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ORDER THREE GENERATED GRAPHS FROM PATHWAYS USING GENERALIZED MEAN LABELING

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Abstract. Third order generalized mean labelling of a graph $G(V, E)$ with p vertices and q edges is a function f. If $f : V(G) \to \{1, 2, 3, \ldots, q+1\}$ is injective and the induced function $f^* : E(G) \to \{1, 2, 3, \ldots, q\}$ defined as $f^*(uv) = \left\lfloor \left(\frac{f(u)^3+f(v)^3}{2}\right)^{\frac{1}{3}} \right\rfloor$ for all $uv \in E(G)$, is bijective. Third order generalized mean graph refers to a graph that allows third order generalized mean labelling. I presented the third order generalized mean labelling of the route P_n in this publication, the graph $P_n \circ S_m$, the graph $P_n \circ K_2$, the graph $TW(P_n)$ and the graph $S(P_n \circ K_1)$.

1. Introduction

In this study, a finite, undirected, and simple graph is referred to as a graph. Assume that $G(V, E)$ is a graph with p edges and q vertices. We refer to [2] for notations and nomenclature. We refer to [3] for a thorough study of graph labeling.

Path on n vertices is denoted by P_n. A star graph S_n is the complete bipartite graph $K_{1,n}$. The graph $G \circ S_m$ is obtained from G by attaching m pendant vertices to each vertex of G. A twig $TW(P_n), n \geq 4$ is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertex of the path P_n. An edge of a graph G can be subdivided by a vertex to create a subdivision of the graph, represented as $S(G)$.

Arockiaraj et al. presented and developed the idea of root square mean labeling in [3]. The F-Heronian mean labeling of graphs was first developed by Saratha Devi and Durai Baskar in [6]. I provide a novel kind of labeling known as the *third order generalized mean labeling graph*, which is inspired by the writings of several scholars in the field of graph labeling.

The term "third order generalized mean labeling" refers to a function f that labels a graph $G(V, E)$ with p vertices and q edges. If

$$f : V(G) \to \{1, 2, 3, \ldots, q+1\}$$

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is injective and the induced function

$$f^* : E(G) \to \{1, 2, 3, \ldots, q\}, \quad f^*(uv) = \left\lfloor \left(\frac{f(u)^3 + f(v)^3}{2} \right)^{\frac{1}{3}} \right\rfloor$$

for all $uv \in E(G)$, is bijective. A graph that admits a third order generalized mean labeling is called a *third order generalized mean graph.*

In this paper, I introduced the third order generalized mean labeling of the path P_n, the graph $P_n \circ S_m$, the graph $P_n \circ K_2$, the graph $TW(P_n)$ and the graph $S(P_n \circ K_1)$.

Main Results

Theorem 1.1. *P_n is a third order generalized mean graph for each path.*

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the path P_n. Define $f : V(G) \to \{1, 2, 3, \ldots, n\}$ as $f(v_i) = i$, for $1 \leq i \leq n$. Then the induced edge labeling is $f^*(v_i v_{i+1}) = i$, for $1 \leq i \leq n - 1$. Thus, f is the path P_n's third order generalized mean labeling. Hence, P_n is a third order generalized mean graph. □

Theorem 1.2. *The graph $P_n \circ S_m$ is a third order generalized mean graph for $n \geq 1$ and $m \leq 4$.*

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the path P_n and $u_1^{(i)}, u_2^{(i)}, u_3^{(i)}, \ldots, u_m^{(i)}$ be the pendant vertices at each v_i, for $1 \leq i \leq n$.

Case (i). $m = 1$. Define $f : V(P_n \circ S_1) \to \{1, 2, 3, \ldots, 2n\}$ as follows:

$$f(v_i) = 2i - 1, \quad 1 \leq i \leq n,$$

$$f(u_1^{(i)}) = 2i, \quad 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows:

$$f^*(v_i v_{i+1}) = 2i, \quad 1 \leq i \leq n - 1,$$

$$f^*(v_i u_1^{(i)}) = 2i - 1, \quad 1 \leq i \leq n.$$

Case (ii). $m = 2$. Define $f : V(P_n \circ S_2) \to \{1, 2, 3, \ldots, 3n\}$ as follows:

$$f(v_i) = \begin{cases} 2i, & 1 \leq i \leq 2, \\ 3i - 1, & 3 \leq i \leq n, \end{cases}$$

$$f(u_1^{(i)}) = \begin{cases} 4i - 3, & 1 \leq i \leq 2, \\ 3i - 2, & 3 \leq i \leq n, \end{cases} \qquad f(u_2^{(i)}) = 3i, \quad 1 \leq i \leq n.$$

Then the induced edge labeling is obtained as follows:

$$f^*(v_i v_{i+1}) = 3i, \quad 1 \leq i \leq n - 1,$$

$$f^*(v_i u_1^{(i)}) = 3i - 2, \quad 1 \leq i \leq n,$$

$$f^*(v_i u_2^{(i)}) = 3i - 1, \quad 1 \leq i \leq n.$$

Case (iii). $m = 3$. Define $f : V(P_n \circ S_3) \to \{1, 2, 3, \ldots, 4n\}$ as follows:

$$f(v_i) = 4i - 2, \quad 1 \leq i \leq n,$$

$$f(u_1^{(i)}) = 4i-3, \quad 1 \le i \le n,$$
$$f(u_2^{(i)}) = 4i-1, \quad 1 \le i \le n,$$
$$f(u_3^{(i)}) = 4i, \quad 1 \le i \le n.$$

Next, the induced edge labeling is:

$$f^*(v_i v_{i+1}) = 4i, \quad 1 \le i \le n-1,$$
$$f^*(v_i u_1^{(i)}) = 4i-3, \quad 1 \le i \le n,$$
$$f^*(v_i u_2^{(i)}) = 4i-2, \quad 1 \le i \le n,$$
$$f^*(v_i u_3^{(i)}) = 4i-1, \quad 1 \le i \le n.$$

Case (iv). $m = 4$. Define $f : V(P_n \circ S_4) \to \{1, 2, 3, \ldots, 5n\}$ as follows:

$$f(v_i) = \begin{cases} 5i-3, & 1 \le i \le 2, \\ 5i-2, & 3 \le i \le n, \end{cases}$$

$$f(u_1^{(1)}) = 1, \qquad f(u_1^{(i)}) = 5i-5, \quad 2 \le i \le n,$$

$$f(u_2^{(1)}) = 3, \qquad f(u_2^{(i)}) = \begin{cases} 5i-2, & 1 \le i \le 2, \\ 5i-3, & 3 \le i \le n, \end{cases}$$

$$f(u_3^{(i)}) = 5i-1, \quad 1 \le i \le n,$$
$$f(u_4^{(i)}) = 5i+1, \quad 1 \le i \le n-1, \qquad f(u_4^{(n)}) = 5n.$$

Next, the induced edge labeling is:

$$f^*(v_i v_{i+1}) = 5i, \quad 1 \le i \le n-1,$$
$$f^*(v_i u_1^{(i)}) = 5i-4, \quad 1 \le i \le n,$$
$$f^*(v_i u_2^{(i)}) = 5i-3, \quad 1 \le i \le n,$$
$$f^*(v_i u_3^{(i)}) = 5i-2, \quad 1 \le i \le n,$$
$$f^*(v_i u_4^{(i)}) = 5i-1, \quad 1 \le i \le n.$$

As a result, f is the third order generalized mean labeling of the path $P_n \circ S_m$. For $n \ge 1$ and $m \le 4$, the graph $P_n \circ S_m$ is a third order generalized mean graph.

□

Theorem 1.3. *For $n \ge 1$, the graph $P_n \circ K_2$ is a third order generalized mean graph.*

Proof.

Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the path P_n and $u_i^{(1)}, u_i^{(2)}$ be the vertices of the i^{th} copy of K_2 attached with v_i, $1 \le i \le n$.

Define

$f : V(P_n \circ K_2) \to \{1, 2, 3, \ldots, 4n\}$ as follows:

$$f(v_i) = \begin{cases} 2i+1, & 1 \le i \le 2, \\ 4i-2, & 3 \le i \le n, \end{cases}$$

$$f(u_i^{(1)}) = \begin{cases} 6i-5, & 1 \le i \le 2, \\ 4i-3, & 3 \le i \le n, \end{cases}$$

$$f(u_i^{(2)}) = \begin{cases} 5, & i = 1, \\ 4i, & 2 \le i \le n. \end{cases}$$

Next, the following is the induced edge labeling:

$$f^*(v_i v_{i+1}) = \begin{cases} 2, & i = 1, \\ 4i, & 2 \le i \le n-1, \end{cases}$$

$$f^*(u_i^{(1)} u_i^{(2)}) = \begin{cases} 4i-1, & 1 \le i \le 2, \\ 4i-2, & 3 \le i \le n, \end{cases}$$

$$f^*(v_i v_{i+1}) = 4i, \quad 1 \le i \le n-1,$$

$$f^*(u_i^{(1)} u_i^{(2)}) = 4i-2, \quad 1 \le i \le n,$$

$$f^*(u_i^{(1)} v_i) = 4i-3, \quad 1 \le i \le n,$$

$$f^*(u_i^{(2)} v_i) = 4i-1, \quad 1 \le i \le n.$$

Thus, f represents the third order generalized mean labeling of the path $P_n \circ K_2$. Consequently, for $n \ge 1$, the graph $P_n \circ K_2$ is a third order generalized mean graph.

□

Theorem 1.4. *For $n \ge 4$, the twig graph $TW(P_n)$ is a third order generalized mean graph.*

Proof. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of the path P_n

Define

$$f : V(TW(P_n)) \to \{1, 2, 3, \dots, 3n-4\} \text{ as follows:}$$

$$f(v_1) = 1, \quad f(v_2) = 2, \quad f(v_i) = 3i-3,\ 3 \le i \le n-1, \quad f(v_n) = 3n-4,$$

$$f(u_1^{(2)}) = 3, \quad f(u_1^{(i)}) = 3i-4,\ 3 \le i \le n-1, \quad f(u_2^{(i)}) = 3i-2,\ 2 \le i \le n-1.$$

Next, the following is the induced edge labeling:

$$f^*(v_i v_{i+1}) = 3i-2,\ 1 \le i \le n-1,$$

$$f^*(v_i u_1^{(i)}) = 3i-4,\ 2 \le i \le n-1,$$

$$f^*(v_i u_2^{(i)}) = 3i-3,\ 2 \le i \le n-1.$$

Therefore, f represents the third order generalized mean labeling of the path $TW(P_n)$. Hence, for $n \ge 1$, the graph $TW(P_n)$ is a third order generalized mean graph.

□

Theorem 1.5. *For $n \ge 1$, the subdivision $S(P_n \circ K_1)$ is a third order generalized mean graph.*

Proof. In $P_n \circ K_1$, let u_i, $1 \leq i \leq n$, be the vertices on the path P_n and let v_i be the vertex attached at each vertex u_i, $1 \leq i \leq n$.

Let x_i be the vertex which divides the edge u_iv_i, for $1 \leq i \leq n$, and let y_i be the vertex which divides the edge u_iu_{i+1}, for $1 \leq i \leq n-1$. Then,

$$V(S(P_n \circ K_1)) = \{u_i, v_i, x_i, y_j \ ; \ 1 \leq i \leq n, \ 1 \leq j \leq n-1\},$$
$$E(S(P_n \circ K_1)) = \{u_ix_i, \ v_iy_i \ ; \ 1 \leq i \leq n\} \cup \{u_iy_i, \ y_iu_{i+1} \ ; \ 1 \leq i \leq n-1\}.$$

Define $f : V(S(P_n \circ K_1)) \to \{1, 2, 3, \ldots, 4n-1\}$ as follows:

$$f(u_i) = 4i-3, \quad 1 \leq i \leq n,$$
$$f(y_i) = 4i-1, \quad 1 \leq i \leq n-1,$$
$$f(x_i) = 4i-2, \quad 1 \leq i \leq n,$$
$$f(v_i) = 4i, \quad 1 \leq i \leq n-1, \qquad f(v_n) = 4n-1.$$

Next, the induced edge labeling is obtained as follows:

$$f^*(u_iy_i) = 4i-2, \quad 1 \leq i \leq n-1,$$
$$f^*(y_iu_{i+1}) = 4i, \quad 1 \leq i \leq n-1,$$
$$f^*(u_ix_i) = 4i-3, \quad 1 \leq i \leq n,$$
$$f^*(x_iv_i) = 4i-1, \quad 1 \leq i \leq n-1, \qquad f^*(x_nv_n) = 4n-2.$$

Therefore, f is $S(P_n \circ K_1)$'s third order generalized mean labeling. Consequently, for $n \geq 1$, the graph $S(P_n \circ K_1)$ is a third order generalized mean graph. □

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