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Fifth-order Convergent Iterative Method for Solving Nonlinear Equations using Quadrature Formula

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Abstract: In this paper, we suggest and analyze a new two-step iterative method for solving nonlinear equations using the three-pint Gaussian quadrature formula. We prove that this new method has fifth-order convergence under certain conditions. Several examples are given to illustrate the efficiency of the new iterative method and its comparison with other similar methods. Our results can be considered as an alternative to Newton methods and other similar methods.

Kewwords: Iterative method, Newton method, Quadrature rule, Convergence.

1. INTRODUCTION

In recent years, much attention has been given to develop several iterative methods for solving nonlinear equations. These methods have been suggested by using Taylor's series, decomposition, homotopy and quadrature formulas, see [1-12]. It is well known that the quadrature rule plays an important and significant rule in the evaluation of the integrals It has been shown [3, 5, 9, 10, 12] that these quadrature formulas can be used to develop some iterative methods for solving nonlinear equations arising in the engineering and optimal control sciences. This interaction among these different branches of applied sciences has played an important and significant role in developing several iterative methods. Motivated and inspired by the recent research activities in this direction, we suggest and analyze a new iterative method for solving nonlinear equations by using the three-point Gaussian quadrature formula, see [11]. This method is an implicit-type method. To implement this, we use Newton method as a predictor and the new method as a corrector. The resultant method can be considered a predictor-corrector iterative method or two-step iterative method. It has been shown that this two-step iterative method is of fifth-order convergence under certain conditions. A comparison between this new method and other similar methods is given. Several examples are given to illustrate the efficiency and advantage of the suggested two-step method. Our method can be considered as an alternative iterative method to recently suggested methods.

2. ITERATIVE METHOD

Let us take r be a simple zero of a sufficiently differentiable function. We consider the numerical solution of the equation $f(x) = 0$. It is known that

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t)dt. \tag{1}$$

Using the three-point Gaussian quadrature rule [11], we have

$$\int_{x_n}^{x} f'(t)dt = \frac{x-x_n}{18}\left\{5f'\left(\frac{x+x_n}{2} - \frac{x-x_n}{2}\sqrt{\frac{3}{5}}\right) + 8f'\left(\frac{x_n+x}{2}\right) + 5f'\left(\frac{x+x_n}{2} + \frac{x-x_n}{2}\sqrt{\frac{3}{5}}\right)\right\}. \tag{2}$$

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From (1) and (2), we have

$$x = x_n - \frac{18f(x_n)}{5f'\left(\frac{x+x_n}{2} - \frac{x-x_n}{2}\sqrt{\frac{3}{5}}\right) + 8f'\left(\frac{x_n+x}{2}\right) + 5f'\left(\frac{x+x_n}{2} + \frac{x-x_n}{2}\sqrt{\frac{3}{5}}\right)}.$$

This fixed point formulation enables us to suggest the following implicit iterative method.

Algorithm 2.1: For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme.

$$x_{k+1} = x_n - \frac{18f(x_n)}{5f'\left(\frac{x_{n+1}+x_n}{2} - \frac{x_{n+1}-x_n}{2}\sqrt{\frac{3}{5}}\right) + 8f'\left(\frac{x_n+x_{n+1}}{2}\right) + 5f'\left(\frac{x_{n+1}+x_n}{2} + \frac{x_{n+1}-x_n}{2}\sqrt{\frac{3}{5}}\right)}$$

In order to implement this method, we use the predictor-corrector technique. For this purpose, we need the following one-step method.

Algorithm 2.2 (Newton Method): For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \qquad k = 0, 1, 2, \ldots$$

It is well known [1] that the Newton method has quadratic convergence.

Using the Newton method as a predictor, we suggest the following new iterative method, which is the main motivation of this paper.

Algorithm 2.3: For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme.

Predictor step:

$$y_k = x_k - \frac{f(x_k)}{f'(x_k)}, \tag{3}$$

Corrector step:

$$x_{k+1} = x_n - \frac{18f(x_n)}{5f'\left(\frac{y_n+x_n}{2} - \frac{y_n-x_n}{2}\sqrt{\frac{3}{5}}\right) + 8f'\left(\frac{x_n+y_n}{2}\right) + 5f'\left(\frac{y_n+x_n}{2} + \frac{y_n-x_n}{2}\sqrt{\frac{3}{5}}\right)}, \tag{4}$$

this is called the two-step method or predictor-corrector iterative method.

We now consider the convergence criteria of Algorithm 2.3 using essentially the technique of Noor [8,9] and Cordero and Rorregrosa [3]..

Theorem 2.1: Let $r \in I$ be a simple zero of sufficiently differentiable function $f : I \subseteq R \to R$ for an open interval I. If x_0 is sufficiently close to r, then Algorithm 2.3 has third order convergence. Moreover, if f verifies that $f''(r) = 0$ then Algorithm 2.3 has 5^{th} order convergence.

Proof. Let r be a simple zero of f. Since f is sufficiently differentiable, by expanding $f(x_n)$ and $f'(x_n)$ about r, we get

$$f\left(x_n\right)=f'\left(r\right)\left[e_n+c_2e_n^2+c_3e_n^3+c_4e_n^4+\cdots\right] \tag{5}$$

$$f'\left(x_n\right)=f'\left(r\right)\left[1+2c_2e_n+3c_3 e_n^2+4 c_4 e_n^3+5c_5e_n^4\cdots\right] \tag{6}$$

where $c_k=\frac{1}{k!}\frac{f^{(k)}(r)}{f'(r)},\quad k=1,2,3,...$ and $e_n=x_n-r$.

Now, from (5) and (6), we have

$$\frac{f(x_n)}{f'(x_n)}=e_n-c_2e_n^2+2(c_2^2-c_3)e_n^3+(7c_2c_3-4c_2^3-3c_4)e_n^4+\cdots \tag{7}$$

Then from (3) and (7), we get

$$y_n=\left[r+c_2e_n^2+2\left(c_3-c_2^2\right)e_n^3+\left(-7c_2c_3+4c_2^3+3c_4\right)e_n^4+\cdots\right], \tag{8}$$

this implies that

$$f'(y_n)=f'(r)\left[1+2c_2\left(y_n-r\right)+3c_3\left(y_n-r\right)^2+4c_4\left(y_n-r\right)^3+5c_5\left(y_n-r\right)^4+\cdots\right]$$

$$=f'(r)\left[1+2c_2^2e_n^2+4\left(c_2c_3-c_2^3\right)e_n^3+\left(-11c_2^2c_3+8c_2^4+6c_2c_4\right)e_n^4+\cdots\right] \tag{9}$$

From (8) and $x_n=e_n+r$, we get

$$k_1=\frac{x_n+y_n}{2}-\sqrt{\frac{3}{5}}\frac{x_n-y_n}{2}=r+\left(-\frac{1}{10}\sqrt{15}+\frac{1}{2}\right)e_n+\left(\frac{1}{10}\sqrt{15}+\frac{1}{2}\right)c_2\,e_n^2$$

$$+\left(\left(\frac{1}{5}\sqrt{5}+1\right)c_3+\left(-1-\frac{1}{5}\sqrt{5}\right)c_2^2\right)e_n^3+\left(-\frac{7}{2}-\frac{7}{10}\sqrt{15}\right)c_2c_3e_n^4$$

$$+2c_2^3+\left(\frac{3}{10}\sqrt{15}+\frac{3}{2}\right)c_4e_n^4+\{-5c_2c_4-\frac{3}{5}\sqrt{15}c_3^2-\frac{4}{5}\sqrt{15}c_2^4+\frac{2}{5}\sqrt{15}c_5$$

$$-3c_3^2+10c_3c_2^2+2c_5+2\sqrt{15}c_3c_2^2-4c_2^4-\sqrt{15}c_2c_4\}e_n^5+\cdots\}$$

From (10), we have

$$f'\left(k_1\right)=f'(r)\{1+2c_2\left(k_1-r\right)+3c_3\left(k_1-r\right)^2+4c_4\left(k_1-r\right)^3+5c_5\left(k_1-r\right)^4+\cdots$$

$$=f'(r)\{1+\left(1-\frac{1}{5}\sqrt{15}\right)c_2e_n+\left(\left(\frac{6}{5}-\frac{3}{10}\sqrt{15}\right)c_3+\left(1+\frac{1}{5}\sqrt{15}\right)c_2^2\right)e_n^2$$

$$+\left(\left(\frac{13}{5}+\frac{2}{5}\sqrt{15}\right)c_2c_3-\left(2+\frac{2}{5}\sqrt{15}\right)c_2^3+\left(\frac{7}{4}-\frac{9}{25}\sqrt{15}\right)c_4\right)e_n^3$$

$$+\{\left(4+\frac{4}{5}\sqrt{15}\right)c_2^4+\left(\frac{31}{20}-\frac{2}{5}\sqrt{15}\right)c_5-\left(7+\frac{11}{10}\sqrt{15}\right)c_2^2c_3$$

$$+\frac{6}{5}c_3^2+\left(\frac{18}{5}+\frac{12}{25}\sqrt{15}\right)c_2c_4\}e_n^4+\cdots\} \qquad (11)$$

From (8) and $x_n = e_n + r$, we get

$$\frac{x_n+y_n}{2}=r+\frac{1}{2}e_n+\frac{1}{2}c_2e_n^2+\left(c_3-c_2^2\right)e_n^3+\left(-\frac{7}{2}c_2c_3+2c_2^3+\frac{3}{2}c_4\right)e_n^4+\cdots, \qquad (12)$$

this implies that

$$f'\left(\frac{x_n+y_n}{2}\right)=f'(r)\{1+2c_2\left(\frac{x_n+y_n}{2}-r\right)+3c_3\left(\frac{x_n+y_n}{2}-r\right)^2$$

$$+4c_4\left(\frac{x_n+y_n}{2}-r\right)^3+5c_5\left(\frac{x_n+y_n}{2}-r\right)^4+\cdots\}$$

$$=f'(r)\{1+c_2e_n+\left(\frac{3}{4}c_3+c_2^2\right)e_n^2+\left(\frac{7}{2}c_2c_3-2c_2^3+\frac{1}{2}c_4\right)e_n^3$$

$$+\left(4c_2^4+\frac{5}{16}c_5-\frac{37}{4}c_2^2c_3+3c_3^2+\frac{9}{2}c_2c_4\right)e_n^4+\cdots\} \qquad (13)$$

From (8) and $x_n = e_n + r$, we have

$$k_2=\frac{x_n+y_n}{2}+\sqrt{\frac{3}{5}}\frac{x_n-y_n}{2}=r+\left(\frac{1}{10}\sqrt{15}+\frac{1}{2}\right)e_n+\left(-\frac{1}{10}\sqrt{15}+\frac{1}{2}\right)c_2e_n^2$$

$$+\left(\left(-\frac{1}{5}\sqrt{5}+1\right)c_3+\left(-1+\frac{1}{5}\sqrt{5}\right)c_2^2\right)e_n^3+\{\left(-\frac{7}{2}+\frac{7}{10}\sqrt{15}\right)c_2c_3$$

$$+\left(2-\frac{2}{5}\sqrt{15}\right)c_2^3+\left(-\frac{3}{10}\sqrt{15}+\frac{3}{2}\right)c_4\}e_n^4+\{\left(-3+\frac{3}{5}\sqrt{15}\right)c_3^2$$

$$+\left(-\frac{2}{5}\sqrt{15}+2\right)c_5+\left(\sqrt{15}-5\right)c_2c_4+\left(10-2\sqrt{15}\right)c_3+\left(\frac{4}{5}\sqrt{15}-4\right)c_2^4\}+e_n^5+\cdots\} \qquad (14)$$

Thus

$$
\begin{aligned}
f'(k_2) &= f'(r)\{1+2c_2(k_2-r)+3c_3(k_2-r)^2+4c_4(k-r)^3+5c_5(k_2-r)^4+\cdots \\
&= f'(r)\{1+\left(1+\frac{1}{5}\sqrt{15}\right)c_2e_n+\left(\left(\frac{6}{5}+\frac{3}{10}\sqrt{15}\right)c_3+\left(1-\frac{1}{5}\sqrt{15}\right)c_2^2\right)e_n^2 \\
&\quad +\left(\left(\frac{13}{5}-\frac{2}{5}\sqrt{15}\right)c_2c_3+\left(-2+\frac{2}{5}\sqrt{15}\right)c_2^3+\left(\frac{7}{5}+\frac{9}{25}\sqrt{15}\right)c_4\right)e_n^3 \\
&\quad +\{\left(4-\frac{4}{5}\sqrt{15}\right)c_2^4+\left(\frac{31}{20}+\frac{2}{5}\sqrt{15}\right)c_5+\left(-7+\frac{11}{10}\sqrt{15}\right)c_2^2c_3 \\
&\quad +\frac{6}{5}c_3^2+\left(\frac{18}{5}-\frac{12}{25}\sqrt{15}\right)c_2c_4\}e_n^4+\cdots\}
\end{aligned}
\tag{15}
$$

From (6), (9), (11), (13) and (15), we have

$$
\begin{aligned}
\left[5f'(k_1)+8f'\left(\frac{x_n+y_n}{2}\right)+5f'(k_2)\right] &= f'(r)\{18+18c_2e_n+18\left(c_2^2+c_3\right)e_n^2 \\
&\quad +18\left(c_4+3c_2c_3-2c_2^3\right)e_n^3+18\left(2c_3^2-8c_3c_2^2+4c_2^4+4c_2c_4+c_5\right)e_n^4 \\
&\quad +\{360c_3c_2^3+90c_3c_4-162c_2c_3^2+90c_2c_5-144c_2^5-198c_2^2c_4\}e_n^5+\cdots\}\}
\end{aligned}
\tag{16}
$$

Then from (4), (16) and $e_{n+1} = x_{n+1} - r$, we obtain

$$
e_{n+1} = c_2^2\ e_n^3+3\left(c_2c_3-c_2^3\right)\ e_n^4+\left(6c_2^4+4c_2c_4+2c_3^2-12c_3c_2^2\right)\ e_n^5+\cdots,
$$

from which it follows that Algorithm 2.3 has a cubic convergence. We also note that, if $c_2 = 0$, that is, if $f''(r) = 0$, then Algorithm 2.2 has 5th order convergence

4. NUMERICAL EXAMPLES

We now present some examples to illustrate the efficiency of the new developed two-step iterative methods, see Table 1. We compare the Newton method (NM), the method of Weerakoon and Fernando (HN[12]), the method of Corderos and Torregrosa (WN[3]), method of Osban (ON[10]), method of Noor (NR2 [9]) and the Algorithm 2.2(NR1) introduced in this present paper. We use $\varepsilon = 10^{-15}$. The following stopping criteria is used for computer programs:

$$
(i)\quad |x_{n+1}-x_n| < \varepsilon. \qquad\qquad (ii)\quad |f(x_{n+1})| < \varepsilon.
$$

The examples are the same as in Corderos and Torregrosa [3].

$$
f_1 = x^3-9x^2+28x-30 \qquad\qquad f_2 = \sin(x)+x\cos(x)
$$

$$
f_3 = e^{x^2}-e^{\sqrt{2}x} \qquad\qquad f_4 = \left(\sin(x)-\frac{x}{2}\right)^2
$$

$$f_5 = \cos(x) - x \qquad f_6 = \tan^{-1}(x)$$

$$f_7 = (x-1)^6 - 1 \qquad f_8 = 4\sin(x) - x + 1$$

$$f_7 = (x-1)^6 - 1$$

As for the convergence criteria, it was required that the distance of two consecutive approximations δ for the zero was les than 10^{-15}. Also displayed are the number of iterations to approximate the zero (IT), the approximate zero x_n and the value $f(x_n)$.

Table 1
Numerical Examples and Comparison

	IT	x_n	$f(x_n)$	δ
f_1, $x_0 = 1$				
NM	8	3.0000000000000000000000000000	0	1.79e-38
WN	6	3.0000000000000000000000000000	-3.10e-61	8.64e-21
ON	5	3.0000000000000000000000000000	1.00e-62	2.69e-16
HN	6	3.0000000000000000000000000000	0	4.34e-36
NR2	5	3.0000000000000000000000000000	1.00e-62	2.69e-16
NR1	5	3.0000000000000000000000000000	0	2.69e-16
f_2, $x_0 = 0.5$				
NM	5	0	-2.44e-86	2.63e-29
WN	4	0	1.87e-53	3.83e-18
ON	4	0	0	3.88e-58
HN	4	0	8.29e-50	6.29e-17
NR2	4	0	0	4.93e-58
NR1	4	0	0	4.93e-58
f_2, $x_0 = 1$				
NM	6	-4.9131804394348836888378206686	-2.05e-48	1.45e-24
WN	5	-4.9131804394348836888378206686	7.15e-57	2.78e-19
ON	5	-4.9131804394348836888378206686	0	1.67e-72
HN	4	-4.9131804394348836888378206686	-1.20e-63	2.66e-30
NR2	6	-4.9131804394348836888378206686	1.71e-62	4.79e-29
NR1	5	-4.9131804394348836888378206686	3.81e-29	1.57e-62
f_2, $x_0 = 3$				
NM	6	2.0287578381104342235769711247	-1.79e-34	1.41e-17
WN	5	2.0287578381104342235769711247	2.27e-41	1.20e-63
ON	4	2.0287578381104342235769711247	1.07e-18	3.64e-55
HN	5	2.0287578381104342235769711247	1.20e-63	2.29e-28
NR2	4	2.0287578381104342235769711247	9.68e-56	6.88e-19
NR1	4	2.0287578381104342235769711247	6.86e-19	9.60e-56
f_3, $x_0 = 0.5$				
NM	5	0	3.00e-64	3.78e-44
WN	4	0	0	1.51e-25
ON	4	0	0	7.07e-65
HN	4	0	0	1.41e-27
NR2	4	0	0	0
NR1	4	0	0	0

	IT	x_n	$f(x_n)$	δ
f_3, $x_0 = 3$				
NM	14	1.4142135623730950488016887242	1.14e-16	3.82e-31
WN	10	1.4142135623730950488016887242	0	1.78e-22
ON	10	1.4142135623730950488016887242	0	1.78e-22
HN	9	1.4142135623730950488016887242	0	3.74e-39
NR2	10	1.4142135623730950488016887242	0	4.81e-38
NR1	10	1.4142135623730950488016887242	0	4.80e-38
f_4, $x_0 = 0.5$				
NM	49	.0000000000000006729758308981	1.13e-31	6.73e-16
WN	32	.0000000000000006729758308981	7.00e-33	3.35e-16
ON	32	.0000000000000006729758308981	0	4.05e-38
HN	25	.0000000000000006729758308981	1.34e-32	6.94e-16
NR2	32	.0000000000000006729758308981	1.08e-32	4.16e-16
NR1	32	.0000000000000006729758308981	1.08e-32	4.16e-16
f_5, $x_0 = 0.6$				
NM	5	.7390851332151606416553120877	8.78e-24	–2.85e-47
WN	4	.7390851332151606416553120877	0	2.23e-45
ON	4	.7390851332151606416553120877	1.08e-32	4.16e-16
HN	4	.7390851332151606416553120877	0	1.03e-42
NR2	4	.7390851332151606416553120877	0	4.45e-40
NR1	4	.7390851332151606416553120877	0	4.45e-40
f_5, $x_0 = 2$				
NM	5	.7390851332151606416553120877	-8.45e-48	4.78e-24
WN	4	.7390851332151606416553120877	4.77e-51	5.72e-17
ON	4	.7390851332151606416553120877	0	4.44e-40
HN	4	.7390851332151606416553120877	-4.61e-47	9.36e-16
NR2	4	.7390851332151606416553120877	0	4.17e-35
NR1	4	.7390851332151606416553120877	0	4.00e-35
f_6, $x_0 = 0.5$				
NM	5	0	0	1.06e-32
WN	4	0	1.21e-58	9.00e-20
ON	4	0	0	1.08e-32
HN	4	0	4.87e-55	1.43e-18
NR2	4	0	0	8.32e-62
NR1	4	0	0	8.39e-62
f_6, $x_0 = 1.5$				
NM	Fails	–	–	–
WN	Fails	–	–	–
ON	5	0	0	2.63e-62
HN	Fails	–	–	–

	IT	x_n	$f(x_n)$	δ
NR2	4	0	5.58e-29	
NR1	4	0	0	1.26e-51
f_6, $x_0 = 1.7$				
NM	Fails	-	-	-
WN	Fails	-	-	-
ON	5	0	0	1.83e-60
HN	Fails	-	-	-
NR2	5	-	-	-
NR1	5	0	0	4.53e-45
f_7, $x_0 = 1.7$				
NM	9	2	6.80e-33	2.13e-17
WN	8	2	0	2.27e-28
ON	7	2	0	5.71e-28
HN	5	2	9.15e-58	4.51e-20
NR2	7	2	0	1.03e-27
NR1	7	2	0	1.03e-27
f_7, $x_0 = 1.5$				
NM	16	2	6.89e-30	6.78e-16
WN	468	2	-7.64e-59	1.17e-20
ON	172	2	0	7.88e-36
HN	8	2	2.60e-52	2.96e-18
NR2	183	2	0	3.19e-24
NR1	183	2	0	3.19e-24
f_8, $x_0 = 0$				
NM	5	-.3421850529244582209950829677	1.95e-42	1.71e-21
WN	4	-.3421850529244582209950829677	1.59e-27	-1.00e-63
ON	4	-.3421850529244582209950829677	-1.00e-63	1.08e-35
HN	4	-.3421850529244582209950829677	0	9.30e-26
NR2	4	-.3421850529244582209950829677	0	1.11e-35
NR1	4	-.3421850529244582209950829677	0	1.11e-35
f_8, $x_0 = -2$				
NM	6	-2.2100839440926608962059466053	1.02e-59	2.52e-30
WN	4	-2.2100839440926608962059466053	1.00e-63	1.24e-25
ON	4	-2.2100839440926608962059466053	1.00e-63	1.24e-25
HN	4	-2.2100839440926608962059466053	1.00e-63	2.10e-32
NR2	4	-2.2100839440926608962059466053	1.00e-63	1.24e-25
NR1	4	-2.2100839440926608962059466053	1.00e-63	1.24e-25
f_8, $x_0 = 2.5$				
NM	5	2.7020613733260402218069499993	-8.76e-43	1.01e-21
WN	4	2.7020613733260402218069499993	1.29e-36	1.00e-63
ON	4	2.7020613733260402218069499993	1.00e-63	1.29e-36
HN	4	2.7020613733260402218069499993	1.00e-63	2.24e-34
NR2	4	2.7020613733260402218069499993	1.00e-63	1.29e-36
NR1	4	2.7020613733260402218069499993	1.00e-63	1.29e-36

Form the table 1, we see that Algorithm 2.2 is comparable with other methods. It is clear from the table that for the examples f_6 and f_7, Newton method, methods of Cordero et al [3] and Weerakoon and Fernando [12] fail, while our methods find the approximate solutions. In view of this fact, one can consider our method as an alternative to Newton method and the methods of [3,12] for solving nonlinear equations. Using the technique of this paper, one can obtain a cubic convergent method for solving a system of nonlinear equations and this is an interesting topic of future research.

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