_{i+1,j+1}(\lambda+\lambda\Delta yp_r)-\theta_{i,j+1}(\lambda\Delta yp_r+2\lambda+2p_r)+\theta_{i-1,j+1}(\lambda)$$
$$=-\theta_{i+1,j}(\lambda+\lambda\Delta yp_r)+\theta_{i,j}(\lambda\Delta yp_r+2\lambda-2p_r)-\theta_{i-1,j}(\lambda)+R_{i,j} \tag{9}$$

where, $\quad R_{i,j}=-\lambda P_rE_c\left((u_{i+1,j}-u_{i,j})^2+(u_{i+1,j+1}-u_{i,j+1})^2\right)$

and $\quad \lambda=\frac{\Delta t}{(\Delta y)^2}$, Δt and Δy are mesh sizes.

In equations (8) and (9), the unknowns $u_{i,j+1}$ and $\theta_{i,j+1}$ are not expressed explicitly in terms of known quantities namely $u_{i-1,j}$, $u_{i,j}$, $u_{i+1,j}$ and $\theta_{i-1,j}$, $\theta_{i,j}$, $\theta_{i+1,j}$ at the time level j. The equation (8) and (9), written at all the interior mesh points form a tridiagonal system of linear algebraic equations, with the following initial and boundary conditions for velocity and temperature fields;

$$u_{i,1} = 0 \text{ and } \theta_{i,1} = 0,\ i = 1, 2, 3, ..., q+1 \tag{10}$$

$$\left.\begin{array}{r} u_{1,j} = 1,\ u_{q+1,j} = 0 \\ \theta_{2,j} - \theta_{1,j} = -\Delta y,\ \theta_{q+1,j} = 0 \end{array}\right\},\ j = 2, 3, 4, ..., p+1 \tag{11}$$

To find the numerical solution for the present problem, we divided the computational domains $0 < t < \infty$ and $0 < y < \infty$ into intervals with step sizes $\Delta t = 0.001$ and $\Delta y = 0.005$ for time t and space y, respectively. If we carry out the computations by changing step sizes slightly to check the stability of finite difference schemes, we observed that there is no significant change is seen in the results obtained by the numerical scheme. It is also known that implicit Crank-Nicolson scheme is convergent and stable for all values of λ.

3. Discussion

In paper the unsteady MHD flow and Casson fluid through porous medium along a vertical porous sheet with constant heat flux is studied. Velocity and temperature analysis solutions are obtained by implicit Crank-Nicolson method. The effects of various parameters on the fluid velocity and temperature are shown through figures.

Figure [1-4] illustrate the effect of various parameters on fluid velocity. These figures shows that the velocity of the fluid decreases with the increase of α, Casson parameter (γ) and Hartman number (M). While fluid velocity increases as we increases permeability parameter (K).

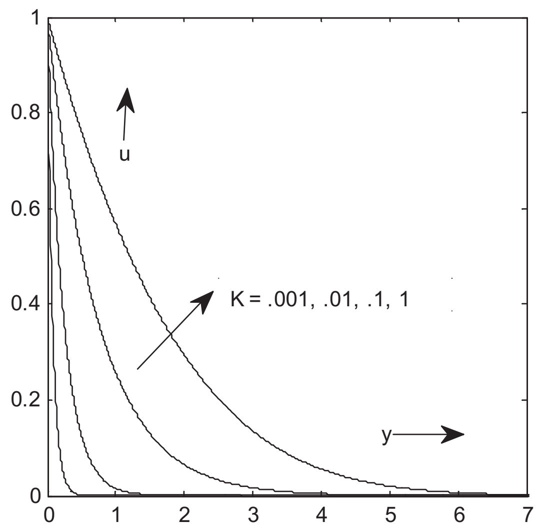

Figure 1. **Velocity profile versus** y **when** $\gamma = .2$**, M = 1 and** $\alpha = \pi/6$

Figure [5-8]depicts that the temperature of the fluid decreases with the increase of permeability parameter (K), and Prandtl number (Pr). Temperature profile shows enhancement as we increases Eckert parameter (Ec) and Casson parameter (γ).

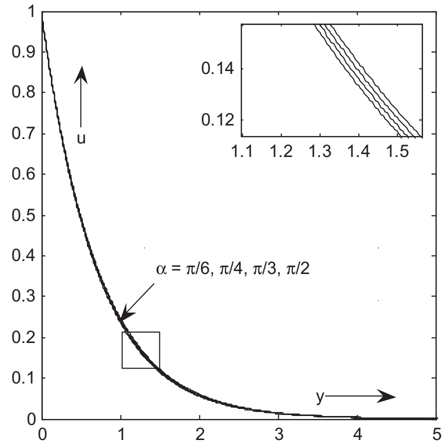

Figure 2. Velocity profile versus y when $\gamma = .2$, M = 1 and K = .1

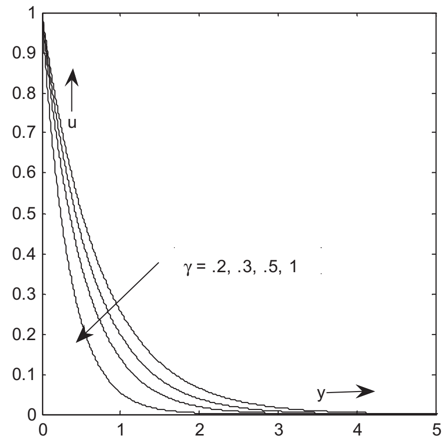

Figure 3. Velocity profile versus y when K = .1, M = 1 and $\alpha = \pi/6$

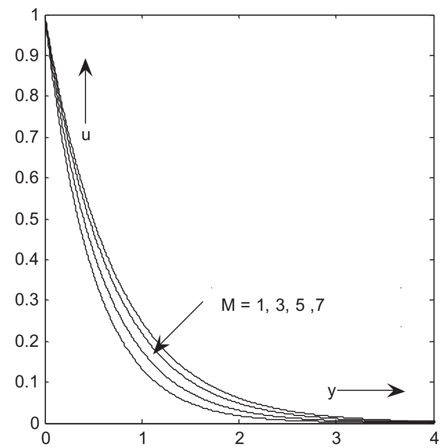

Figure 4. Velocity profile versus y when $\gamma = .2$, $\alpha = \pi/6$ and K = .1

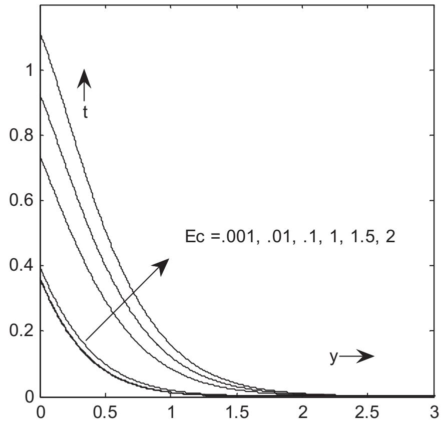

Figure 5. Temperature profile versus y when K = .1, M = 1, Pr = 2, γ = .2 and α = 621/6.

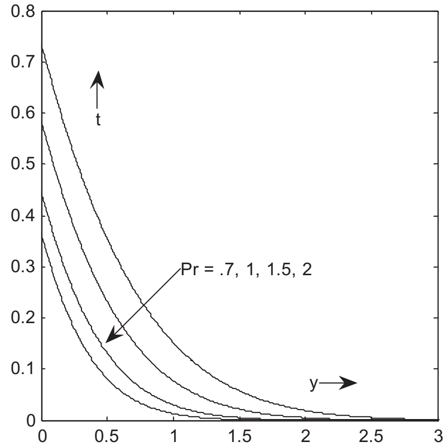

Figure 6. Temperature profile versus y when $\alpha = \pi/6$, M = 1, Ec = .1, γ = .2 and K = .1.

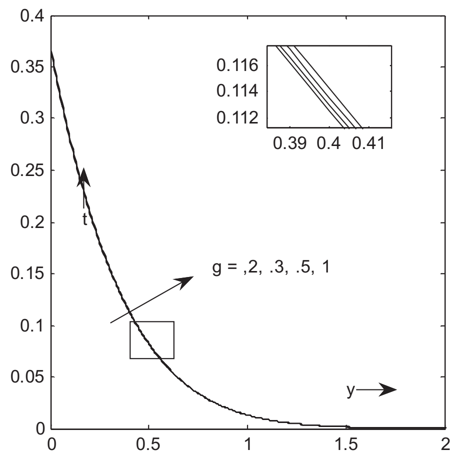

Figure 7. Temperature profile versus y when K = .1, M = 1, $\alpha = \pi/6$, Pr = 2 and Ec = .1

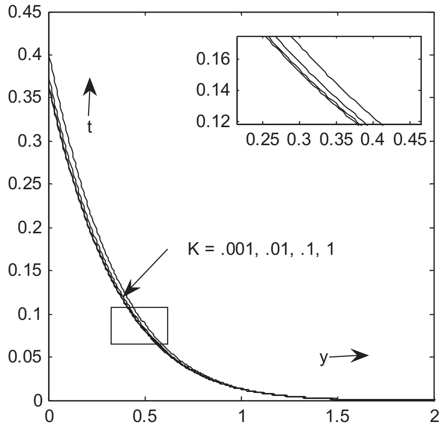

Figure 8. Temperature profile versus y when $\gamma = .2$, M = 1, $\alpha = \pi/6$, Pr = 2 and Ec = .1

4. Conclusion

From this investigation we have observed that:

(i) Velocity increases when permeability parameter (K) increases.

(ii) Velocity decreases when α, Casson parameter (γ) and Hartman number (M) increases.

(iii) Temperature increases when Casson parameter (γ) and Eckert parameter (Ec) increases.

(iv) Temperature decreases when permeability parameter (K) and Prandtl number (Pr) increases.

References

1. L. Crane, Flow past a stretching plate, Zeitschrift fur Angewandte Mathematik und Physik 21, 645-647, 1970.
2. B.C. Sakiadis, Boundary layer behavior on continous solid surfaces. II. Boundary layer on a continous flat surface, Journal American Institute of Chemical Engineers 7, 221-225, 1961.
3. F.K. Tsou, E.M. Sparrow, R.J. Goldstein, Flow and heat transfer in the boundary layer on a continous moving surface, International Journal of Heat and Mass Transfer 10, 219-235, 1967.
4. S. Nadeem, R. Ul Haq, N.S. Akbar, Z.H. Khan, MHD three- dimentional Casson fluid flow past a porous linearly streatching sheet, Alexendria Engineering Journal 52, 577-582, 2013.
5. S. Shaw, R.S.R. Gorla, P.V.S.N. Murthy, C-O Ng, Pulasatile Casson fluid flow through a stenosed bifurcated artery, International Journal of fluid mechanics Research 36(1), 43-63.
6. H. Alfven, Existence of electromagnetic-hydrodynamic waves, Nature, 150, No. 3805, 405-406, 1942.
7. C.B. Singh, Magneto hydrodynamic steady flow of liquid between two parallel plates, In: Proc. Of first conference of Kenya Mathematical society, 24-26, 1993.
8. S. Mahmud and R. A. Fraser, Analysis of mixed convection- Radiation interaction in a vertical channel: Entropy generation, Exergy an Int. J.n 2, 330-339, 2002.
9. A. Raptis and N. Kafoussias, Heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of magnetic field. Energy Res., 6, 241-245, 1982.
10. A.J. Chamkha, MHD free convection from a vertical plate in saturated porous medium. Appl. Math. Modeling, 21, 603-609, 1997.

11. S. Jain and R. Choudhary, "Effects of MHD on boundary layer flow in porous medium due to Exponentially Shrinking Sheet with Slip" in Procedia Engineering (Elsevier Journal, I.F: 0.69), 127, 1203-1210, 2015.
12. R. Nazar, N. Amin, D. Filip, I. Pop, Stagnation point flow of a micropolar fluid towards a stretching sheet, International Journal of Non-Linear Mechanics 39, 1227-1235, 2004.
13. A. Ishak, R. Nazar, I. Pop, Mixed convection boundary layers in the stagnation point flow toward a stretching vertical sheet, Meccanica 41, 509-518, 2006.
14. D.S. Chauhan and V. Kumar, "Exact solution of unsteady couttee flow due to an oscillating wall with slip condition in a parallel plate channel filled by a hyper porous medium", J. Rajasthan Acad. Phy. Sci., Vol. 8(2), 219-230, 2009.

Department of Mathematics & Statistics, Manipal University Jaipur, Jaipur-303007, India

E-mail address: **shalini.jain@jaipur.manipal.edu**