Stochastic Modeling and Applications
Vol.26 No. 1 (January-June, 2022)
ISSN: 0972-3641

Received: 14th December 2021 *Revised: 30th January 2022* *Selected: 03rd February 2022*

FUZZY CONTROL CHART BASED ON SIX SIGMA FOR MEAN USING RANGE

M.R. PAVITHRA AND DR. P. BALAMURUGAN

Abstract. The Quality has established over a number of points such as inspection, quality control, quality assurance, and total quality control and the effects produced by the above phases are used to check and develop the production/service procedure. Statistical process control (SPC) is a powerful collection of problem solving tools valuable in attaining process steadiness and enlightening capability through the decline of variability. Fuzzy set theory is a utilitarian tool to succeed the uncertainty environmental circumstances and the Fuzzy control limits provide a more accurate and flexible rating than the traditional control charts. The purpose of this research article is to construct the six sigma based fuzzy mean using range control chart under moderate distribution with the assistance of process capability.

1. Introduction

The major contribution of fuzzy set theory is its capability of representing vague data (Zadeh, 1965). Fuzzy logic offers a systematic base in dealing with situations, which are ambiguous, or not well defined (Murat Gulbay and Cengiz Kahraman, 2006).

Fuzzy data exist ubiquitously in the modern manufacturing process, and two alternative approaches to fuzzy control charts are developed by Khademi and Amirzadeh (2014) for monitoring sample averages and range. These approaches are based on 'fuzzy mode' and 'fuzzy rules' methods, when the measures are expressed by non-symmetric triangular fuzzy numbers. In contrast to the existing fuzzy control charts, their approach does not require the use of the defuzzification and this prevents the loss of information included in samples, Sevil Senturk and Nihal Erginel (2009).

The measures of central tendency in descriptive Statistics are used in variable control charts. These measures can be used to convert fuzzy sets into scalars which are -level fuzzy midrange, and fuzzy average. There is no theoretical basis to select the appropriate fuzzy measures among these four (Pandurangan and Varadharajan, 2011). As mentioned, according to the real-world data and information are vague and imprecise expression, this research article with development of Fuzzy control charts, trying to overcome the problem of inaccurate and ambiguous information. Thus, the main objective of this research article is to calculate the six sigma based fuzzy control chart for mean using

2000 *Mathematics Subject Classification.* Primary ; Secondary .
Key words and phrases. Fuzzy control chart, Moderate distribution, Process capability and Six sigma.

range under moderate distribution with the help of process capability, which is more sensitive than the existing control charts.

2. Methods and Materials

The Shewhart (1924) control limits for mean based on sample range ($\overline{X} - R$) is given below:

$$UCL_{\overline{X}-R} = \overline{\overline{X}} + \left[\frac{3}{\sqrt{n}}\ \frac{\overline{R}}{d_2}\right]$$
$$CL_{\overline{X}-R} = \overline{\overline{X}}$$
$$UCL_{\overline{X}-R} = \overline{\overline{X}} - \left[\frac{3}{\sqrt{n}}\ \frac{\overline{R}}{d_2}\right]$$

Where d_2 is a control chart co-efficient (Appendix I: Table A), $\overline{R}$ is the average of R_i and n is the size of sample.

In fuzzy control chart, each sample or subgroup is represented by a trapezoidal fuzzy number (a, b, c, d) as shown in below.

A fuzzy set $X = (a, b, c, d)$ is said to trapezoidal fuzzy number if its membership function is given by where $a \leq b \leq c \leq d$.

$$\mu_\Delta(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{(x-a)}{(b-a)} & \text{for } a \leq x \leq b \\ 1 & \text{for } b \leq x \leq c \\ \dfrac{(d-x)}{(d-c)} & \text{for } c \leq x \leq d \\ 0 & \text{for } x > d \end{cases}$$

In this study, trapezoidal fuzzy numbers are represented as (X_a, X_b, X_c, X_d) for each observation (Ponnivalavan and Pathinathan, 2015).

The centre line $C\tilde{L}$ is the arithmetic mean of the fuzzy sample means, which are represented by $\left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right)$.

$$C\tilde{L} = \left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right) = \left(\frac{\sum_{j=1}^{m} \overline{X}_{aj}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{bj}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{cj}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{dj}}{m}\right)$$

for $j = 1, 2, 3, ..., m$. Where $\overline{\overline{X}}_r = \dfrac{\sum_{j=1}^{m} \overline{X}_{rj}}{m}$, $r = a, b, c, d$ and $j = 1, 2, ..., m$.

$$\overline{\overline{X}}_{rj} = \frac{\sum_{i=1}^{n} \overline{X}_{rij}}{n}, \ r = a, b, c, d;\ i = 1, 2, ..., n \text{ and } j = 1, 2, ..., m.$$

'n' is the fuzzy sample size
'm' is the number of fuzzy samples and

$C\tilde{L}$ is the fuzzy centre line for $\tilde{\bar{\bar{X}}} - R$ fuzzy control chart.

The average of range for the trapezoidal fuzzy numbers are represented as $(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$.

$$(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d) = \left(\frac{\sum_{j=1}^{m} R_{aj}}{m}, \frac{\sum_{j=1}^{m} R_{bj}}{m}, \frac{\sum_{j=1}^{m} R_{cj}}{m}, \frac{\sum_{j=1}^{m} R_{dj}}{m} \right),$$

where $\overline{R}_r = \frac{\sum_{j=1}^{m} R_{rj}}{m}$, $r = a, b, c, d$ and $j = 1, 2, ...m$.

$R_{rj} = (X_{\max .aj} - X_{\min .dj}, X_{\max .bj} - X_{\min .cj}, X_{\max .cj} - X_{\min .bj}, X_{\max .dj} - X_{\min .aj})|$ $j = 1, 2, ...m$ $(X_{\max .aj}, X_{\max .bj}, X_{\max .cj}, X_{\max .dj})$ are the maximum trapezoidal fuzzy numbers for each sample and $(X_{\min .aj}, X_{\min .bj}, X_{\min .cj}, X_{\min .dj})$ are the minimum trapezoidal fuzzy numbers for each sample.

The fuzzy control limits for $\tilde{\bar{\bar{X}}} - R$ are given below:

$$\begin{aligned}
U\tilde{C}L_{\tilde{\bar{\bar{X}}}-R} &= \left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right) + \left[\frac{3}{d_2\sqrt{n}}(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)\right] \\
&= \left(\overline{\overline{X}}_a + \frac{3}{d_2\sqrt{n}}\overline{R}_a, \overline{\overline{X}}_b + \frac{3}{d_2\sqrt{n}}\overline{R}_b, \overline{\overline{X}}_c + \frac{3}{d_2\sqrt{n}}\overline{R}_c, \overline{\overline{X}}_d + \frac{3}{d_2\sqrt{n}}\overline{R}_d\right) \\
&= \left(U\tilde{C}L_{a.\tilde{\bar{\bar{X}}}-R}, U\tilde{C}L_{b.\tilde{\bar{\bar{X}}}-R}, U\tilde{C}L_{c.\tilde{\bar{\bar{X}}}-R}, U\tilde{C}L_{d.\tilde{\bar{\bar{X}}}-R}\right) \\
C\tilde{L}_{\tilde{\bar{\bar{X}}}-R} &= \left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right) \\
L\tilde{C}L_{\tilde{\bar{\bar{X}}}-R} &= \left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right) - \left[\frac{3}{d_2\sqrt{n}}(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)\right] \\
&= \left(\overline{\overline{X}}_a - \frac{3}{d_2\sqrt{n}}\overline{R}_a, \overline{\overline{X}}_b - \frac{3}{d_2\sqrt{n}}\overline{R}_b, \overline{\overline{X}}_c - \frac{3}{d_2\sqrt{n}}\overline{R}_c, \overline{\overline{X}}_d - \frac{3}{d_2\sqrt{n}}\overline{R}_d\right) \\
&= \left(L\tilde{C}L_{a.\tilde{\bar{\bar{X}}}-R}, L\tilde{C}L_{b.\tilde{\bar{\bar{X}}}-R}, L\tilde{C}L_{c.\tilde{\bar{\bar{X}}}-R}, L\tilde{C}L_{d.\tilde{\bar{\bar{X}}}-R}\right)
\end{aligned}$$

The proposed standard deviation $(\sigma_{r.FMD:6\sigma},\ r = a, b, c, d)$ for fuzzy $\tilde{\bar{\bar{X}}} - R$ fuzzy control chart with the help of process capability

$$C_p = \frac{USL_{r.RFC_p} - LSL_{r.RFC_p}}{6\sigma}, \quad r = a, b, c, d$$

using a JAVA script (Radhakrishnan and Balamurugan, 2012) under moderate distribution is to calculate by the specified tolerance level from the relation

$$\frac{\frac{\sum_{j=1}^{m} R_{rj}}{m}}{d_2},$$

for $r = a, b, c, d$ and $j = 1, 2, ..., m$.

Apply the value of $\sigma_{FMD:6\sigma}$ in the control limits, to get the six sigma based fuzzy control limits for mean using range under Moderate distribution. The value of $A_{FMD:6\sigma}$ is obtained using $p(z \leq z_{6\sigma}) = 1 - \alpha_1$, $\alpha_1 = 3.4 \times 10^{-6}$ and z is a standard moderate variate.

Therefore the resultant of proposed six sigma fuzzy control limits under moderate distribution for $\tilde{\bar{X}} - R$ is given below:

$$
\begin{aligned}
U\tilde{C}L_{\tilde{\bar{X}}-R:C_p} &= \Bigg(\overline{\overline{X}}_a + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{a.FMDC_p},\ \overline{\overline{X}}_b + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{b.FMDC_p}, \\
&\qquad \overline{\overline{X}}_c + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{c.FMDC_p},\ \overline{\overline{X}}_d + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{d.FMDC_p}\Bigg) \\
&= \left(U\tilde{C}L_{a.\tilde{\bar{X}}-R:C_p}, U\tilde{C}L_{b.\tilde{\bar{X}}-R:C_p}, U\tilde{C}L_{c.\tilde{\bar{X}}-R:C_p}, U\tilde{C}L_{d.\tilde{\bar{X}}-R:C_p}\right) \\
C\tilde{L}_{\tilde{\bar{X}}-R} &= \left(\overline{\overline{X}}_{a.\tilde{\bar{X}}-R}, \overline{\overline{X}}_{b.\tilde{\bar{X}}-R}, \overline{\overline{X}}_{c.\tilde{\bar{X}}-R}, \overline{\overline{X}}_{d.\tilde{\bar{X}}-R}\right) \\
L\tilde{C}L_{\tilde{\bar{X}}-R:C_p} &= \Bigg(\overline{\overline{X}}_a - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{a.FMDC_p},\ \overline{\overline{X}}_b - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{b.FMDC_p}, \\
&\qquad \overline{\overline{X}}_c - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{c.FMDC_p},\ \overline{\overline{X}}_d - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{d.FMDC_p}\Bigg) \\
&= \left(L\tilde{C}L_{a.\tilde{\bar{X}}-R:C_p}, L\tilde{C}L_{b.\tilde{\bar{X}}-R:C_p}, L\tilde{C}L_{c.\tilde{\bar{X}}-R:C_p}, L\tilde{C}L_{d.\tilde{\bar{X}}-R:C_p}\right)
\end{aligned}
$$

3. Application

Consider a process by which coils are manufactured by a company in Salem District. The primary data collected and presented in Table-1 have been used the samples of size 5 are randomly selected from the process, and the 'between' measurements resistance values (in ohms) of the coils are measured for the application. These measurements are then converted into trapezoidal fuzzy numbers (TFN) using computer program and are given in Table-2.

Table 1: Resistance values (in ohms) of coils

Sample No.	X_1		X_2		X_3		X_4		X_5	
1	0.52	0.55	0.49	0.51	0.56	0.57	0.49	0.50	0.52	0.53
2	0.52	0.53	0.51	0.52	0.52	0.54	0.51	0.53	0.45	0.47
3	0.51	0.52	0.53	0.54	0.54	0.55	0.49	0.55	0.50	0.51
4	0.42	0.45	0.42	0.44	0.46	0.56	0.51	0.54	0.53	0.54
5	0.48	0.50	0.47	0.51	0.50	0.50	0.57	0.58	0.52	0.53
6	0.55	0.56	0.49	0.50	0.50	0.52	0.48	0.49	0.50	0.51
7	0.49	0.53	0.52	0.54	0.49	0.55	0.46	0.48	0.49	0.50
8	0.43	0.46	0.49	0.52	0.49	0.51	0.50	0.53	0.50	0.51
9	0.54	0.55	0.48	0.53	0.51	0.53	0.47	0.49	0.48	0.50
10	0.50	0.52	0.49	0.51	0.48	0.49	0.48	0.52	0.46	0.47
11	0.47	0.50	0.55	0.57	0.50	0.52	0.49	0.50	0.48	0.50
12	0.50	0.54	0.56	0.57	0.47	0.51	0.48	0.51	0.48	0.50
13	0.46	0.48	0.52	0.54	0.50	0.51	0.53	0.54	0.50	0.51
14	0.50	0.51	0.48	0.51	0.48	0.52	0.51	0.53	0.44	0.45
15	0.49	0.51	0.50	0.51	0.54	0.55	0.48	0.52	0.49	0.50

Table 2: Trapezoidal Fuzzy measurement resistance values of coils

Sample No.	X_1				X_2				X_3			
1	0.45	0.52	0.55	0.58	0.48	0.49	0.51	0.53	0.55	0.56	0.57	0.58
2	0.50	0.52	0.53	0.55	0.45	0.51	0.52	0.55	0.50	0.52	0.54	0.56
3	0.47	0.51	0.52	0.56	0.52	0.53	0.54	0.58	0.53	0.54	0.55	0.56
4	0.40	0.42	0.45	0.48	0.41	0.42	0.44	0.49	0.41	0.46	0.56	0.57
5	0.45	0.48	0.50	0.53	0.45	0.47	0.51	0.54	0.46	0.50	0.50	0.52
6	0.52	0.55	0.56	0.57	0.47	0.49	0.50	0.53	0.47	0.50	0.52	0.53
7	0.46	0.49	0.53	0.56	0.51	0.52	0.54	0.55	0.45	0.49	0.55	0.56
8	0.40	0.43	0.46	0.47	0.44	0.49	0.52	0.55	0.46	0.49	0.51	0.54
9	0.53	0.54	0.55	0.57	0.41	0.48	0.53	0.56	0.46	0.51	0.53	0.55
10	0.49	0.50	0.52	0.54	0.44	0.49	0.51	0.54	0.45	0.48	0.49	0.52
11	0.41	0.47	0.50	0.53	0.54	0.55	0.57	0.58	0.46	0.50	0.52	0.55
12	0.48	0.50	0.54	0.56	0.54	0.56	0.57	0.59	0.42	0.47	0.51	0.53
13	0.44	0.46	0.48	0.50	0.51	0.52	0.54	0.55	0.48	0.50	0.51	0.56
14	0.47	0.50	0.51	0.53	0.45	0.48	0.51	0.57	0.46	0.48	0.52	0.58
15	0.46	0.49	0.51	0.55	0.48	0.50	0.51	0.56	0.53	0.54	0.55	0.56

Table 2: Continued...

Sample No.	X_4				X_5			
1	0.47	0.49	0.50	0.52	0.49	0.52	0.53	0.54
2	0.49	0.51	0.53	0.55	0.42	0.45	0.47	0.49
3	0.48	0.49	0.55	0.58	0.48	0.50	0.51	0.52
4	0.46	0.51	0.54	0.57	0.50	0.53	0.54	0.55
5	0.56	0.57	0.58	0.59	0.49	0.52	0.53	0.54
6	0.46	0.48	0.49	0.50	0.48	0.50	0.51	0.52
7	0.44	0.46	0.48	0.51	0.47	0.49	0.50	0.55
8	0.48	0.50	0.53	0.55	0.49	0.50	0.51	0.53
9	0.46	0.47	0.49	0.54	0.45	0.48	0.50	0.52
10	0.45	0.48	0.52	0.56	0.42	0.46	0.47	0.48
11	0.41	0.49	0.50	0.54	0.45	0.48	0.50	0.52
12	0.45	0.48	0.51	0.55	0.47	0.48	0.50	0.52
13	0.48	0.53	0.54	0.55	0.48	0.50	0.51	0.53
14	0.50	0.51	0.53	0.54	0.42	0.44	0.45	0.48
15	0.46	0.48	0.52	0.55	0.41	0.49	0.50	0.51

The centre line $C\tilde{L}$ is the arithmetic mean of the fuzzy sample means, which are represented by $\left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right)$

$$\begin{aligned}
\overline{X}_{a1} &= \frac{X_{a11} + X_{a21} + X_{a31} + X_{a41} + X_{a51}}{5} \\
&= \frac{0.45 + 0.48 + 0.55 + 0.47 + 0.49}{5} = 0.488 \\
\overline{X}_{a2} &= \frac{X_{a12} + X_{a22} + X_{a32} + X_{a42} + X_{a52}}{5} \\
&= \frac{0.50 + 0.45 + 0.50 + 0.49 + 0.42}{5} = 0.472
\end{aligned}$$

and so on....

$$\begin{aligned}
\overline{\overline{X}}_a &= \frac{\overline{X}_{a1} + \overline{X}_{a2} + ... + \overline{X}_{a15}}{15} = \frac{0.488 + 0.472 + ... + 0.468}{15} = 0.4679 \\
\overline{\overline{X}}_b &= \frac{\overline{X}_{b1} + \overline{X}_{b2} + ... + \overline{X}_{b15}}{15} = \frac{0.516 + 0.502 + ... + 0.500}{15} = 0.4961 \\
\overline{\overline{X}}_c &= \frac{\overline{X}_{c1} + \overline{X}_{c2} + ... + \overline{X}_{c15}}{15} = \frac{0.532 + 0.518 + ... + 0.518}{15} = 0.5173
\end{aligned}$$

and

$$\begin{aligned}
\overline{\overline{X}}_d &= \frac{\overline{X}_{d1} + \overline{X}_{d2} + ... + \overline{X}_{d15}}{15} = \frac{0.550 + 0.540 + ... + 0.546}{15} = 0.5416 \\
C\tilde{L} &= \left(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d\right) = (0.4679, 0.4961, 0.5173, 0.5416)
\end{aligned}$$

The average of range for the trapezoidal fuzzy numbers are represented as $(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$

$$\begin{aligned}R_{a1} &= (X_{\max .aj} - X_{\min .dj}, X_{\max .bj} - X_{\min .cj}, X_{\max .cj} - X_{\min .bj}, X_{\max .dj} - X_{\min .aj})\\ &= (0.55 - 0.52, 0.56 - 0.50, 0.57 - 0.49, 0.58 - 0.45)\\ R_{a2} &= (0.50 - 0.49, 0.52 - 0.47, 0.54 - 0.45, 0.56 - 0.42)\end{aligned}$$

and so on....

$$\begin{aligned}\overline{R}_a &= \frac{R_{a1} + R_{a2} + ... + R_{a15}}{15} = \frac{0.03 + 0.01 + ... + 0.02}{15} = 0.0173\\ \overline{R}_b &= \frac{R_{b1} + R_{b2} + ... + R_{b15}}{15} = \frac{0.06 + 0.05 + ... + 0.04}{15} = 0.0520\\ \overline{R}_c &= \frac{R_{c1} + R_{c2} + ... + R_{c15}}{15} = \frac{0.08 + 0.09 + ... + 0.07}{15} = 0.0887\\ \overline{R}_d &= \frac{R_{d1} + R_{d2} + ... + R_{d15}}{15} = \frac{0.13 + 0.14 + ... + 0.15}{15} = 0.1427\\ (\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d) &= (0.0173, 0.0520, 0.0887, 0.1427)\end{aligned}$$

The fuzzy control limits for $\tilde{\overline{\overline{X}}} - R$ are given below:

$$\begin{aligned}U\tilde{C}L_{\tilde{\overline{\overline{X}}}-R} &= \left(\overline{\overline{X}}_a + \frac{3}{d_2\sqrt{n}}\overline{R}_a, \overline{\overline{X}}_b + \frac{3}{d_2\sqrt{n}}\overline{R}_b, \overline{\overline{X}}_c + \frac{3}{d_2\sqrt{n}}\overline{R}_c, \overline{\overline{X}}_d + \frac{3}{d_2\sqrt{n}}\overline{R}_d\right)\\ &= (0.4679, 0.4961, 0.5173, 0.5416) + \left[\frac{3}{2.326\sqrt{5}}(0.0173, 0.0520, 0.0887, 0.1427)\right]\\ &= \left(0.4679 + \frac{3}{2.326\sqrt{5}}0.0173,\ 0.4961 + \frac{3}{2.326\sqrt{5}}0.0520,\right.\\ &\qquad \left. 0.5173 + \frac{3}{2.326\sqrt{5}}0.0887,\ 0.5416 + \frac{3}{2.326\sqrt{5}}0.1427\right)\\ &= (0.4779, 0.5261, 0.5685, 0.6239)\end{aligned}$$

$$\begin{aligned}C\tilde{L}_{\tilde{\overline{\overline{X}}}-R} &= (0.4679, 0.4961, 0.5173, 0.5416)\\ L\tilde{C}L_{\tilde{\overline{\overline{X}}}-R} &= \left(\overline{\overline{X}}_a - \frac{3}{d_2\sqrt{n}}\overline{R}_a,\ \overline{\overline{X}}_b - \frac{3}{d_2\sqrt{n}}\overline{R}_b,\ \overline{\overline{X}}_c - \frac{3}{d_2\sqrt{n}}\overline{R}_c,\ \overline{\overline{X}}_d - \frac{3}{d_2\sqrt{n}}\overline{R}_d\right)\\ &= (0.4679, 0.4961, 0.5173, 0.5416) - \left[\frac{3}{2.326\sqrt{5}}(0.0173, 0.0520, 0.0887, 0.1427)\right]\\ &= \left(0.4679 - \frac{3}{2.326\sqrt{5}}0.0173,\ 0.4961 - \frac{3}{2.326\sqrt{5}}0.0520,\right.\\ &\qquad \left. 0.5173 - \frac{3}{2.326\sqrt{5}}0.0887,\ 0.5416 - \frac{3}{2.326\sqrt{5}}0.1427\right)\\ &= (0.4579, 0.4661, 0.4662, 0.4593)\end{aligned}$$

The proposed standard deviation

$$\begin{aligned}
USL_{a.RFC_p} - LSL_{a.RFC_p} &= 0.01720 - 0.00000 \quad \Rightarrow \quad \tilde{\sigma}_{a.FMDC_p} = 0.00143\\
USL_{b.RFC_p} - LSL_{b.RFC_p} &= 0.03869 - 0.01290 \quad \Rightarrow \quad \tilde{\sigma}_{b.FMDC_p} = 0.00215\\
USL_{c.RFC_p} - LSL_{c.RFC_p} &= 0.06019 - 0.02580 \quad \Rightarrow \quad \tilde{\sigma}_{c.FMDC_p} = 0.00287\\
USL_{d.RFC_p} - LSL_{d.RFC_p} &= 0.07309 - 0.04729 \quad \Rightarrow \quad \tilde{\sigma}_{d.FMDC_p} = 0.00215
\end{aligned}$$

Therefore the resultant of proposed six sigma fuzzy control limits under moderate distribution for $\tilde{\bar{X}} - R$ using process capability is given below:

$$\begin{aligned}
U\tilde{C}L_{\tilde{\bar{X}}-R:C_p} &= \Bigg(\overline{\overline{X}}_a + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{a.FMDC_p},\ \overline{\overline{X}}_b + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{b.FMDC_p}\\
&\qquad \overline{\overline{X}}_c + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{c.FMDC_p},\ \overline{\overline{X}}_d + \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{d.FMDC_p}\Bigg)\\
&= \Bigg(0.4679 + \frac{4.5}{\sqrt{5}}\ 0.00143,\ 0.4961 + \frac{4.5}{\sqrt{5}}\ 0.00215,\\
&\qquad 0.5173 + \frac{4.5}{\sqrt{5}}\ 0.00287,\ 0.5416 + \frac{4.5}{\sqrt{5}}\ 0.00215\Bigg)\\
&= (0.4708, 0.5005, 0.5231, 0.5459)
\end{aligned}$$

$$C\tilde{L}_{\tilde{\bar{X}}-R} = (0.4679, 0.4961, 0.5173, 0.5416)$$

$$\begin{aligned}
L\tilde{C}L_{\tilde{\bar{X}}-R:C_p} &= \Bigg(\overline{\overline{X}}_a - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{a.FMDC_p},\ \overline{\overline{X}}_b - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{b.FMDC_p}\\
&\qquad \overline{\overline{X}}_c - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{c.FMDC_p},\ \overline{\overline{X}}_d - \frac{A_{FMD:6\sigma}}{\sqrt{n}}\tilde{\sigma}_{d.FMDC_p}\Bigg)\\
&= \Bigg(0.4679 - \frac{4.5}{\sqrt{5}}\ 0.00143,\ 0.4961 - \frac{4.5}{\sqrt{5}}\ 0.00215,\\
&\qquad 0.5173 - \frac{4.5}{\sqrt{5}}\ 0.00287,\ 0.5416 - \frac{4.5}{\sqrt{5}}\ 0.00215\Bigg)\\
&= (0.4650, 0.4918, 0.5116, 0.5373)
\end{aligned}$$

4. Conclusion

The constructed six sigma based control chart for fuzzy mean using range under moderateness with the help of process capability, procedures adopted and discussed in the research article by taking the process capability (C_p) as the base only. In this article offers the possibility of using six sigma based fuzzy mean using range control chart, which rules out the weaknesses compared to the existing control charts. Specifically, it presents one of the six sigma based fuzzy mean using range control chart under moderate distribution and on the real-life data illustrates the simplicity of its usage in practice.

References

1. Sevil Senturk, A, Nihal Erginel: Development of fuzzy and control charts $\alpha-$ Cuts, *Information Sciences* **179** (2009) 1542–1551.

2. Ponnivalavan, K, Pathinathan, T.: Ranking of a Pentagonal Fuzzy Number and Its Applications, *Journal of Computer and Mathematical Sciences* **6 (11)** (2015) 571–584.
3. Gulbay, M, Kahraman, C.: Design of Fuzzy Process Control Charts for Linguistic and im-precise Data, *StudFuzz* **201** (2006) 59–88.
4. Khademi, M, Amirzadeh, V.: Fuzzy rules for fuzzy and R control charts, *Iranian Journal of Fuzzy Systems* **11 (5)** (2014) 55–66.
5. Pandurangan, Varadharajan, R.: Construction of $\alpha-$cut fuzzy and control charts using fuzzy trapezoidal number, *International Journal of Recent Research and Applied Studies* **9 (1)** (2011) 100-111.
6. Radhakrishnan, R, Balamurugan, P.: Construction of control charts based on six sigma Initiatives for Fraction Defectives with varying sample size, *Journal of Statistics & anagement Systems* **5 (4 , 5)** (2012) 405–413.
7. Shewhart, W.A.: *Economic Control of Quality of Manufactured Product*, Van Nostrand, New York, (1924).
8. Zadeh, L.A.: Fuzzy sets, *Information and Control* **8 (3)** (1965) 338–353.

M.R. Pavithra: Ph.D Research Scholar, Department of Statistics, Periyar University, Salem–636011, Tamil Nadu, India.

M.R. Pavithra: Assistant Professor, Operations & Analytics, Great Lakes Institute of Management, Chennai–600096. Tamil Nadu, India.
E-mail address: pavistats@gmail.com

Dr. P. Balamurugan: Assistant Professor, Department of Statistics, Government Arts College (Autonomous), Salem–636007. Tamil Nadu, India.
E-mail address: pbalamuruganstat@gmail.com