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RELIABILITY MODELLING ON FINITE CAPACITY $M/PH/1$ QUEUEING SYSTEM WITH BREAKDOWN AND RECOVERY POLICIES

S.R. SRUTHI AND P.R.JAYASHREE

Abstract. In this paper, the Reliability, Availability and Maintainability (RAM) analysis of the finite capacity $M/PH/1$, optional service queueing model are studied in this paper. The arrival process of the machines are according to Poisson process with parameter λ with two phases of service, essential and optional services. Apart from the Phase-type distribution the environmental states such as working and working-breakdown for the queueing systems are also considered in this paper. The Ordinary-Differential Difference Equations for the queueing system with working and working-breakdown policies are formed. The transient probabilities are obtained from the system of equations for the special case of $N = 4$ which are solved by using numerical method. The RAM model for $M/PH/1$ has been analysed numerically and shown graphically for the special case. The Reliability measures like MTBF, MTTR are also analysed for the model. Sensitivity Analysis is carried out for different parametric values, to find the changes in the Reliability (R(t)), Availability (A(t)) and Maintainability (M(t)) of the system.

1. Introduction

The most widely used subject, Stochastic process otherwise known as the Random process is nothing but the mathematical study of random variables which are usually interpreted by representing the numerical values of a system for randomly varying time. The Stochastic process is classified into two categories with respect to time, which are referred as the Discreate-time and Continuous time Stochastic process. Markov chain model is a part of the stochastic model is used for measuring Continuous-time stochastic process. Christian R. Shelton, Gianfranco Ciardo [3] has given a detail study on Structured Continuous-time Markov Process.

The CTMC is most commonly used in Queueing theory with Phase-type distributions. The Queueing models with optional phase service system has been used by researchers to solve industrial problems and to improve their grade of service. In Optional Phase service queueing system, each customer is served in two phases out of which some are essential phases whereas, others are optional phases which are provided to the customer based on their own choice. The most common distributions which are used for phase-type are exponential and Poisson distributions. S. Mocanu, C. Commault [7] formulates Sparse representations of

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phase-type distributions are studied as an extension of the monocyclic distributions. He has proved that if the phase-type distribution is triangular it is possible to find an upper triangular markovian representation for this distribution. Tian, N. and Zhang, Z. G. [12]have studied the performance measures of the multi-server queueing system with multiple exponential vacations. Madan [6] had analysed an $M/G/1$ model with optional service where the server provides essential service first, to all the arriving customers and then optional service is provided to the customers who are in need additional service.

Reliability, Availability and Maintainability (RAM) is an important metric that are used to measure the performance of the system and are also considered to be a good starting point to improve the system. Hence, researchers will put huge efforts to study the performance modelling and availability analysis on various industrial systems. Aggarwal, et. al. [1], applied the concept of fuzzy in system Reliability in order to analyse the failure and repair rates in the subsystems of the sugar plant problem. An Overview of reliability, availability, maintainability and supportability (RAMS) engineering are provided by the authors Saraswat and G.S. Yadava [10].This paper provides details regarding the current information along with the past information in the RAMS engineering research and industry.

Sensitivity Analysis is also known as 'what-if' analysis is used to find the effect of a set of independent variables on a set of dependent variables under specific conditions. This type of analysis can be used for analysing any activity or system within certain boundaries. Neetu Singh [8] deals with sensitivity Analysis of a Markovian Queue along with Threshold policies and additional servers. It has been found in this model that, the optimal threshold policy for the Markovian queueing model is obtained by having additional servers along with the permanent server.

Based on the literatures reviewed, not much work has been done on the Reliability analysis of Phase-type queuing system with working and working-breakdown states. Thus, the goal of this paper is to investigate the Reliability (R(t)), Availability (A(t)) and Maintainability (M(t)) analysis of the finite capacity $M/PH/1$ optional service queueing system with working and working-breakdown states. The Differential-Difference equations has been formed for the general case. As a special case of $N = 4$ the equations are solved with by using numerical method. The Reliability,Availability and Maintainability of the model is illustrated numerically and shown graphically. The Sensitivity Analysis has been carried out for different parametric values in analysing the robustness of the Phase-type Queueing model.

2. Description of the System

In this paper, a time-dependent, finite capacity $M/PH/1$ queueing model with working and working-breakdown states are considered. The mean arrival time of the machines are calculated according to the Poisson process with constant parameter λ There are two Phases of Service, in which the first service is an essential phase whereas the second service service is an optional phase which are exponentially distributed with four service rates (i.e., μ_1, μ_{11}, μ_2 and μ_{21}). With probability p the machine goes through the First Essential Service before it departs, and if there is a need for additional service the machine goes through an optional service facility with probability $q = 1 - p$.

The Assumptions and Notations used for this model are given below.

2.1. Assumptions.

- The machines arrive independently according to the Poisson process with a constant parameter.
- There are two service phases, which follows a First Come First Service (FCFS) discipline.
- Once the service is completed in the system it is inspected. With probability p the service is found to be satisfactory in the machines during inspection and the machine departs. Whereas with probability $q = 1 - p$, the machine needs some additional service before it departs.
- When the system is not empty (i.e., there should be at least one machine) breakdown occurs at the service facility which is exponentially distributed. When the system is in breakdown state the service is performed at a slower rate.
- Whenever breakdown occurs in the system it is immediately recovered in the recovery state which is also exponentially distributed. Once the system recovers it performs its activity at a normal service rate.
- All inter-arrival, Phase type optional services, working state and the working-breakdown state are independent of each other.

2.2. Notations. The notations that are used in this paper are as follows:

$N(t)$: Number of machines in the system at time t

PH : Phase type distribution

$S(t)$: Environmental state at any instant of time t which is given by:

$$S(t) = \begin{cases} 0, & \text{if the server is in the working environment state for Phase 1 \& 2} \\ 1, & \text{if the server is in the working breakdown state for Phase 1 \& 2} \end{cases}$$

λ : Arrival rate

μ_1 : First Phase of service for working state

μ_2 : First phase of service for working-breakdown state ($\mu_1 < \mu_2$)

μ_{11} : Optional phase of service for working state

μ_{21} : Optional phase of service for working-breakdown state ($\mu_{11} < \mu_{21}$)

α_1 : Failure rate for First phase of service

α_2 : Failure rate for Second phase of service ($\alpha_1 < \alpha_2$).

β_1 : Recovery rate for First phase of service

β_2 : Recovery rate for Second phase of service ($\beta_1 < \beta_2$).

$P_{0,0,0}(t)$ = Probability that there are no machines in the system.

$P_{n,i,j}(t)$ = Probability that there is$n$ ($n \geq 1$) machines in the i^{th} phase of the system with j^{th} state

The state transition diagram for the finite capacity Markovian Phase-Type queueing model is shown in Figure 1

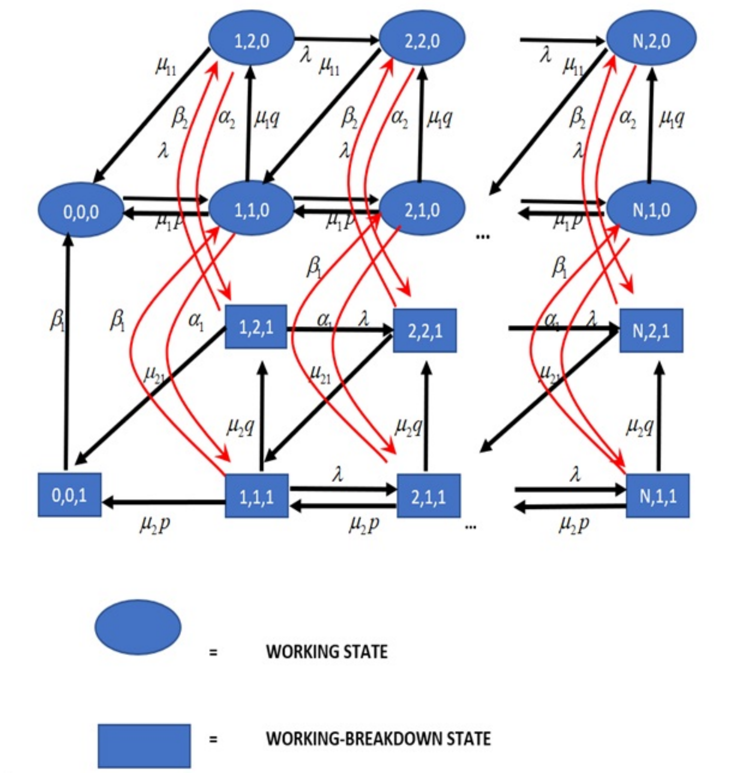

FIGURE 1. State Transition Diagram for finite capacity Phase-Type Markovian queueing model

3. Governing Equations

The differential-difference equations for the finite Phase-type queueing system under the working and working-breakdown states are given below:

Working State

$$\frac{dP_{0,0,0}(t)}{dt} = \mu_1 p P_{1,1,0}(t) + \mu_{11} P_{1,2,0}(t) - \lambda P_{0,0,0}(t), n = 0 \tag{3.1}$$

$$\frac{dP_{n,1,0}(t)}{dt} = \lambda P_{n-1,1,0}(t) + \mu_1 p P_{n+1,1,0}(t) + \beta_1 P_{n,1,1}(t) - (\lambda + \mu_1 p + \mu_1 q + \alpha_1) P_{n,1,0}(t), n \geq 1 \tag{3.2}$$

$$\frac{dP_{n,2,0}(t)}{dt} = \lambda P_{n-1,2,0}(t) + \mu_1 q P_{n,1,0}(t) + \beta_2 P_{n,2,1}(t) - (\lambda + \mu_{11} + \alpha_2) P_{n,2,0}(t), n \geq 1 \tag{3.3}$$

$$\frac{dP_{N,1,0}(t)}{dt} = \lambda P_{N-1,1,0}(t) + \beta_1 P_{N,1,1}(t) - (\mu_1 p + \mu_1 q + \alpha_1) P_{N,1,0}(t), n = N \tag{3.4}$$

$$\frac{dP_{N,2,0}(t)}{dt} = \lambda P_{N-1,2,0}(t) + \mu_1 q P_{N,1,0}(t) + \beta_2 P_{N,2,1}(t) - (\mu_{11} + \alpha_2) P_{N,2,0}(t), n = N \tag{3.5}$$

Working-Breakdown State

$$\frac{dP_{0,0,1}(t)}{dt} = \mu_2 p P_{1,1,1}(t) + \mu_{21} P_{1,2,1}(t) - (\lambda + \beta_1) P_{0,0,1}(t), n = 0 \tag{3.6}$$

$$\frac{dP_{n,1,1}(t)}{dt} = \lambda P_{n-1,1,1}(t) + \mu_2 p P_{n+1,1,1}(t) + \alpha_1 P_{n,1,0}(t) - (\lambda + \mu_2 p + \mu_2 q + \beta_1) P_{n,1,1}(t), n \geq 1 \tag{3.7}$$

$$\frac{dP_{n,2,1}(t)}{dt} = \lambda P_{n-1,2,1}(t) + \mu_2 q P_{n,1,1}(t) + \alpha_2 P_{n,2,0}(t) - (\lambda + \mu_{21} + \beta_2) P_{n,2,1}(t), n \geq 1 \tag{3.8}$$

$$\frac{dP_{N,1,1}(t)}{dt} = \lambda P_{N-1,1,1}(t) + \alpha_1 P_{N,1,0}(t) - (\mu_2 p + \mu_2 q + \beta_1) P_{N,1,1}(t), n = N \tag{3.9}$$

$$\frac{dP_{N,2,1}(t)}{dt} = \lambda P_{N-1,2,1}(t) + \mu_2 q P_{N,1,1}(t) + \alpha_2 P_{N,2,0}(t) - (\mu_{21} + \beta_2) P_{N,2,1}(t), n = N \tag{3.10}$$

without loss of generality the initial state conditions are given by
$P_{0,0,0}(0) = 0,\ P_{n,i,j}(0) = 0,\ \forall\ n = 1, 2, \ldots, N;\ i = 1, 2;\ j = 0, 1$

The above equations can be solved for transient case by using Fourth-Order Runge Kutta numerical method.

The system reliability at time t is calculated as follows:

$$R(t) = \sum_{n=0}^{N} \sum_{i=1}^{2} \sum_{j=0}^{1} P_{n,i,j} \tag{3.11}$$

The system Availability at time t is calculated by considering up all the working states as follows:

$$A(t) = \sum_{n=0}^{N} \sum_{i=1}^{2} \sum_{j=0}^{1} P_{n,i,j} \tag{3.12}$$

The system Maintainability at time t is calculated by considering working-breakdown state which is calculated as follows:

$$M(t) = \sum_{n=0}^{N} \sum_{i=1}^{2} \sum_{j=0}^{1} P_{n,i,j} \tag{3.13}$$

The metrics such as MTBF (Mean time between failures) and MTTR (Mean Time Till Recovery) are calculated as follows:

$$MTBF = \frac{1}{(\alpha_1 + \alpha_2)} \tag{3.14}$$

$$MTTR = \frac{1}{(\beta_1 + \beta_2)} \tag{3.15}$$

3.1. Special Case. As a special case for $N = 4$ for the finite capacity Phase-Type optional queueing system, the system of differential equations of the model is obtained as

Working State

$$\frac{dP_{0,0,0}(t)}{dt} = \mu_1 p P_{1,1,0}(t) + \mu_{11} P_{1,2,0}(t) - \lambda P_{0,0,0}(t) \tag{3.16}$$

$$\frac{dP_{1,1,0}(t)}{dt} = \lambda P_{0,0,0}(t) + \mu_1 p P_{2,1,0}(t) + \beta_1 P_{1,1,1}(t) - (\lambda + \mu_1 p + \mu_1 q + \alpha_1) P_{1,1,0}(t) \tag{3.17}$$

$$\frac{dP_{2,1,0}(t)}{dt} = \lambda P_{1,1,0}(t) + \mu_1 p P_{3,1,0}(t) + \beta_1 P_{2,1,1}(t) - (\lambda + \mu_1 p + \mu_1 q + \alpha_1) P_{2,1,0}(t) \tag{3.18}$$

$$\frac{dP_{3,1,0}(t)}{dt} = \lambda P_{2,1,0}(t) + \mu_1 p P_{4,1,0}(t) + \beta_1 P_{3,1,1}(t) - (\lambda + \mu_1 p + \mu_1 q + \alpha_1) P_{3,1,0}(t) \tag{3.19}$$

$$\frac{dP_{4,1,0}(t)}{dt} = \lambda P_{3,1,0}(t) + \beta_1 P_{4,1,1}(t) - (\mu_1 p + \mu_1 q + \alpha_1) P_{4,1,0}(t) \tag{3.20}$$

$$\frac{dP_{1,2,0}(t)}{dt} = \mu_1 q P_{1,1,0}(t) + \beta_2 P_{1,2,1}(t) - (\lambda + \mu_{11} + \alpha_2) P_{1,2,0}(t) \tag{3.21}$$

$$\frac{dP_{2,2,0}(t)}{dt} = \lambda P_{1,2,0}(t) + \mu_1 p P_{2,1,0}(t) + \beta_2 P_{2,2,1}(t) - (\lambda + \mu_{11} + \alpha_2) P_{2,2,0}(t) \tag{3.22}$$

$$\frac{dP_{3,2,0}(t)}{dt} = \lambda P_{2,2,0}(t) + \mu_1 q P_{3,1,0}(t) + \beta_2 P_{3,2,1}(t) - (\lambda + \mu_{11} + \alpha_2) P_{3,2,0}(t) \tag{3.23}$$

$$\frac{dP_{4,2,0}(t)}{dt} = \lambda P_{3,2,0}(t) + \mu_1 q P_{3,1,0}(t) + \beta_2 P_{4,2,1}(t) - (\mu_{11} + \alpha_2) P_{4,2,0}(t) \tag{3.24}$$

Working-Breakdown State

$$\frac{dP_{0,0,1}(t)}{dt} = \mu_2 p P_{1,1,1}(t) + \mu_{21} P_{1,2,1}(t) - (\lambda + \beta_1) P_{0,0,1}(t) \tag{3.25}$$

$$\frac{dP_{1,1,1}(t)}{dt} = \lambda P_{0,1,1}(t) + \mu_2 p P_{2,1,1}(t) + \alpha_1 P_{1,1,0}(t) - (\lambda + \mu_2 p + \mu_2 q + \beta_1) P_{1,1,1}(t) + \mu_{21} P_{2,2,1} \tag{3.26}$$

$$\frac{dP_{2,1,1}(t)}{dt} = \lambda P_{1,1,1}(t) + \mu_2 p P_{3,1,1}(t) + \alpha_1 P_{2,1,0}(t) - (\lambda + \mu_2 p + \mu_2 q + \beta_1) P_{2,1,1}(t) + \mu_{21} P_{3,2,1} \tag{3.27}$$

$$\frac{dP_{3,1,1}(t)}{dt} = \lambda P_{2,1,1}(t) + \mu_2 p P_{4,1,1}(t) + \alpha_1 P_{3,1,0}(t) - (\lambda + \mu_2 p + \mu_2 q + \beta_1) P_{3,1,1}(t) + \mu_{21} P_{4,2,1} \tag{3.28}$$

$$\frac{dP_{4,1,1}(t)}{dt} = \lambda P_{3,1,1}(t) + \alpha_1 P_{4,1,0}(t) - (\mu_2 p + \mu_2 q + \beta_1) P_{4,1,1}(t) \tag{3.29}$$

$$\frac{dP_{1,2,1}(t)}{dt} = \mu_2 q P_{1,2,0}(t) + \alpha_2 P_{1,2,0}(t) - (\lambda + \mu_{21} + \beta_2) P_{1,2,1}(t) \tag{3.30}$$

$$\frac{dP_{2,2,1}(t)}{dt} = \lambda P_{1,2,1}(t) + \mu_2 q P_{2,1,1}(t) + \alpha_2 P_{2,2,0}(t) - (\lambda + \mu_{21} + \beta_2) P_{2,2,1}(t) \tag{3.31}$$

$$\frac{dP_{3,2,1}(t)}{dt} = \lambda P_{2,2,1}(t) + \mu_2 q P_{3,1,1}(t) + \alpha_2 P_{3,2,0}(t) - (\lambda + \mu_{21} + \beta_2) P_{3,2,1}(t) \tag{3.32}$$

$$\frac{dP_{4,2,1}(t)}{dt} = \lambda P_{3,2,1}(t) + \mu_2 q P_{4,1,1}(t) + \alpha_2 P_{4,2,0}(t) - (\mu_{21} + \beta_2) P_{4,2,1}(t) \tag{3.33}$$

4. Numerical Illustration

The main focus of this paper, is to analyse RAM using the transient behaviour of the Markovian Phase-type finite capacity queueing model. Considering the time range from $t = 0$ to $t = 200$ (in hours), and the parameter values as $\lambda = 0.04$, $\mu_1 = 0.07$, $\mu_2 = 0.04$, $\mu_{11} = 0.02$, $\mu_{21} = 0.015$, $\alpha_1 = 0.04$, $\alpha_2 = 0.02$, $\beta_1 = 0.03$, $\beta_2 = 0.01$ and $p = 0.7$. The system of equations (3.16)-(3.33) are solved for time-dependent probabilities $P_n(t)$, for n customers using Fourth-Order Runge-Kutta method.

The probability curves that are displayed in Figure 2, shows the time dependent total system size probability distribution $P_n(t)$ of the total number of machines in the system.

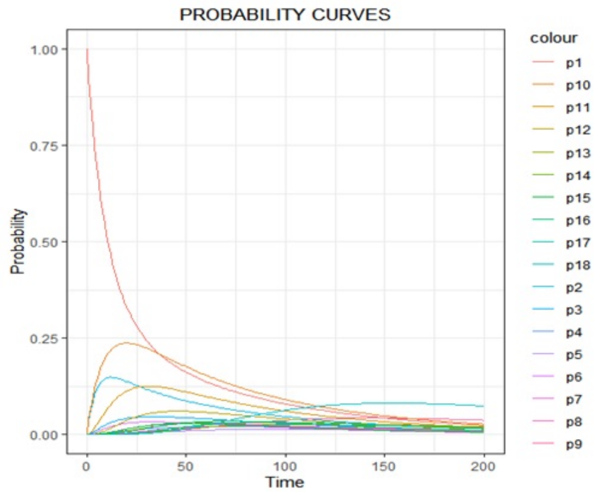

FIGURE 2. Probability Curves

Figure 3, depicts the Reliability of the Phase-Type Queueing system and it is found that,the reliability of the system decreases with increase in time and the system reliability after 200 hours is found to be 12%.

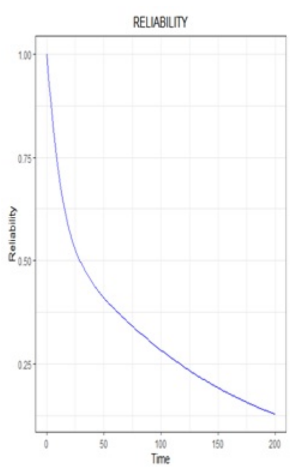

FIGURE 3. Relliability of Phase-type Queueing system

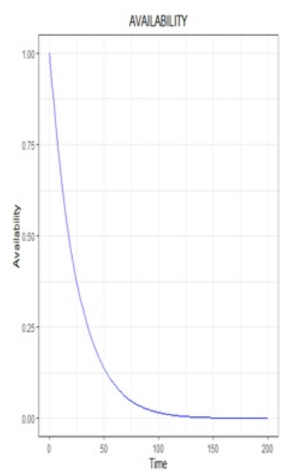

FIGURE 4. Availability of Phase-type Queueing system

Figure 4, represents the Availability of the system for the Phase-Type Queueing model. It is proved that as time increases the Availability of the system decreases.

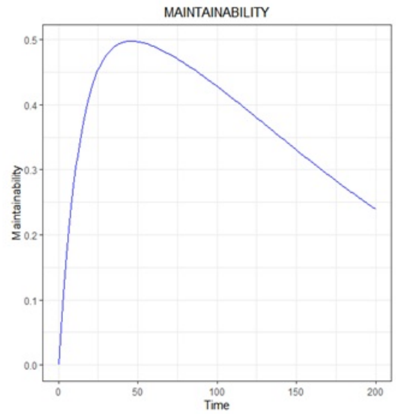

FIGURE 5. Maintainability of Phase-type Queueing system

Figure 5, Illustrates the maintainability of the system with respect to time for the Phase -Type queueing model. It is shown that Maintainability of the Queueing system increases with respect to time, until 47 hours and starts decreasing. the $M(t)$ decreases and at 200 operating hours it is found out to be 23%. The values of MTBF and MTTR are found to be 17 hours/failure and 25 hours/recovery respectively.

5. Sensitivity Analysis

The Sensitivity Analysis is carried out for Phase-type queueing system for different parametric values. Figure 6, 7 and 8 shows the variation in $R(t)$, $A(t)$ and $M(t)$ for different sets of Failure rate values (α_1 and α_2). By fixing the other parameters, we observe that for increasing values of $\alpha_1(0.05, 0.06, 0.07)$ and $\alpha_2(0.03, 0.04, 0.05)$ Reliability and Availability decreases. After 100 operating hours the Availability for different failure rates becomes constant. Maintainability increases at the beginning and as time increases it decreases.

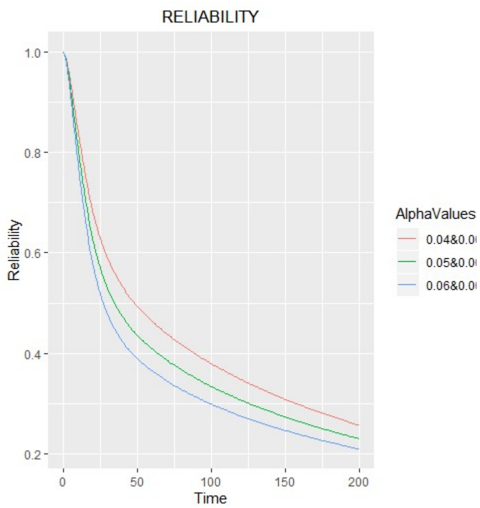

FIGURE 6. Sensitivity Analysis for Failure Rate values of Reliability

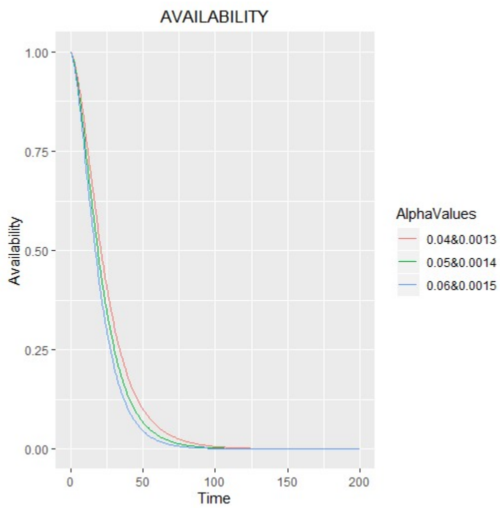

FIGURE 7. Sensitivity Analysis for Failure Rate values of Availability

In figure 9 and 10, illustrates the variation in $R(t)$ and $M(t)$ for different sets of $\beta_1 = (0.04, 0.05, 0.06)$ and $\beta_2 = (0.02, 0.03, 0.04)$. By fixing the other parameters such as $\lambda, \mu_1, \mu_2, \mu_{11}, \mu_{21}, \alpha_1, \alpha_2, p$ as constant, it is observed that for increasing values of β_1 and β_2, Reliability increases. Figure 10 shows the variation in $M(t)$ for different sets of β_1 and β_2, it is observed that for increasing values of β_1 and β_2, Maintainability decreases.

Table 1 shows the changes in RAM for different values of Failure rate and the Service rate. It is seen that as the values of the Failure rate increases keeping the service rate constant there is a decrease in Reliability and Availability values whereas the value of the Maintainability increases.

Table 2 displays the changes in RAM for different values of Arrival rates and the Service rates. It is clearly visible that as arrival rate increases keeping the Service rate constant the Reliability and Availability decreases but Maintainability increases.

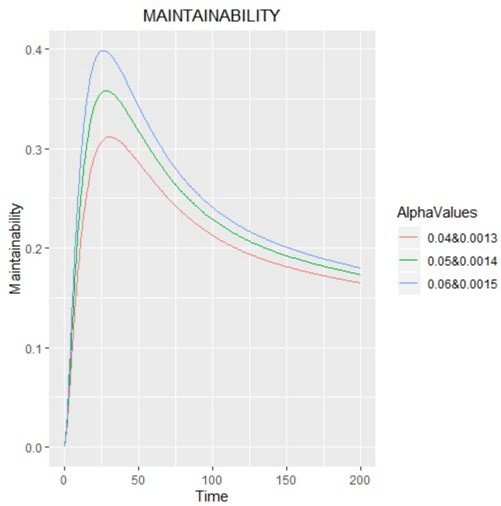

FIGURE 8. Sensitivity Analysis for Failure Rate values of Maintainability

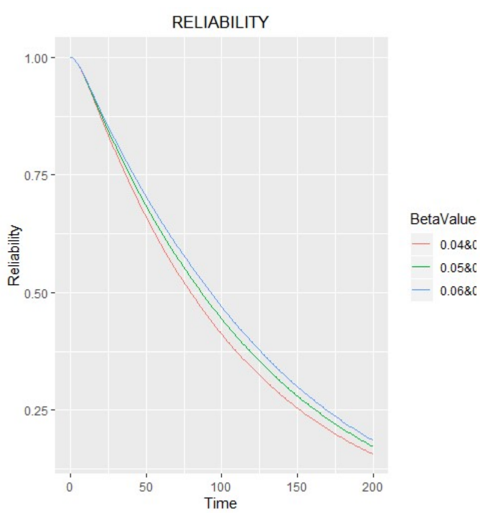

FIGURE 9. Sensitivity Analysis for Recovery Rate values of Reliability

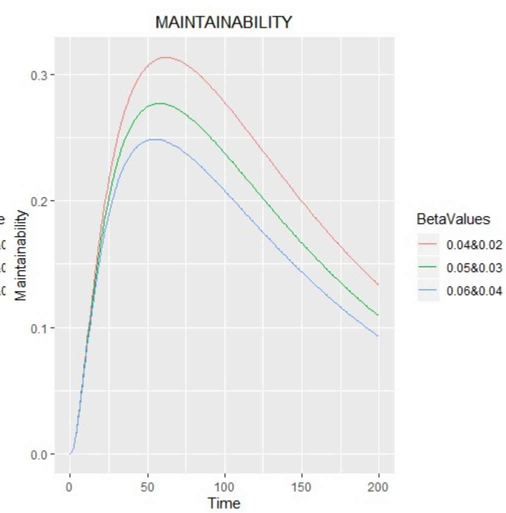

FIGURE 10. Sensitivity Analysis for Recovery Rate values of Maintainability

Table 3 depicts the changes in RAM for different values of Failure rates and Arrival rates. It is proven that there are variations in Failure rate and Arrival rates values. It is seen that as Failure rate increases Reliability and Availability decreases whereas Maintainability increases.

Table 4 shows the changes in RAM for different values of Arrival rate and Recovery rate. It is found out that for different values of Failure rate with respect to Recovery rate, Reliability decreases whereas Maintainability also decreases.

Table 5 shows the comparison between Working state Service rates and the Recovery rates for different values has been calculated for N=4. It is predicted that for different Recovery rates and Service rates the Reliability decreases, and Maintainability increases.

TABLE 1. Sensitivity Analysis for Different values of Failure rate (α_1, α_2) and Service rates (μ_1, μ_{11})

Time		$\mu1 = 0.04, \mu11 = 0.03$			$\mu1 = 0.09, \mu11 = 0.04$			$\mu1 = 0.10, \mu11 = 0.05$		
		$R(t)$	$M(t)$	$A(t)$	$R(t)$	$M(t)$	$A(t)$	$R(t)$	$M(t)$	$A(t)$
40	$\alpha1 = 0.05$, $\alpha2 = 0.03$	0.662	0.3617	0.5871	0.6683	0.3482	0.5964	0.6747	0.3354	0.6154
	$\alpha1 = 0.06$, $\alpha2 = 0.04$	0.6228	0.41	0.544	0.6306	0.3958	0.5544	0.6383	0.3823	0.5645
	$\alpha1 = 0.07$, $\alpha2 = 0.05$	0.5893	0.4512	0.5085	0.5981	0.4366	0.5196	0.6067	0.4227	0.5305
80	$\alpha1 = 0.05$, $\alpha2 = 0.03$	0.4052	0.3947	0.2718	0.4134	0.3785	0.2859	0.4231	0.3638	0.3004
	$\alpha1 = 0.06$, $\alpha2 = 0.04$	0.3633	0.4361	0.2331	0.3733	0.4204	0.2473	0.3841	0.4059	0.2617
	$\alpha1 = 0.07$, $\alpha2 = 0.05$	0.3296	0.4676	0.2043	0.3404	0.4366	0.2181	0.3518	0.4392	0.2319
120	$\alpha1 = 0.05$, $\alpha2 = 0.03$	0.2539	0.3248	0.1214	0.2608	0.3104	0.1337	0.2701	0.2984	0.1466
	$\alpha1 = 0.06$, $\alpha2 = 0.04$	0.2208	0.3528	0.0972	0.2287	0.3395	0.1084	0.2383	0.3282	0.1199
	$\alpha1 = 0.07$, $\alpha2 = 0.05$	0.1947	0.3721	0.0806	0.203	0.3603	0.0904	02124	0.3501	0.1006
160	$\alpha1 = 0.05$, $\alpha2 = 0.03$	0.1624	0.2445	0.0538	0.1671	0.2325	0.0623	0.1745	0.2235	0.0715
	$\alpha1 = 0.06$, $\alpha2 = 0.05$	0.1384	0.2628	0.0404	0.1434	0.2517	0.0474	0.1506	0.2434	0.0549
	$\alpha1 = 0.07$, $\alpha2 = 0.05$	0.1198	0.2743	0.0317	0.1249	0.2646	0.0374	0.1315	0.2571	0.0436
200	$\alpha1 = 0.05$, $\alpha2 = 0.03$	0.1053	0.1768	0.0238	0.1079	0.1669	0.0291	0.1135	0.1605	0.0348
	$\alpha1 = 0.06$, $\alpha2 = 0.04$	0.0886	0.1889	0.0168	0.0913	0.1796	0.0207	0.0961	0.1794	0.0251
	$\alpha1 = 0.07$, $\alpha2 = 0.05$	0.0757	0.1959	0.0125	0.0784	0.1876	0.0155	0.0827	0.1819	0.0189

TABLE 2. Sensitivity Analysis for Different values of Arrival rate (λ) and Service rates (μ_1, μ_{11})

Time		$\boldsymbol{\mu 1 = 0.08, \mu 11 = 0.03}$			$\boldsymbol{\mu 1 = 0.09, \mu 11 = 0.04}$			$\boldsymbol{\mu 1 = 0.10, \mu 11 = 0.05}$		
		$\boldsymbol{R(t)}$	$\boldsymbol{M(t)}$	$\boldsymbol{A(t)}$	$\boldsymbol{R(t)}$	$\boldsymbol{M(t)}$	$\boldsymbol{A(t)}$	$\boldsymbol{R(t)}$	$\boldsymbol{M(t)}$	$\boldsymbol{A(t)}$
40	$\lambda = 0.05$	0.6583	0.3265	0.5791	0.6607	0.3145	0.5849	0.6635	0.303	0.5911
	$\lambda = 0.06$	0.6151	0.339	0.5273	0.6151	0.3275	0.5313	0.6162	0.31651	0.5359
	$\lambda = 0.07$	0.5779	0.3453	0.4831	0.5750	0.3344	0.4851	0.5744	0.3239	0.4878
80	$\lambda = 0.05$	0.3893	0.3338	002511	0.3892	0.3191	0.2597	0.3926	0.3061	0.2701
	$\lambda = 0.06$	0.3365	0.3222	0.1963	0.3314	0.3081	0.2012	0.3308	0.2958	0.2083
	$\lambda = 0.07$	0.2961	0.3092	0.1546	0.2867	0.2952	0.1578	0.2823	0.2832	0.1619
120	$\lambda = 0.05$	0.2319	0.2605	0.1014	0.2302	0.2465	0.1093	0.2333	0.2356	0.1189
	$\lambda = 0.06$	0.1876	0.2389	0.0665	0.1808	0.2246	0.0707	0.1793	0.2136	0.0767
	$\lambda = 0.07$	0.1574	0.2211	0.0449	0.1467	0.2059	0.0465	0.1418	0.1945	0.0499
160	$\lambda = 0.05$	0.1407	0.1867	0.04	0.1379	0.1743	0.0455	0.1401	0.1658	0.0521
	$\lambda = 0.06$	0.1083	0.1645	0.0217	0.1014	0.151	0.0244	0.0993	0.142	0.0279
	$\lambda = 0.07$	0.0885	0.1482	0.01227	0.0788	0.1337	0.0133	0.0742	0.1239	0.0151
200	$\lambda = 0.05$	0.0868	0.1285	0.0157	0.0836	0.1181	0.0189	0.0846	0.1117	0.0228
	$\lambda = 0.06$	0.0646	0.194	0.0069	0.0582	0.0979	0.0084	0.0561	0.0907	0.0101
	$\lambda = 0.07$	0.0521	0.0966	0.0033	0.0442	0.0842	0.0038	0.0402	0.0763	0.0046

TABLE 3. Sensitivity Analysis for Different values of Arrival rate (λ) and Failure rates (α_1, α_2)

Time		**$\alpha 1 = 0.05, \alpha 11 = 0.03$**			**$\alpha 1 = 0.06, \alpha 11 = 0.04$**			**$\alpha 1 = 0.07, \alpha 11 = 0.05$**		
		$R(t)$	**$M(t)$**	**$A(t)$**	**$R(t)$**	**$M(t)$**	**$A(t)$**	**$R(t)$**	**$M(t)$**	**$A(t)$**
40	$\lambda = 0.05$	0.6021	0.4007	0.5125	0.557	0.4516	0.4638	0.5191	0.4941	0.4245
	$\lambda = 0.06$	0.5572	0.4137	0.4584	0.5091	0.4651	0.4067	0.4689	0.5076	0.3655
	$\lambda = 0.07$	0.5197	0.4195	0.4132	0.4694	0.4705	0.3596	0.4278	0.5122	0.3173
80	$\lambda = 0.05$	0.3369	0.4029	0.1909	0.2934	0.4399	0.1545	0.2596	0.4664	0.1293
	$\lambda = 0.06$	0.2913	0.3876	0.144	0.2496	0.4212	0.1111	0.2178	0.4446	0.0892
	$\lambda = 0.07$	0.2579	0.3718	0.1112	0.2182	0.4026	0.0817	0.1882	0.4233	0.0627
120	$\lambda = 0.05$	0.1987	0.3155	0.0661	0.1677	0.3377	0.0486	0.1443	0.3514	0.0377
	$\lambda = 0.06$	0.1649	0.2916	0.0411	0.1377	0.3111	0.028	0.1174	0.3227	0.0204
	$\lambda = 0.07$	0.1427	0.2728	0.0266	0.1185	0.2905	0.0167	0.1006	0.3007	0.0113
160	$\lambda = 0.05$	0.1233	0.2309	0.0223	0.1026	0.2447	0.0151	0.0871	0.2521	0.0109
	$\lambda = 0.06$	0.1005	0.2081	0.0112	0.0835	0.2206	0.0068	0.0708	0.2273	0.0046
	$\lambda = 0.07$	0.0869	0.1921	0.006	0.0724	0.2057	0.0032	0.0615	0.2102	0.002
200	$\lambda = 0.05$	0.0794	0.1645	0.0074	0.0659	0.1738	0.0046	0.0557	0.1784	0.0031
	$\lambda = 0.06$	0.0645	0.1458	0.003	0.0539	0.1547	0.0033	0.0458	0.1592	0.001
	$\lambda = 0.07$	0.0562	0.1334	0.0013	0.0473	0.1422	0.0006	0.0405	0.1468	0.0003

TABLE 4. Sensitivity Analysis for Different Values of Arrival rate (λ) and Recovery rates ($\beta 1, \beta 2$)

Time		$\boldsymbol{\beta 1 = 0.04, \beta 2 = 0.02}$		$\boldsymbol{\beta 1 = 0.05, \beta 2 = 0.03}$		$\boldsymbol{\beta 1 = 0.06, \beta 2 = 0.04}$	
		$\boldsymbol{R(t)}$	$\boldsymbol{M(t)}$	$\boldsymbol{R(t)}$	$\boldsymbol{M(t)}$	$\boldsymbol{R(t)}$	$\boldsymbol{M(t)}$
40	$\lambda = 0.05$	0.6808	0.3061	0.7019	0.2782	0.7201	0.2544
	$\lambda = 0.06$	0.6429	0.3169	0.6662	0.2881	0.6863	0.2636
	$\lambda = 0.07$	0.6106	0.3219	0.6358	0.2928	0.6575	0.2681
80	$\lambda = 0.05$	0.4374	0.2981	0.4711	0.2591	0.4982	0.2291
	$\lambda = 0.06$	0.3919	0.2881	0.4265	0.2507	0.4542	0.2219
	$\lambda = 0.07$	0.3573	0.2772	0.3924	0.2412	0.4204	0.2135
120	$\lambda = 0.05$	0.2818	0.2281	0.3115	0.1931	0.3345	0.1677
	$\lambda = 0.06$	0.2422	0.2108	0.2706	0.1781	0.2925	0.1545
	$\lambda = 0.07$	0.2149	0.1966	0.2422	0.1656	0.2631	0.1434
160	$\lambda = 0.05$	0.182	0.1621	0.203	0.1342	0.219	0.1149
	$\lambda = 0.06$	0.1516	0.1447	0.1704	0.119	0.1844	0.1013
	$\lambda = 0.07$	0.1324	0.1321	0.1494	0.1079	0.1621	0.0914
200	$\lambda = 0.05$	0.1177	0.1113	0.131	0.0901	0.1409	0.0761
	$\lambda = 0.06$	0.0958	0.0966	0.1067	0.0772	0.1147	0.0645
	$\lambda = 0.07$	0.083	0.0867	0.0923	0.0686	0.0989	0.0568

TABLE 5. Sensitivity Analysis for Different values of Service rates (μ_1, μ_{11}) and Recovery rates $(\beta 1, \beta 2)$

Time		$\beta 1 = 0.04, \beta 2 = 0.02$		$\beta 1 = 0.05, \beta 2 = 0.03$		$\beta 1 = 0.06, \beta 2 = 0.04$	
		$R(t)$	$M(t)$	$R(t)$	$M(t)$	$R(t)$	$M(t)$
40	$\mu 1 = 0.08, \mu 11 = 0.03$	0.7287	0.2714	0.7461	0.2486	0.7611	0.2271
	$\mu 1 = 0.09, \mu 11 = 0.04$	0.732	0.2626	0.7485	0.2378	0.7628	0.2169
	$\mu 1 = 0.1, \mu 11 = 0.05$	0.7356	0.2518	0.7514	0.2278	0.7649	0.2074
80	$\mu 1 = 0.08, \mu 11 = 0.03$	0.4949	0.2866	0.5235	0.24759	0.5464	0.2177
	$\mu 1 = 0.09, \mu 11 = 0.04$	0.4968	0.2721	0.5228	0.2341	0.5437	0.2053
	$\mu 1 = 0.1, \mu 11 = 0.05$	0.5018	0.2593	0.5259	0.2224	0.5452	0.1944
120	$\mu 1 = 0.08, \mu 11 = 0.03$	0.3319	0.2289	0.3577	0.1921	0.3778	0.1658
	$\mu 1 = 0.09, \mu 11 = 0.04$	0.3327	0.2154	0.3556	0.1797	0.3735	0.1544
	$\mu 1 = 0.1, \mu 11 = 0.05$	0.3387	0.2048	0.3598	0.1701	0.3764	0.1458
160	$\mu 1 = 0.08, \mu 11 = 0.03$	0.2206	0.16742	0.2395	0.1374	0.2542	0.1171
	$\mu 1 = 0.09, \mu 11 = 0.04$	0.2208	0.1562	0.2375	0.1273	0.2507	0.1081
	$\mu 1 = 0.1, \mu 11 = 0.05$	0.2269	0.1485	0.2426	0.1207	0.2553	0.1023
200	$\mu 1 = 0.08, \mu 11 = 0.03$	0.1454	0.1172	0.158	0.0944	0.168	0.0796
	$\mu 1 = 0.09, \mu 11 = 0.04$	0.1454	0.1085	0.1568	0.0869	0.1661	0.0732
	$\mu 1 = 0.1, \mu 11 = 0.05$	0.1511	0.1036	0.1622	0.0828	0.1716	0.0699

6. Conclusion

The Reliability, Availability and Maintainability (RAM) analysis of the time-dependent finite capacity Markovian Phase-Type queueing system with breakdown and recovery policies are studied in this paper. By using the differential-difference equations the transient system is obtained for the general case. As a special case $N = 4$ is formulated and are solved by using Fourth-Order Runge-Kutta numerical method. The Reliability, Availability and Maintainability are illustrated numerically and shown graphically. It is found that as time increases Reliability, Availability decreases but Maintainability initially increases after time period it decreases. The other metrics such as MTBF and MTTR were also calculated. By using Sensitivity Analysis, the Reliability, Availability and Maintainability graphs are obtained for different sets of values for Failure and Recovery rates. It is also carried out to find the values of $R(t)$, $A(t)$ and $M(t)$ for different sets of parameter values.

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S.R. Sruthi: Research Scholar, Department of Statistics, Presidency College (Autonomous), Chennai-05.
Email address: srsruthi1986@gmail.com

P.R.Jayashree: Assistant Professor, Department of Statistics, Presidency College (Autonomous), Chennai-05.
Email address: vprjaya@gmail.com