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POWER OF CONTROL CHART FOR SINGLY TRUNCATED BINOMIAL DISTRIBUTION UNDER INSPECTION ERROR

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Abstract: In this paper an expression for the power of control chart with standardized normal variate for Singly-Truncated Binomial Distribution (STBD) is obtained. Furthermore, numerical calculation is also done to see the effect of inspection error on the power curve. Also, average run length (ARL) is measured to gain the knowledge of the sensitivity of screening procedure.

1. Introduction

Binomial distribution is exercised to create control chart for attributes, either$p$-chart or d-chart when fraction defective or the number of defective is taken into consideration. Probability distributions frequently occur in practices which are of binomial nature, but for several causes zero value is overlooked. For example, assume that the variable understudy characterize the number of defective items in a manufactured lot of n items and r defects are to be anticipated and not more than n are observed, then remaining defects may follow a singly truncated binomial distribution. A special case, when $r = 1$ means singly-truncated binomial distribution (STBD). Inspection, however regarded as infertile, still remains one of the major activities in most quality control set ups in different types of industries. The concept of inspection error was first introduced by Lavin (1946) and then the analysis of inspection error continues to be an important area of research in statistical quality control. No inspection is perfect all the time. Indeed, it is generally distinguished that 100 percent inspection is much less than 100 percent effective in screening out defective items. This error can be of two types: false-positive, classifying non-defective as defective, and false-negative, classifying defective as non-defective. Juran and Gryna (1970) have indicated that in the face of monotony and fatigue, only about 80 percent of the defective will be detected. Case *et. al.*, (1973) have studied the continuous sampling plan under inspection error.

A statistical power analysis is either retrospective or prospective in which prospective analysis is frequently exercised to conclude a required sample size to accomplish target statistical power, while a retrospective analysis computes the statistical power of a test for a given sample size and effect size. Reynolds (1975) has approximated the average run length in cumulative sum control (CUSUM) charts. Robinson and Ho(1978) have studied average run length of geometric moving average chats by numerical methods. Cohen (1988) says, it is the probability that it will result in the conclusion that the phenomenon exists. Hawkins (1992) evaluated

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the average run length of CUSUM charts for an arbitrary data distribution. Greene (2000) considered the power of a statistical test is the probability that it will correctly show the way to the rejection of a false null hypothesis. High (2000) concluded the statistical power is the ability of a test to identify an effect, if the effect actually exists.

The ARL, which is defined as the average number of samples before the chart signals out-of-control process, has been usually employed as a performance indicator to estimate the effectiveness of various control schemes, provided that the sampling interval remains constant.

In this paper an attempt has been made to obtain the power of control chart with for STBD under inspection error. Furthermore, an expression for the power of control chart with standardized normal variate for STBD is obtained. Also, ARL is measured to gain the knowledge of the sensitivity of monitoring procedure.

1.1. Power of Control Chart for STBD under Inspection Error. STBD is a modified form of a binomial distribution. A random variable $x$is said to follow STBD if it assumes only non-negative values and its probability mass function is given by:

$$f(x) = \frac{\binom{n}{x} p^x q^{n-x}}{\left(1 - q^n\right)}, \tag{1}$$

where, $x \in \{1, 2, ..., n\}$.

The mean and variance of $STBD$ is given by:

$$\text{Mean} = \mu_p = \frac{np}{\left(1-q^n\right)}, \tag{2}$$

$$\text{Variance} = \sigma_p^2 = \frac{1}{\left(1-q^n\right)}\left[npq + n^2p^2 - \frac{n^2p^2}{\left(1-q^n\right)}\right]. \tag{3}$$

Moreover, inspection of attributes are distinguished by two decision variables in which each and every item is scrutinized and categorized as good or faulty. Two types of error are probable, an item which is good but classified as faulty (Type-I error), e_1, or an item which is faulty but classified as good (Type-II error), e_2. If p is true fraction defective and p' is the apparent fraction defective, then we set:

$$p' = p(1 - e_2) + (1 - p)e_1 \tag{4}$$

with both e_1 and e_2 estimated.

In the development of the power and ARL for equation (1), the following assumptions are made and notations are used:

The inspection of items is utilized to determine the number of defects in a lot. The process has STBD with mean μ_p and variance σ_p^2. When the inspection error is considered the process has STBD with mean $\mu_{p'} = \dfrac{np'}{(1-q^n)}$ and varianc

$\sigma^2_{p'} = \frac{1}{\left(1 - q'^n\right)}\left[np'q + n^2p'^2 - \frac{n^2p'^2}{\left(1 - q'^n\right)}\right]$. Inspection of the items is received to categorize the produced units into defective and non-defective ones. The process is in a situation of statistical control at the time of resolving the control limits and the same evaluating instrument is used for later evaluation. Under the above assumptions, She whart control limits for *STBD* will be:

$$\text{UCL} = \mu_p + 3\sigma_p;\ \text{CL} = \mu_p;\ \text{LCL} = \mu_p - 3\sigma_p. \tag{5}$$

If we assume that $x$is a STB variate with mean $\mu_{p'}$ and variance $\sigma^2_{p'}$ then, the power of detecting the change of process parameter for *STBD* is given by:

$$\text{P}_{p'} = \text{P}\{x \geq \mu_{p'} + 3\sigma_{p'}\} + \text{P}\{x \leq \mu_{p'} - 3\sigma_{p'}\} \tag{6}$$

$$\text{P}_{p'} = \text{P}\left\{x \geq \frac{np'}{(1 - q'^n)} + 3\,\frac{1}{(1 - q^n)}\left[npq + n^2p^2 - \frac{n^2p^2}{(1 - q^n)}\right]\right\} + \text{P}\left\{x \leq \frac{n}{(1 - q^n)} - 3\,\frac{1}{(1 - q'^n)}\left[np'q + n^2p'^2 - \frac{n^2p'^2}{(1 - q'^n)}\right]\right\} \tag{7}$$

$$\text{P}_{p'} = 1 - \sum_{x = \text{LCL}}^{\text{UCL}} \frac{\binom{n}{x} p^x q^{n-x}}{(1 - q^n)}. \tag{8}$$

The calculation and graphical representation of $\text{P}_{p'}$ for equation (8) is shown below in Table 1 and Figure 1 respectively.

1.2. Power of Control Chart for Standardized STBD. We can standardize the variates which can be plotted accordingly instead of plotting the number of defects in the control chart, this stabilizes the variables and the resulting control chart. When the process parameter shifts, then data is approach from STBD with mean μ'_p and variance σ'^2_p. Thus, equation (6) can be expressed in terms of standardized normal variable Z as:

$$\text{Z} = \frac{x - \mu'}{\sqrt{\sigma^2_{p'}}} \tag{9}$$

When the process parameter changes from μ to μ', the power of the control chart for STBD is:

$$\text{P}_{p'} = \text{P}\left\{\frac{x - \mu'}{\sqrt{\sigma'^2_p}} \geq \frac{(\mu' - \mu)}{\sqrt{\sigma'^2_p}} + 3\frac{\sqrt{\sigma^2_p}}{\sqrt{\sigma'^2_p}}\right\} + \text{P}\left\{\frac{x - \mu'}{\sqrt{\sigma'^2_p}} \leq \frac{(\mu' - \mu)}{\sqrt{\sigma'^2_p}} - 3\,\frac{\sqrt{\sigma^2_p}}{\sqrt{\sigma'^2_p}}\right\} \tag{10}$$

$$\text{P}_{p'} = \text{P}\left\{\text{Z} \geq \frac{(\mu' - \mu)}{\sqrt{\sigma'^2_p}} + 3\,\frac{\sqrt{\sigma^2_p}}{\sqrt{\sigma'^2_p}}\right\} + \text{P}\left\{\text{Z} \leq \frac{(\mu' - \mu)}{\sqrt{\sigma'^2_p}} - 3\,\frac{\sqrt{\sigma^2_p}}{\sqrt{\sigma'^2_p}}\right\} \tag{11}$$

$$\text{P}_{p'} = \text{P}\{\text{Z} \geq -d + 3k^{-1}\} + \text{P}\{\text{Z} \leq -d - 3k^{-1}\} \tag{12}$$

where, $d = \dfrac{(\mu - \mu')}{\sigma_{p'}}$, $k^2 = \dfrac{\sigma_p'^2}{\sigma_p^2}$.

The average run length (ARL) is the average number of points plotted on the chart until an out-of-control condition is signaled. It is the projected value of the run length distribution.

$$\text{ARL} = \frac{1}{\text{power}} = \frac{1}{\text{P}_{p'}} \tag{13}$$

The values of ARL obtained by using (13) and its diagrammatical representation are shown in Table 2 and Figure 2 respectively.

2. Numerical Illustration

For the purpose of numerical illustration, we will consider five cases as: $(e_1, e_2) = (0, 0), (0.03, 0.3), (0.03, 0.1), (0.01, 0.05), (0.005, 0.02)$. The first case corresponds to sampling without inspection error while the other four represent different error rates. Also, to calculate the power function and ARL given by the equation (8) and (13) respectively, we have considered a process with a targeted value or currently operating value of $p = 0.6$, so that UCL and LCL is given by equation (5) as:

$$\text{UCL} = 9.0 + 3\sqrt{3.6} = 14.6920 = 15$$

$$\text{CL} = 9.0$$

$$\text{LCL} = 9.0 - 3\sqrt{3.6} = 3.3070 = 3$$

Table 1 and Figure 1 illustrates the power function and power curve respectively, corresponding to the above five cases.

From Table 1, one can see that when both errors are operative together, the type-I error has more influence on the power function for low fraction defective, while the type-II error dominates the effect on the power function for high fraction defective. The reason is that when the actual process fraction defective is quite low, there is little opportunity for the realization of a type-II error. However, as p increases, the effect of this error becomes increasingly dominant.

Table 1. Calculation of Power for STBD when $n = 15$

	Power				
p	$(e_1, e_2) = (0, 0)$	$(e_1, e_2) =$ $(0.03, 0.30)$	$(e_1, e_2) =$ $(0.03, 0.10)$	$(e_1, e_2) =$ $(0.01, 0.05)$	$(e_1, e_2) =$ $(0.005, 0.02)$
0.4	0.0267	0.1259	0.0387	0.0339	0.0291
0.5	0.0037	0.0477	0.0078	0.0057	0.0043
0.6	0.0003	0.0149	0.0010	0.0006	0.0004
0.7	0.0000	0.0037	0.0001	0.0000	0.0000
0.8	0.0000	0.0007	0.0000	0.0000	0.0000

Table 2. Calculation of ARL for STBD when $n = 15$

		ARL			
p	$(e_1, e_2) = (0, 0)$	$(e_1, e_2) = (0.03, 0.30)$	$(e_1, e_2) = (0.03, 0.10)$	$(e_1, e_2) = (0.01, 0.05)$	$(e_1, e_2) = (0.005, 0.02)$
0.4	38	8	26	30	34
0.5	273	21	129	176	231
0.6	3599	67	956	1718	2690
0.7	114876	267	12204	33471	69806
0.8	17538838	1383	349441	1997295	7090927

From Table 2, it can be clearly observed that the type-II error has more influence on the ARL for low fraction defective, while the type-I error leads the effect on the ARL for high fraction defective. Come across the visual comparison through Figure 1 and Figure 2 reveals that inspection errors shifted the power and ARL curves to the right of the true curve. Error rate of (0.03, 0.30) has highly affected the power curve.

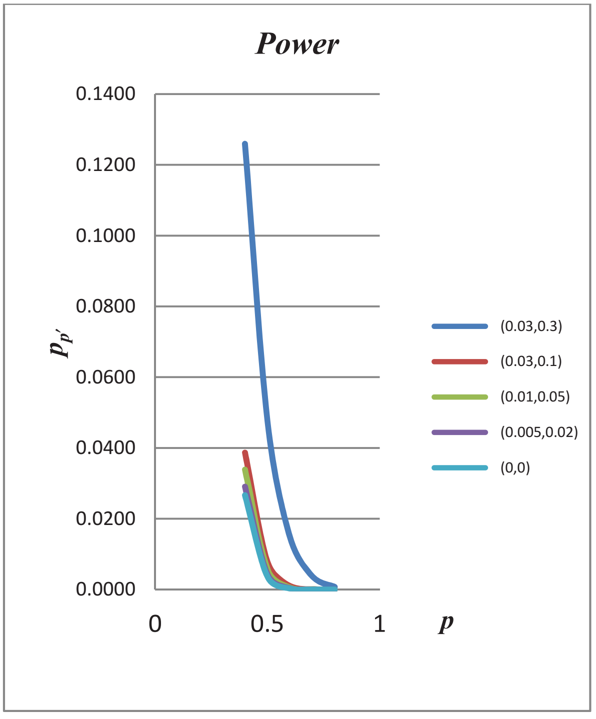

Figure 1. Diagrammatic representation of power for $n = 15$

In general, inspection error results in Power and ARL curves extensively unlike from that obtained under error free inspection.

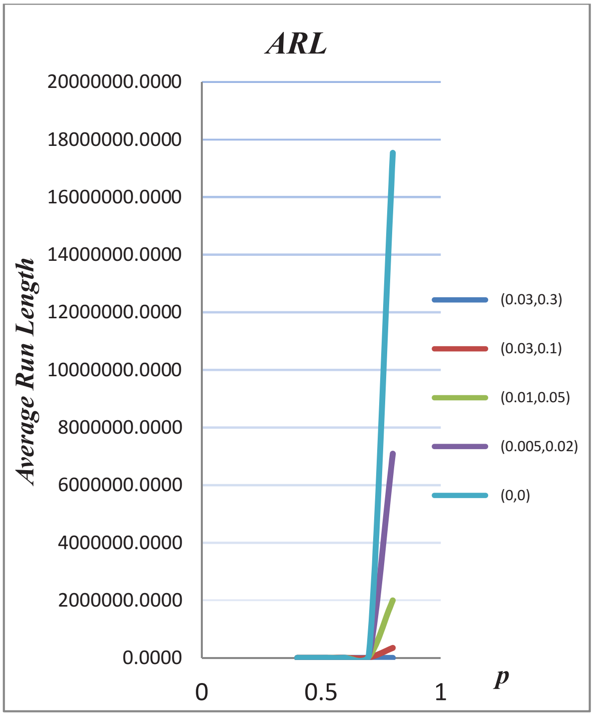

Figure 2: Diagrammatic representation of ARL for $n = 15$

3. Conclusion

Inspection error is an essential part of any process control, where inspections are taken in order to make sure that the monitored process is under control. Specifically, in SPC the existence of inspection error are certain to affect the effectiveness of the employed scheme. This paper has revealed that inspection error rates that are reasonable in industry seriously affect the power curve of a control chart. Especially, when the center line and control limits are based on a target value, the process can very effortlessly be moderated in-control when, indeed, it is not. When the control chart is based upon data obtained under inspection error, the power curve is again imprecise, but not nearly so critically. The effect of non-constant inspection error was considered. The increasing type-I error and the decreasing type-II error, as a function of process fraction defective, tend to moderate the effect of inspection error on the power curve. So far, the degree of inconsistency between the desired and achieved power curves will vary with the error rates encountered.

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