Global and Stochastic Analysis
Vol. 7 No. 1 (January-June, 2020)

MUK PUBLICATIONS
Open Access Publisher

SOLUTION OF PROBLEMS FOR THE RELAXATION FILTRATION MODEL PROCEEDING UNDER LINEAR DARCY'S LAW USING MONTE-CARLO AND PROBABILISTIC-DIFFERENCE METHODS

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Abstract. In the work three-dimensional filtration problems are considered, namely, the problems: Dirichlet, Neumann an A mixed problem in a relaxation-compressible porous medium carried out under linear Darcy's law. This indicates that the disturbance front instantly passes through the entire considered area, making it instantly a filtration area. All problems are mathematically posed correctly. The obtained results are used in the oil industry to determine the pressure at a particular point of the oil reservoir.

1. Introduction

Multidimensional problems for models of relaxation filtration in a relaxation-compressible porous medium carried out under linear Darcy's law have not been considered before. Moreover, these problems were not solved by Monte-Carlo and probabilistic-difference methods. The physical formulation of the problem itself promotes (encourages) the use of probabilistic methods for solving these problems, since many physical processes, as well as their parameters in underground hydromechanics are random. For this reason, the relevance of solving the initial problems by Monte-Carlo and probabilistic-difference methods is not in doubt.

2. Preliminary

Following the work [1], we consider the main principles and equations of relaxation filtration and features of filtering with a finite velocity of the propagation disturbances.

1. The equations of conservation of momentum of resistance forces and liquid mass

The law of conservation of momentum of resistance forces has the form

$$\frac{d}{dt}\int_V \mathbf{J}dV - \int_S p\mathbf{n}dS = 0, \tag{1}$$

Received 2019-8-26; Accepted 2019-10-14;
Keywords: Coin-Carlo method, Darcy's law, linear relaxation filtration, Neumann problem, Dirichlet problem.

and the law of conservation of the liquid mass is written in integral form

$$\frac{d}{dt}\int_V m\rho\, dV + \int_S \rho W_n dS = 0, \tag{2}$$

where t – time, $\mathbf{J}$ – impulse density of resistance forces, V – arbitrary fixed in space volume, S – surface limiting V, p – pressure, W_n – projection of the filtration velocity $\mathbf{W}$ to the normal $\mathbf{n}$, m – porosity, ρ – liquid density. The system of equations (1), (2) are open with respect to the quantities m, ρ, p, W, $\mathbf{J}$. To closure these equations, it is necessary to introduce three constitutive relations taking into account the properties of liquids and rocks from which the porous skeleton is composed, i.e. to write the dependence m and ρ on pressure p as well as to express the impulse density of the forces J through the filtration velocity.

2. The constitutive relations for the impulse of resistance forces and the liquid mass. Basic principles of linear relaxation filtration

The force impulses $\boldsymbol{J}$ at some fixed point are due to the action of viscous forces in the interaction of a moving fluid with a fixed porous skeleton and, therefore, depends on the filtration velocity $\boldsymbol{W}$ at this point. The simplest relationship between the quantities $\boldsymbol{J}$ and $\boldsymbol{W}$ can be represented as

$$\mathbf{D} = \frac{\partial \mathbf{J}}{\partial t} = -c\left(|\mathbf{W}|\right)\mathbf{W}, \tag{3}$$

where $\mathbf{D}$ – density of the resistance force, c – constant. Using this dependence leads to the well-known nonlinear law of filtration of the form

$$\operatorname{grad}\left(p\right) = -c\left(|\mathbf{W}|\right)\mathbf{W}, \tag{4}$$

Which at constant $c\left(|\mathbf{W}|\right)\mathbf{W} = \frac{\mu}{\kappa}$ goes into Darcy's low, where μ – liquid viscosity, κ – permeability coefficient. From relation (3), the following determining relation between $\boldsymbol{W}$ and $\boldsymbol{D}$ is obtained:

$$\mathbf{W} = \frac{\kappa}{\mu}\frac{\tau_p}{\tau_W}\mathbf{D} + \frac{\kappa}{\mu}\frac{\tau_p - \tau_W}{\tau_W}\int_{-\infty}^{t}\mathbf{D}\left(\mathbf{x},\tau\right)\cdot e^{-\frac{t-\tau}{\tau_W}}\cdot\frac{1}{\tau_W}\cdot d\tau. \tag{5}$$

Here τ_p and τ_W are the non-negative relaxation time constants, respectively of pressure and filtration velocity.

Now we introduce the constitutive relation for the impulse of resistance forces taking into account the effects of relaxation, indicate the basic principles that are common to many sections of continuum mechanics.

The principle of determinism – PD. The magnitude of the impulse of resistance forces $\boldsymbol{J}$ at some fixed point of the ground is determined by the entire history of movement up to the moment t. This means that $\boldsymbol{J}$ at a given moment of time it depends only on past and present movements and does not depend on future ones.

The principle of local action – PLA. The value $\boldsymbol{J}$ at a given point is uniquely determined by the history of movement in an arbitrarily small neighborhood of this point. Thus, the PLA excludes the influence on the magnitude $\boldsymbol{J}$ of the history of the motion of particles lying far from the measurement point $\boldsymbol{J}$.

Note that this principle can mean, in essence, a refusal to transfer the history of motion by a substance (convection) due to its smallness.

The principle of superposition – PS. The resulting displacement of the liquid $\mathbf{W} = \sum_i \mathbf{W}_i$ causes an impulse of resistance forces equal to the sum of the impulses of these forces caused by the component displacements, i.e. $\mathbf{J}(\mathbf{W}) = \sum_i \mathbf{J}(\mathbf{W}_i)$.

The principle formulated above is analogous to the Boltzmann's superposition principle for viscoelastic materials and the Hobkinson's superposition principle for electric circuits.

The principle of fading memory – PFM. A decrease in the rate of change in the filtration velocity to zero determines the asymptotic tendency of dependence $\boldsymbol{J}$ on to the «equilibrium¸ Darcy's law

$$\mathrm{grad(p)} = \mathbf{D} = \frac{\partial \mathbf{J}}{\partial t} \sim -\frac{\mu}{k}\mathbf{W} \tag{6}$$

In practice, this principle establishes the existence of some characteristic relaxation time τ_1; if T – the time of a significant change in the filtration velocity is much longer τ_1, then relaxation can be neglected. In other words, the asymptotic equality (6) should hold for $\frac{T}{\tau_1} \to \infty$. Obviously, for Darcy's law itself $\tau_1 = 0$.

3. The mathematical formulation of the above principles

For an arbitrary movement of a liquid with a filtration velocity $\mathbf{W}(t')$ that changes stepwise at a time moment $t' = \tau$ by an amount [$\mathbf{W}$], the integral formula of the resulting impulse of resistance forces for isotropic media can be written as

$$\mathbf{J}(\mathbf{x},t) = -F(t-\tau)[\mathbf{W}]_\tau - \int_{-\infty}^{t} F(t-t')\frac{d\mathbf{W}(\mathbf{x},t-t')}{dt'}dt' =$$
$$-F(0)\mathbf{W}(\mathbf{x},t) - \int_{-\infty}^{t} \frac{dF(t-t')}{d(t-t')}\mathbf{W}(\mathbf{x},t')dt' =$$
$$-F(0)\mathbf{W}(\mathbf{x},t) - \int_{0}^{\infty} \frac{dF(t')}{d(t-t')}\mathbf{W}(\mathbf{x},t-t')dt', \tag{7}$$

where $F(0) = \lim_{t\to 0+0} F(t)$.

Applying the symbolism of convolution algebra, differentiating (7) with respect to time *t*, using the determination of the generalized derivative with respect to time (point over a function), relation (7) can be briefly written as follows:

$$\mathbf{J} = -F * \dot{\mathbf{W}} = -\dot{F} * \dot{\mathbf{W}}, \tag{8}$$

where $*$ – means the convolution operation, • – means the generalized derivative with respect to time.

4. Constitutive relation for the quantity $m\rho$

The rock and liquid are slightly compressible, i.e. dependence m and ρ on pressure with a high degree of accuracy can be described by linear relationships of the form $m = m_0 + \beta_e(p - p_0)$ and $\rho = \rho_0(1 + \beta_f(p - p_0))$, where m_0 and ρ_0 – liquid density and porosity in undisturbed reservoir conditions, β_e – the compressibility coefficient of the porous medium and β_f – liquid coefficient, each of β_e and β_f has an order of about 10^{-5} I/at. Then it is obvious that the constitutive relation for the quantity can be written with good accuracy in the form

$$m\rho = m_0\rho_0 + \rho_0\beta(p - p_0), \tag{9}$$

where $\beta = \beta_e + m_0\beta_f$ – reservoir elastic capacity coefficient. This dependence is widely used in the theory of «equilibrium¸ non-stationary filtration. Note that relation (9) has the disadvantage that it is based on the hypothesis of the instantaneous establishment of an «equilibrium¸ correspondence between $m\rho$ and *p*. The practice of piezometry of formation indicates that the hypothesis mentioned above is often violated, i.e. relation (9) is not realized. In this case, to describe the filtration in relaxation-compressible media the constitutive relation of the form can be used

$$m\rho = m_0\rho_0 + \rho_0\left(\beta_*\left(p-p_0\right)+\frac{\lambda_m-\lambda_p}{\lambda_m}\beta_e\int_0^t\left(p-p_0\right)(\mathbf{x},t')\exp\left(-\frac{t-t'}{\lambda_m}\right)\frac{dt'}{\lambda_m}\right), \tag{10}$$

where $\beta_* = m_0\beta_f + \beta_e\lambda_p / \lambda_m$ – dynamic reservoir elastic capacity coefficient. By generalizing relation (10) similarly to (8), we obtain the constitutive relation in the form

$$m\rho - m_0\rho_0 = \Phi(0)\left(p-p_0\right)+\int_0^\infty\frac{d\Phi(t')}{dt'}\left(p-p_0\right)(\mathbf{x},t-t')dt' = \\ = \Phi*\left(\dot{\overline{p-p_0}}\right) = \dot{\Phi}*\left(p-p_0\right), \tag{11}$$

where $\Phi = \Phi(t) \geq 0$ at $t \geq 0$, $\Phi(t) = 0$ at $t < 0$.

5. A closed system of equations of linear relaxation filtration

Linear relaxation filtration will be described by the law of conservation of momentum of resistance forces (1), the linearized law of conservation of mass of liquid (2), the constitutive relations for the impulse of resistance forces (7) and liquid mass (11), by the system of equations

$$\frac{d}{dt}\int_V \mathbf{J}dV - \int_S p\mathbf{n}dS = 0,$$

$$\frac{d}{dt}\int_V m\rho dV + \int_S \rho W_n dS = 0,$$

$$\mathbf{J}(\mathbf{x},t) = -F(0)\mathbf{W}(\mathbf{x},t) - \int_0^\infty \frac{dF(t')}{d(t-t')}\mathbf{W}(\mathbf{x},t-t')dt',$$

$$m\rho - m_0\rho_0 = \Phi(0)\left(p-p_0\right)+\int_0^\infty\frac{d\Phi(t')}{dt'}\left(p-p_0\right)(\mathbf{x},t-t')dt'. \tag{12}$$

Obviously, this system of equations is closed.

In underground hydromechanics, it is generally to consider only pressure and filtration velocity; therefore, it is more convenient to exclude $\boldsymbol{J}$ and $\boldsymbol{m\rho}$ from system (12). It is easy to verify that after such an exception we obtain the following system of equations closed with respect to the quantities p and $\boldsymbol{W}$ describing relaxation filtration in the region of continuity of pressure fields and filtration velocity

$$\Delta p(\mathbf{x},t) = \frac{F(0)\Phi(0)}{\rho_0}\frac{\partial^2 p(\mathbf{x},t)}{\partial t^2} +$$
$$\int_0^{\infty}\left(\frac{F(0)}{\rho_0}\frac{d\Phi(t')}{dt'} + \frac{\Phi(0)}{\rho_0}\frac{dF(t')}{dt'} + \frac{1}{\rho_0}\int_0^{t'}\frac{dF(\tau)}{d\tau}\frac{d\Phi(t'-\tau)}{d(t'-\tau)}d\tau\right)\frac{\partial^2 p(\mathbf{x},t-t')}{\partial (t-t')^2}dt',$$
$$\operatorname{grad} p(\mathbf{x},t) = -F(0)\frac{\partial \mathbf{W}(\mathbf{x},t)}{\partial t} - \int_0^{\infty}\frac{dF(t')}{dt'}\frac{\partial \mathbf{W}(\mathbf{x},t-t')}{\partial (t-t')}dt' . \quad (13)$$

Note that any linear relaxation filtration model is completely characterized by two functions of time $F(t)$ and $\Phi(t)$, called relaxation kernels of the filtration law and liquid mass, respectively.

6. Examples

6.1 Classical model of elastic filtration mode.

The basis of the classical elastic filtration mode is Darcy's law (6) and a linear relationship between the change in the amount of liquid in the elementary volume and the pressure in it (9). These two relations correspond to relaxation kernels of the form

$$F(t) = \frac{\mu}{\kappa}t\eta(t), \quad \Phi(t) = \rho_0\beta\eta(t), \quad (14)$$

where $\eta(t)$ – Heaviside function,

$$\eta(t) = \{1 \text{ при } t>0,\ 1/2 \text{ при } t=0,\ 0 \text{ при } t<0\}.$$

In this case, system (13) takes the form

$$\frac{\partial p(\mathbf{x},t)}{\partial t} - \chi\Delta p(\mathbf{x},t) = 0, \quad \mathbf{W}(\mathbf{x},t) = -\frac{\kappa}{\mu}\operatorname{grad} p(\mathbf{x},t), \quad (15)$$

where χ – piezo conductivity coefficient of formation, $\chi = \dfrac{\kappa}{\mu\beta}$.

6.2 The simplest model of filtration with a constant propagation velocity of disturbances.

This model is determined by the following relaxation kernels.

$$F(t) = \frac{\mu}{\kappa}(t+\tau)\eta(t), \quad \Phi(t) = \rho_0\beta\eta(t). \quad (16)$$

System (13) is transformed to

$$\tau \frac{\partial^2 p(\mathbf{x},t)}{\partial t^2} + \frac{\partial p(\mathbf{x},t)}{\partial t} - \chi \Delta p(\mathbf{x},t) = 0,$$

$$\tau \frac{\partial \mathbf{W}(\mathbf{x},t)}{\partial t} + \mathbf{W}(\mathbf{x},t) = -\frac{\kappa}{\mu} \operatorname{grad} p(\mathbf{x},t). \tag{17}$$

The first equation in (17) is the telegraph equation. It is known that it describes the propagation disturbance, taking into account the discontinuity surface of the hydrodynamic parameters of the flow, which carries the disturbance into the rest region with a constant velocity equal to $v_0 = \sqrt{\frac{\chi}{\tau}}$.

6.3 Darcy's law filtration model in a relaxation-compressible porous medium.

The following relaxation kernels correspond to this type of filtration flow.

$$F(t) = \frac{\mu}{\kappa} t\eta(t), \quad \Phi(t) = \rho_0 \left(\beta - \frac{\lambda_m - \lambda_p}{\lambda_m} \beta_e e^{-\frac{t}{\lambda_m}} \right) \eta(t) \tag{18}$$

The system has the form

6.3.1

$$\frac{\beta_*}{\beta} \frac{\partial p(\mathbf{x},t)}{\partial t} + \frac{\beta_e}{\beta} \frac{\lambda_m - \lambda_p}{\lambda_m^2} (p - p_0)(\mathbf{x},t) -$$

$$\frac{\beta_e}{\beta_*} \frac{\lambda_m - \lambda_\rho}{\lambda_m^2} \int_0^t (p - p_0)(\mathbf{x},t') \exp\left(-\frac{t - t'}{\lambda_m} \right) \frac{dt'}{\lambda_m} - \chi \Delta p(\mathbf{x},t) = 0,$$

$$\mathbf{W}(\mathbf{x},t) = -\frac{\kappa}{\mu} \operatorname{grad} p(\mathbf{x},t). \tag{19}$$

or

$$\frac{\partial}{\partial t} \left(p(\mathbf{x},t) + \lambda_m' \frac{\partial p(\mathbf{x},t)}{\partial t} \right) - \chi \Delta \left(p(\mathbf{x},t) + \lambda_m \frac{\partial p(\mathbf{x},t)}{\partial t} \right) = 0,$$

$$\mathbf{W}(\mathbf{x},t) = -\frac{\kappa}{\mu} \operatorname{grad} p(\mathbf{x},t), \tag{20}$$

where $\lambda_m^{'} = \lambda_m \dfrac{\beta_*}{\beta}$. In special case of an incompressible liquid $\beta_f = 0$ and $\lambda_p = 0$ instead of system (19), we have

6.3.2 $$\chi\Delta p(\mathbf{x},t) - \frac{(p-p_0)(\mathbf{x},t')}{\lambda_m} - \frac{1}{\lambda_m}\int_0^t (p-p_0)(\mathbf{x},t')\exp\left(-\frac{t-t'}{\lambda_m}\right)\frac{dt'}{\lambda_m} = 0,$$

$$\mathbf{W}(\mathbf{x},t) = -\frac{\kappa}{\mu}\operatorname{grad} p(\mathbf{x},t). \tag{21}$$

Instead of the integral form of the first equation in (21), the equivalent differential form is used

$$\frac{\partial p(\mathbf{x},t)}{\partial t} - \chi\Delta\left(p(\mathbf{x},t) + \lambda_m \frac{\partial p(\mathbf{x},t)}{\partial t}\right) = 0. \tag{22}$$

Model (22) describes the filtration of an incompressible liquid in a relaxation-incompressible porous medium at $\lambda_p = 0$, as well as in a fractured-porous medium with infinitely small elastic capacity of cracks and block conductivity.

6.4 Filtration model according to the simplest nonequilibrium law in an elastic porous medium.

Here the relaxation kernels have the form:

$$F(t) = \frac{\mu}{\kappa}\left(t + (\tau_W - \tau_p)\left(1 - \exp\left(-\frac{t}{\tau_p}\right)\right)\right)\eta(t), \quad \Phi(t) = \rho_0\beta\eta(t). \tag{23}$$

System (13) is equated to the following form

$$\chi\Delta p(\mathbf{x},t) = \int_0^\infty \left(1 - \frac{\tau_p - \tau_W}{\tau_p}\exp\left(-\frac{t'}{\tau_p}\right)\right)\frac{\partial^2 p(\mathbf{x},t-t')}{\partial (t-t')^2}dt',$$

$$-\frac{\kappa}{\mu}\operatorname{grad} p(\mathbf{x},t) = \int_0^\infty \left(1 - \frac{\tau_p - \tau_W}{\tau_p}\exp\left(-\frac{t'}{\tau_p}\right)\right)\frac{\partial \mathbf{W}(\mathbf{x},t-t')}{\partial (t-t')}dt'. \tag{24}$$

We can rewrite system (24) in the equivalent differential form

$$\tau_W \frac{\partial^2 p(\mathbf{x},t)}{\partial t^2} + \frac{\partial p(\mathbf{x},t)}{\partial t} - \chi\Delta\left(p(\mathbf{x},t) + \tau_p \frac{\partial p(\mathbf{x},t)}{\partial t} \right) = 0,$$

$$\tau_W \frac{\partial \mathbf{W}(\mathbf{x},t)}{\partial t} + W(\mathbf{x},t) + \frac{\kappa}{\mu}\operatorname{grad}\left(p(\mathbf{x},t) + \tau_p \frac{\partial p(\mathbf{x},t)}{\partial t} \right) = 0. \quad (25)$$

7. Solution of problems for the relaxation filtration model proceeding under linear Darcy's law using Monte-Carlo and probabilistic-difference methods

In a limited area $\Omega \in R^3$ with a boundary $\partial\Omega$ and for $t \in [0, T]$, we consider for the first equation from (20) (Model 6.3.1, the equation for pressure $p(\mathbf{x},t)$), i.e.

$$\frac{\partial}{\partial t}\left(p(\mathbf{x},t) + \lambda'_m \frac{\partial p(\mathbf{x},t)}{\partial t} \right) - \chi\Delta\left(p(\mathbf{x},t) + \lambda_m \frac{\partial p(\mathbf{x},t)}{\partial t} \right) = 0, \quad (26)$$

where $\lambda'_m = \lambda_m \frac{\beta_*}{\beta}$, following problems:

Problem 1 (Dirichlet problem). Find in a limited area $\Omega \in R^3$ with a boundary $\partial\Omega$ and for $t \in [0, T]$ solution of equation (26) satisfying the initial data

$$p(\mathbf{x},t) = 0 \text{ at } t = 0 \quad (27)$$

$$\frac{\partial p(\mathbf{x},t)}{\partial t} = 0 \text{ at } t = 0 \quad (28)$$

and boundary condition

$$p(\mathbf{x},t) = p_1(\mathbf{x},t) \text{ at } \mathbf{x} \in \partial\Omega \times [0, T]. \quad (29)$$

Problem 2 (Newmann problem). Find in a limited area $\Omega \in R^3$ with a boundary $\partial\Omega$ and for $t \in [0, T]$ solution of equation (26) satisfying the initial data (27), (28) and boundary condition

$$\frac{\partial p(\mathbf{x},t)}{\partial \mathbf{n}} = p_2(\mathbf{x},t) \text{ at } \mathbf{x} \in \partial\Omega \times [0,\ T], \tag{30}$$

where $\mathbf{n}$ – inner normal to the boundary $\partial\Omega$.

Problem 3 (mixed problem). Find in a limited area $\Omega \in R^3$ with a boundary $\partial\Omega$ and for $t \in [0,\ T]$ solution of equation (26) satisfying the initial data (27), (28) and boundary condition

$$\alpha_1 p(\mathbf{x},t) + \beta_1 \frac{\partial p(\mathbf{x},t)}{\partial \mathbf{n}} = p_3(\mathbf{x},t) \quad npu \quad \mathbf{x} \in \partial\Omega \times [0,\ T], \tag{31}$$

where α_1 and β_1 – given fixed constants.

Comment 1. Let the matching conditions be satisfied for problems 1 - 3.

In the system of equations (20), the pressure $p(\mathbf{x},t)$ can be found from the first equation, then, after calculating $\operatorname{grad} p(\mathbf{x},t)$, from the second equation of system (20), i.e. from $\mathbf{W}(\mathbf{x},t) = -\frac{\kappa}{\mu}\operatorname{grad} p(\mathbf{x},t)$, we find the vector of filtration velocity $\mathbf{W}(\mathbf{x},t)$.

7.1 Solution of problem 1.

Let the coefficients χ, λ_m, λ'_m be constant positive values. The time interval $t \in [0,T]$ is divided into N equal parts of the length τ. So $t_n = n\tau$, $n = 0,1,2,\ldots,N$, $\tau = \frac{T}{N} > 0$. Now, we write (26) in the form

$$\chi\Delta p(\mathbf{x},t)(\mathbf{x},t) + \chi\lambda_m \frac{\partial}{\partial t}\left(\Delta p(\mathbf{x},t)\right) = \frac{\partial p(\mathbf{x},t)}{\partial t} + \lambda'_m \frac{\partial^2 p(\mathbf{x},t)}{\partial t^2}$$

and discretize only with respect to t by using the implicit scheme. As a result, accounting the value λ'_m, we obtain the equation on time layers t_n

$$\chi\Delta p^{n+1}(\mathbf{x}) + \chi\lambda_m\Delta\left(\frac{p^{n+1}(\mathbf{x}) - p^n(\mathbf{x})}{2\tau}\right) = \left(\frac{p^{n+1}(x) - p^n(x)}{2\tau}\right) + \lambda'_m\left(\frac{p^{n+1}(\mathbf{x}) - 2p^n(\mathbf{x}) + p^{n-1}(\mathbf{x})}{\tau^2}\right)$$

$$\Delta p^{n+1}(\mathbf{x}) - ap^{n+1}(\mathbf{x}) = bp^n(\mathbf{x}) + c\Delta p^{n-1}(\mathbf{x}) + dp^{n-1}(\mathbf{x}), \tag{32}$$

where $a = \frac{m_0\beta_f(\tau + 2\lambda_m) + \beta_e(\tau + 2\lambda_p)}{\wp}$, $b = -\frac{4(m_0\beta_f\lambda_m + \beta_e\lambda_p)}{\wp}$, $c = \frac{\lambda_m}{2\tau + \lambda_m}$,

$$d = \frac{m_0\beta_f(2\lambda_m - \tau) + \beta_e(2\lambda_p - \tau)}{\wp}, \quad \wp = \tau\chi(2\tau + \lambda_m)(\beta_e + m\beta_f).$$

The algorithm of «random walk by spheres».

It is clear that $a > 0$, since the parameters m_0, β_f, τ, λ_m, β_e, λ_p, χ are positive given values. We rewrite (32) as

$$\Delta p^{n+1}(\mathbf{x}) - ap^{n+1}(\mathbf{x}) = f^n(\mathbf{x}), \tag{33}$$

where $f^n(\mathbf{x}) = bp^n(\mathbf{x}) + c\Delta p^{n-1}(\mathbf{x}) + dp^{n-1}(\mathbf{x})$ and attach to it the initial conditions

$$p^0(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega, \tag{34}$$

$$\frac{p^1(\mathbf{x}) - p^0(\mathbf{x})}{\tau} = 0, \quad \mathbf{x} \in \Omega, \tag{35}$$

which are difference analogues of the initial conditions (27) and (28), respectively.

For this problem, the boundary conditions are transformed to

$$p^{n+1}(\mathbf{x}) = p_1^{n+1}(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega. \tag{36}$$

The boundary $\partial\Omega$ and $\partial\Omega_{\varepsilon}$, which corresponds to the Dirichlet condition, is called the absorbing boundary. It is known that the initial-boundary value problem (33) – (36) – the Dirichlet problem for the Helmholtz equation of time layers t_n, is solved using the algorithm of «random walk by spheres¸ by Monte-Carlo methods, i.e., the constructed ε – biased estimation of the solution $p^{n+1}(\mathbf{x})$ using the «random walk by spheres¸ has a uniformly limited variance with respect to ε [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14].

Using the algorithms of «random walk by spheres¸ and «random walk by lattice¸ by the Monte-Carlo and the probabilistic-difference methods, we were able to estimate approximate and discrete solutions to all three considered multidimensional problems.

The results obtained in this work can be used in the oil-producing industry, i.e. in practical measures on determining the pressure at a single point in the oil reservoir. And also, after determining the pressure, can be determined the filtration velocity.

Conclusion

The obtained theoretical results are of great practical use in the oil-producing industry. This is facilitated by those features of the Monte-Carlo methods, such as the ability to evaluate a solution at a particular point without involving solutions at other points and the dependence of the solution estimation on the geometry of the area, as well as the effectiveness of Monte-Carlo methods in solving multidimensional problems in comparison with «classical¸ numerical methods.

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