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OPTIMIZATION OF MULTI-OBJECTIVE, MULTI-STAGE STOCHASTIC TRANSPORTATIONS PROBLEM USING GOAL PROGRAMMING APPROACH

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Abstract. Stochastic multi-objective optimisation has been considered to be significant because of its direct relationship to real-world issues. This work analyses the multi-choice multi-objective transportation problem (MC-MOTP), in which supply an demand parameters are random variables with no preconceived values, and at least one of the objectives has many goal levels to be achieved. In the present work, firstly, to find the specific solution, present problem is transformed to an equivalent deterministic problem using chance-constrained programming approach. Secondly, multi-choice, multi-objective transportation problem (MCMOTP) can be reduced to a multi-objective transportation problem (MOTP) by implementing transformation method by means of binary variables. The reduced problem can be resolved using goal programming by choose from a variety of aspiration levels one to pursue each goal. This research demonstrates that an effective solution to the stochastic multi-objective programming problem was reached by application of two-phase approach. To explain the proposed methodology, a numerical example is provided.

Keywords: Multichoice multi-objective transportation problem, Chance constrained programming, Binary variables, Goal programming, Stochastic multi objective programming problem.

1. Introduction:

A specific kind of linear programming problem known as "transportation problem" (TP) deals with distribution of just one commodity from different supply sources to different demand points in a way that reduces the overall cost of transportation. Parameters of TP are the amounts that are needed at demand points and amounts that are available at the supply points. These parameters are not completely known and/or are not always predictable. The lack of accurate data contributes to these misconceptions. In the recent past, the stochastic programming model has taken into consideration normal, log-normal, and other random variables.

Kantorovich (1960) was the first person to investigate the transportation model and provided an incomplete approach for finding the answer. In 1941, Hitchcock took a first look at the issue of reducing those costs associated with distributing objects from several producers to multiple consumers. He created a process for solving the TPs that is very similar to Dantzig's primal simplex approach (1963). When random variables, as opposed to a deterministic scale, describe any or all of the optimisation problem's parameters, this is known as stochastic programming. sources and destinations, which change according to the kind and character of the problem, are the definition of random variables. In stochastic optimisation, decision-making issues occur when some of the model's optimisation coefficients are either unknown or unfixed. The values are arbitrary in these circumstances. Multi-objective stochastic optimisation techniques have grown in significant popularity in recent times, especially in the fields of technology, transportation, manufacturing, economics, and military applications.

The concepts and techniques for introducing stochastic variations to a mathematical programming problem are at the core of stochastic programming [23]. The parameters in a large number of realistic mathematical programming problems are understood as random variables. "Stochastic programming" is name given to the area of mathematical programming that focuses on the theory and techniques for solving conditional extremum problems when there is insufficient knowledge about the random parameters. The majority of applied mathematics problems can be classified into either of the following classes[24]:

1. Descriptive problems: such involve processing information about the event under investigation using mathematical techniques, with some rules of an event being adopted by others.
2. Optimization Problems where, Optimal solution is selected from a group of workable solutions.

In addition to the division mentioned above, deterministic, and stochastic problems can be further subdivided into applied mathematics problems. The stochastic problem has led to the development of numerous methods in mathematics. However, for a very long period, only descriptive problem categories were solved using probabilistic approaches. The last forty years have seen research on mathematical development of stochastic programming. It has been effectively applied to many kinds of management science real-world issues[26]. Converting the problem's probabilistic nature into an equivalent deterministic scenario is the fundamental concept behind all stochastic programming techniques[25]. According to Goicoechea et al.[13], three techniques for stochastic programming are developed. The two primary techniques are as follows:

a) Chance-constrained programming, proposed by Charnes and Cooper[7], Which has a finite probability of violation and can be applied to solve problems;
b) Two-stage programming, proposed by Dantzig and Infanger[8], which excludes the violation of any constraints.

Stochastic models have been applied when dealing with the probabilistic uncertainty in parameters for several years. A cutting-plane approach was presented by Abbas and Bellahcene[1]to resolve the stochastic integer linear programme with several objectives. Azaron et al[3] and Goh et al[12] looked at the use of stochastic models in supply chain network risk management. A fuzzy solution strategy for a multi-objective stochastic integer programming problem was discussed by Sakawa and Matsui[23]. In their suggested study, they took the basic recourse model into account. Han et al[15] investigated the multi-stage stochastic mixed integer programming model with interval parameters. The research presented an application to inter-basin water in Mathematical Modelling of Engineering Problems (2021), and took into account probabilistic restrictions to deal with the uncertainty. Körpeoğlu and colleagues[19] examined the problem of production scheduling. They used a multi-stage stochastic programming technique deal with scheduling problem. The stochastic programming approach has several uses in the domains of inventory management, production scheduling, and logistics have been discussed by Birge and Louveaux[4]. Under the value-at-risk criteria, Wang and Watada[30] discusses two-step fuzzy stochastic programming. The challenge may have multiple objectives, some of which may be in opposition to one another. For instance, reducing both the cost and the duration of shipping could be the goal. In this case, the two objectives are going in the same direction—minimization—but there is a cost. For instance, shipment by automobile may be less expensive than transporting them by air, but it will require a lot more time. In order to assist the decision maker (DM), goal programming is thus offered to define a multi-aspiration-level goal-programming transportation problem providing the aspiration levels of at least one target several options. The problem of transportation becomes a stochastic multi-aspiration-level goal-programming approach when random variation also affects the supply and demand factors. A multichoice stochastic transportation problem (MCSTP) model has been investigated by Mahapatra[20], where there is an extreme value dispersion in supply and demand dimensions of the constraints. Cost coefficients of different objective function are multichoice in nature. To ensure lowest possible transportation costs, the ideal system would calculate the quantity of units to be delivered while satisfying the needs of both the source and the destination. The present work aims to address the problem from a different perspective by including the concept of goal programming. This will make it possible the model to handle multiple conflicting objectives and establish several aspirations levels for specific goals. With an extreme value distribution, the most recent model transforms into stochastic multi-aspiration-level goal-programming approach based on TP. There are m sources and n destinations when it comes to transportation problems.Let $C_{ij}^{k} of$ the k^{th} objective function could represent the unit of trans potation cost for transporting the unit from $i^{th}(i = 1, 2, 3 \dots m)$ origin to $j^{th}(j = 1, 2, 3 \dots n) destination, x_{ij}$ is the quantity shipped from the i^{th} origin to j^{th} destination, a_i is supply available at origin i and b_j is destination j .In such a situation, all these parameters of all constraints are define as random variable. For a solution close to these uncertainties, a stochastic problem can be created by assuming that random variables have a specified distribution rather than fixed values. To implement the disjoint chance-constrained approach to change the constraints from probabilistic to deterministic, an extreme value distribution will be assumed in this case. When it is necessary to have a distribution that limits a sample of independent random variables to either their maximum or minimum with identical distributions, the extreme value distribution is selected. The extreme value distribution type I[10] probability density function looks like this: Gole programming is a multi-objective optimisation technique that is based on liner programming and is used when there are frequently conflicts between the distinct objectives. Each of these variables has an objective or target value that must be achieved. Significant variations from this configuration of desired values are subsequently reduced using a successful function. Depending on the requirements of the DM or the goal programming type that is used, it might be either a vector or a weighted sum. DM's objectives and their nature defines the kind of goal programming approach that is used. The original goal programming formulations allow for the prioritisation of reducing deviations of the more significant elements by ranking the unwanted deviations according to importance. Lexicographical or non-Archimedean goal programming is the term for this. We will present a novel solution to the transportation problem in this paper, where supply and demand parameter are extreme value-distributed random variables. We can reduce the shipping time, the risk of delivering the items, and the time it takes to arrive, compared to minimising the cost coefficient for the transportation problem. An additionally feature is that each target can have more than one aspiration level. The task now transforms from a multi-objective, multi-choice stochastic transportation problem. In order to get above this challenge, we will first convert the probabilistic restriction into a deterministic one using a stochastic technique. Second, the aspiration level for each target is chosen from a range of levels using a standard transformation composed from binary variables. After that, the reduced problem turns into a MOTP, which goal programming will be implemented to solve.

2. Mathematical Model:

First, the standard transportation problem is taken into considerations. The description of the transportation model is as follows if x_{ij} stands for the quantity transported from the source to the destination.

$$\min z = \sum_{i,j} C_{ij} x_{ij} \tag{2.1}$$

Find x_{ij} $i = 1, 2, 3, , , , m;$ $j = 1, 2, 3 \dots n,$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \qquad \forall i \tag{2.2}$$

$$\sum_{j=1}^{m} x_{ij} \geq b_i, \qquad \forall j \tag{2.3}$$

$$\sum_{i} a_i = \sum_{j} b_j, \tag{2.4}$$

$$x_{ij} \geq 0, \qquad \forall i, j \tag{2.5}$$

where x_{ij} is the amount delivered, a_i is the amount of supply at source i, b_j is the amount of demand at destination j, and C_{ij} is the transportation cost per unit and $x_i j$ is amount delivered[29].

We are now looking at the following mathematical model for a stochastic transportation problem with an extreme value distribution.

$$Lex\min\{n_i p_i\},$$

s.t

$$f_i(x)+n_i-p_i=g_1,g_2,...,g_q. \qquad q=1,2,...,k \tag{2.6}$$

$$P\left(\sum_{j=1}^{n} x_{ij}\le a_i\right)\ge 1-\mu_i, \qquad i=1,2,\ldots,m; 0\le\mu_i\le 1 \tag{2.7}$$

$$P\left(\sum_{j=1}^{m} x_{ij}\ge b_j\right)\ge 1-\rho_j,, \qquad j=1,2,\ldots,m; 0\le\rho_j\le 1 \tag{2.8}$$

$$x_{ij}\ge 0, \quad n_i,p_i\ge 0$$

$$\sum_i a_i=\sum_j b_j, \tag{2.9}$$

Where $f_i(x)$ is the linear function of i^{th} goal, g_i is the aspiration levels of the i^{th} goal, x_{ij} is the quantity of supply at source i, b_j is the amount of demand and destination j, n_i is the negative deviation variable, and p_i is the positive deviation variable. On the right side of the supply and demand limitations, there are three random situations were taken into consideration while transforming the probabilistic constraint into a deterministic constraint using Mahapatra's Disjoint[8], Chance-constrained Method.

(1) The extreme value distribution is only followed by $a_i,\quad i=1,2,\ldots,m$
(2) The extreme value distribution is only followed by $b_j,\quad j=1,2,\ldots,n$
(3) $b_j,\ j=1,2,\ldots,n$ and $a_i,\ i=1,2,\ldots,m$ both indicate an extreme value distribution.

2.1. The extreme value distribution is only followed by $a_i, i=1,2,\ldots,m$. According to this assumption, $a_i,\ i=1,2,\ldots m$ is an independent random variable with an extreme value distribution, while γ_i,δ_i and τ_i represent the location, scale, and form perimeters, respectively, with $\theta_i, 0\le\mu_i\le 1$ providing as the aspiration level. I remember the initial restrictions from the

$$P\left(\sum_{j=1}^{n} x_{ij}\le a_i\right)\ge 1-\mu_i, \qquad i=1,2,\ldots,m; 0\le\mu_i\le 1 \tag{2.10}$$

Or

$$P\left(\sum_{j=1}^{n} x_{ij}\ge a_i\right)\ge 1-\mu_i, \qquad i=1,2,\ldots,m; \tag{2.11}$$

The probability density function for $a_i, i=1,2,\ldots m$ is given by

$$f(a_i)=\frac{\tau_i}{\delta_i}\left(\frac{a_i-\gamma_i}{\delta_i}\right)^{\tau_i-1} e^{-\left[\left(\frac{a_i-\gamma_i}{\delta_i}\right)^{\tau_i}\right]}; \qquad a_i\ge\gamma_i,\gamma_i\in R \; and \; \delta_i\ge 0,\tau_i\ge 0. \tag{2.12}$$

Therefore, the probabilistic constraints can be expressed as

$$\int_{\gamma_i}^{\sum_{j=1}^{n} x_{ij}} f(a_i)\,d(a_i)\le\mu_i \tag{2.13}$$

The above integration can be expressed as

$$\int_{\gamma_i}^{\sum_{j=1}^{n} x_{ij}} \frac{\tau_i}{\delta_i}\left(\frac{a_i-\gamma_i}{\delta_i}\right)^{\tau_i-1} e^{-\left[\left(\frac{a_i-\gamma_i}{\delta_i}\right)^{\tau_i}\right]} d\left(a_i\right) \leq \mu_i \tag{2.14}$$

Let us assume

$$\left(\frac{a_i-\gamma_i}{\delta_i}\right)^{\tau_i} = \omega \tag{2.15}$$

The above integration can be expressed as

$$\int_{0}^{\left(\sum_{j=1}^{n} \frac{x_{ij}-\gamma_i}{\delta_i}\right)^{\tau_i}} e^{-z} d\left(z\right) \leq \ \mu_i \tag{2.16}$$

It can be integrated as;

$$\left[e^{-z}\right]_{0}^{\left(\sum_{j=1}^{n} \frac{x_{ij}-\gamma_i}{\delta_i}\right)^{\tau_i}} \leq \ \mu_i \tag{2.17}$$

On rearrange

$$e^{-\left(\sum_{j=1}^{n} \frac{x_{ij}-\gamma_i}{\delta_i}\right)^{\tau_i}} \geq 1-\mu_i \tag{2.18}$$

Tacking logarithm both sides, I have

$$-\left(\sum_{j=1}^{n} \frac{x_{ij}-\gamma_i}{\delta_i}\right)^{\tau_i} \geq \left[\ln\left(1-\mu_i\right)\right] \tag{2.19}$$

After simply and rearrange, I get

$$\sum_{j=1}^{n} x_{ij} \leq \gamma_i \leq \delta_i[\ln\{-ln(1-\mu_i)\}] \tag{2.20}$$

Therefore, the final probabilistic constraints can be converted into deterministic liner constraints in the following ways;

$$\sum_{j=1}^{n} x_{ij} \leq \gamma_i \leq \delta_i[\ln\{-ln(1-\mu_i)\}] \tag{2.21}$$

Thus, I have obtained a multichoice deterministic model

$$Min; \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{C_{ij}^{1}C_{ij}^{2},\ldots,C_{ij}^{p}\}, \qquad p = 1,2,3,\ldots p \tag{2.22}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq \gamma_i \leq \delta_i[\ln\{-\ln(1-\mu_i)\}] \tag{2.23}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_i, \qquad j = 1,2,3,\ldots n \tag{2.24}$$

$$x_{ij} \geq 0, \qquad \forall\ i\ and\ j$$

Where

$$\sum_{j=1}^{n} \gamma_i - \delta_i\{-\ln(1-\mu_i)\}\frac{1}{\tau_i} \geq \sum_{j=1}^{m} b_j \qquad (\textit{feasibility condition}\ ) \tag{2.25}$$

2.2. The extreme value distribution is only followed by $\mathbf{b}_j,\ j = \mathbf{1}, \mathbf{2}, \ldots, n$. It is generally accepted that $b_j, j = 1, 2, \ldots, n$ are independent random variable this has an extreme value distribution with $\gamma_j, 0 \leq \gamma_j \leq 1$ as the aspiration level and $\gamma'_j \delta'_j, \tau'_j$, and $\gamma_j, 0 \leq \gamma_j \leq 1$ as the location, scale, and shape parameters, respectively.I reformulate the constraints of Model 1 (2.4) as

$$P\left(\sum_{j=1}^{m} x_{ij} \geq b_j\right) \geq 1 - \rho_j, \qquad j = 1, 2, \ldots, m \tag{2.26}$$

The probability distribution function of $b_j,\ j = 1, 2, \ldots, n$ is given by

$$f(b_j) = \frac{\tau'_j}{\delta'_j}\left(\frac{b_j - \gamma'_j}{\delta'_j}\right)^{\tau'_j - 1} e^{-\left[\left(\frac{a_i - \gamma'_j}{\delta'_j}\right)^{\tau'_j}\right]}; \qquad b_j \geq \gamma'_j, \gamma'_j \in R and \delta'_j \geq 0, \tau'_j \geq 0. \tag{2.27}$$

Hence the probabilistic constraints can be present as

$$\int_{\gamma'_j}^{\sum_{j=1}^{n} x_{ij}} f(\ b_j)\, d(\ b_j) \geq 1 - \varphi_j \tag{2.28}$$

In a similar manner, a deterministic liner constraint may be created from the probabilistic constraints (2.4) as,

$$\sum_{j=1}^{m} x_{ij} \geq \gamma'_j - \delta'_j[\ln\{-\ln(\varphi_j)\}] \tag{2.29}$$

As a result, I have the following multi-choice deterministic model:

$$Min; \qquad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{1} C_{ij}^{2}, \ldots, C_{ij}^{p}, \qquad p = 1, 2, 3, \ldots p. \tag{2.30}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \qquad i = 1, 2, 3, \ldots m \tag{2.31}$$

$$\sum_{j=1}^{m} x_{ij} \geq \gamma'_j - \delta'_j[\ln\{-\ln(\varphi_j)\}] \tag{2.32}$$

$$x_{ij} \geq 0, \quad \forall\ i\ and\ j$$

Where

$$\sum_{j=1}^{m} a_i \geq \sum_{j=1}^{m} \gamma'_j - \delta'_j[\ln\{-\ln(\varphi_j)\}] \qquad (\textit{feasibility}\ \ \textit{condition}) \tag{2.33}$$

2.3. $b_j, j = 1, 2, \ldots n$ **and** $a_i, i = 1, 2, \ldots m$ **both indicate an extreme value distribution.**
Here, both $a_i, i = 1, 2, \ldots m$ and $b_j, j = 1, 2, \ldots n$ follows extreme value distribution. Model 1 may be modified to: Model 4:

$$Min; \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{1} C_{ij}^{2}, \ldots, C_{ij}^{p}, \qquad p = 1, 2, 3, \ldots p \tag{2.34}$$

Subject to

$$\sum_{j=1}^{m} x_{ij} \leq \gamma_j \leq \delta_j [\ln\{-\ln(\varphi_j)\}] \tag{2.35}$$

$$\sum_{j=1}^{m} x_{ij} \geq \gamma_j' - \delta_j' [\ln\{-\ln(\varphi_j)\}] \tag{2.36}$$

$$x_{ij} \geq 0. \qquad \forall\ i\ and\ j$$

Where

$$\sum_{j=1}^{n} \gamma_j - \delta_j [\ln\{-\ln(\varphi_j)\}] \geq \sum_{j=1}^{m} \gamma_j' - \delta_j' [\ln\{-\ln(\varphi_j)\}] \ \ (feasibility\ condition) \tag{2.37}$$

3. Conversion of the Goal restriction with-aspiration levels into an equivalent form

Having several aspiration levels and a goal constraint to consider

$$f_i(x) + n_i - p_i = g_{1,} g_{2,\ldots,} g_{q,}, \tag{3.1}$$

In multichoice programming, a binary variable is a fundamental idea for selecting a single option from a group of multiple choices. I have developed the intended model in this research by utilising the auxiliary constraints that hold the binary variables. The function $\frac{\ln(p)}{\ln(2)}$, where p indicate number of choices, determines how many binary variables are present for each choice[6]. An additional constraint is not required to design an over-purpose model for $p = 2, 4, or 8$. For the building of our model, just one auxiliary constraint related to binary variables is required for $p = 3, 7, or 11$. Two auxiliary constraints are required when $p = 6 or 10$, respectively. Three auxiliary restrictions are required when $p = 5 or 9$, respectively. There are four auxiliary constraints required when p equals 12. I get various models of my suggested problem and solve each model based on the total number of options or objectives. Let $x = z_i z_j$, where x satisfy the subsequent inequality,

$$(z_i + z_j - 2) + 1 \leq x \leq (2 - z_i - z_j) + 1 \tag{3.2}$$

$$x \leq z_i, \tag{3.3}$$

$$x \leq z_j \tag{3.4}$$

$$x \geq 0 \tag{3.5}$$

The inequalities are identified:

(1) If $z_i = z_j = 1$ then $x = 1$ from (2.39)
(2) If $z_i z_j = 0$ then $x = 0$ from (2.40 to 2.42)

Therefore, the novel goal restriction will be

$$f_i\left(x\right) + n_i - p_i = \sum_{j=1}^{n} g_{ij} S_{ij}\left(B\right) \tag{3.6}$$

Here the binary serial number's function is represented by $S_{ij}\left(B\right)$. This is an extreme value distribution model goal programming problem that is stochastic and includes multiple stages.

$$Lex \min \{n_i p_i\}$$

s.t

$$\begin{gathered} f_i\left(x\right) + n_i - p_i = \textstyle\sum_{j=1}^{n} g_{ij} S_{ij}\left(B\right) \\ \textstyle\sum_{j=1}^{n} x_{ij} \leq \gamma_i \leq \delta_i [\ln\{-ln(1-\mu_i)\}] \\ \textstyle\sum_{j=1}^{m} x_{ij} \geq \gamma'_j + \delta'_j [\ln\{-\ln\left(\varphi_j\right)\}] \\ x_{ij} \geq 0, \qquad \forall\ i\ and\ j \end{gathered}$$

Where

$$\sum_{j=1}^{n} \gamma_i - \delta_i [\ln\{-ln(1-\mu_i)\}] \geq \sum_{j=1}^{m} \gamma'_j - \delta'_j [\ln\{-\ln\left(\varphi_j\right)\}] \tag{3.7}$$

4. Case Study:

The multi-choice stochastic transportation problem (TP) is represented numerically in example, where cost coefficients of objective have multiple options, and the supply and demand are represented by binary variables. I'll give an example of tea TP here for showing how the approach is applied. Tea is one of the refreshments that gives people in human society more energy. As a result, one of the most important challenges in the transportation of tea is how to do it economically from the bottom of the hills to the various locations. In Darjeeling, West Bengal, India, at the base of the hills, a reputable Tea is transported by a tea supply firm (Darjeeling Black Tea and Darjeeling Green Tea are internationally recognised for their quality and fragrance) from three supply points—Simulbari, Makaibari, and Happyvalley— via 12 routes to the four destination hubs in India: Delhi, Mumbai, Rajasthan, Kolkata. The primary goals are to maximise profit relative to market price at various marketplaces and decrease transportation costs. The total cost of shipping a single unit (25 kg) of tea from its source to its destination is broken down into multiple-choice factors. The problem is not frequently solved if the multi-choice programming technique is not used. The price rates for the transport expenses in each route are attached below due to increases in the tax on road collecting and fuel prices.

The following objectives are being pursued by the tea transportation supply company: reducing transportation costs and time invested in transportation are the first two. The goals are 30,00 to 37,00 Units and 120 or 130 Hours, respectively. Table 1 takes into account the cost coefficient C_{ij} and transportation time cost t_{ij} from each source to each destination. A stochastic multi-aspiration level goal programming goal TP technique has been considered established on variability of the factor given above, where in the supply and demand parameters are distributed according to an extreme value. Specified probability levels and the supply's shape and scale characteristics are listed in Table 2, and the demand's form and scale parameters are listed in Table 3, along with designated probability levels.

Table 1. The transportation cost coefficient (C_{ij}) and time cost (t_{ij}) for each source to each destination

No.	Route x_{ij}	Transportation time cost t_{ij} (hours)	Cost coefficient C_{ij}
1	(1,1): x_{11}	17	26
2	(1,2): x_{12}	20	30
3	(1,3): x_{13}	25	35
4	(1,4): x_{14}	29	39
5	(2,1): x_{21}	21	32
6	(2,2): x_{22}	23	33
7	(2,3): x_{23}	14	20
8	(2,4): x_{24}	22	31
9	(3,1): x_{31}	29	39
10	(3,2): x_{32}	17	29
11	(3,3): x_{33}	30	42
12	(3,4): x_{34}	33	46

Table 2. Location and scale parameter values with $a_i's$ SPL

Shape Parameter	**Scale Parameter**	**SPL (μ)**
$\gamma_1 = 250$	$\delta_1 = 5.8$	$\mu_1 = 0.092$
$\gamma_2 = 200$	$\delta_2 = 6.4$	$\mu_2 = 0.087$
$\gamma_3 = 150$	$\delta_3 = 7.2$	$\mu_3 = 0.074$

TABLE 3. : Location value and scale parameter using $b_i's$ SPL

Shape Parameter	Scale Parameter	SPL (ϕ)
$\gamma_1' = 200$	$\delta_1' = 6.2$	$\phi_1 = 0.081$
$\gamma_2' = 150$	$\delta_2' = 6.7$	$\phi_2 = 0.098$
$\gamma_3' = 120$	$\delta_3' = 7.8$	$\phi_3 = 0.074$
$\gamma_4' = 100$	$\delta_3' = 8.2$	$\phi_3 = 0.93$

$$Lex\ min\{p_1 p_2\}$$

s.t

$$\sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij} + f_i(x) + n_i - p_i = 120z_1 + 130z_1$$

$$\sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij} + f_i(x) + n_i - p_i = 30000z_2 + 35000z_2$$

$$\sum_{i=1}^{4} x_{1j} \leq 256.974$$

$$\sum_{i=1}^{4} x_{2j} \leq 209.324$$

$$\sum_{i=1}^{4} x_{3j} \leq 159.078$$

$$\sum_{j=1}^{3} x_{i1} \geq 207.857$$

$$\sum_{j=1}^{3} x_{i2} \geq 157.844$$

$$\sum_{j=1}^{3} x_{i3} \geq 129.035$$

$$\sum_{j=1}^{3} x_{i4} \geq 109.68$$

$x_{ij},\ n_q,\ d_q \geq 0,\ i = 1,2,3,\ j = 1,2,3,4,\ q = 1,2,\ z_k = 0$ or $1,\ k = 1,2.$
Checking the feasibility condition is satisfied

$$\sum_{j=1}^{n}(\gamma_i - \delta_i[\ln(-\ln(1-\mu_i))]) = 621.258 \geq \sum_{j=1}^{m}(\gamma_j' - \delta_j'[\ln(-\ln(\phi_j))]) = 598.98 \quad (4.1)$$

Next, using GAMS (Software), the deterministic liner mix integer issue is solved, and the optimum solution is

$$x_{12} = 341.851$$
$$x_{13} = 554.254$$
$$x_{14} = 465.128$$
$$x_{22} = 925.842$$
$$x_{23} = 358.685$$
$$p_1 = 0$$
$$p_2 = 0$$
$$n_1 = 245.056$$
$$n_2 = 356.086$$
Where $z_1 = 0$, and $z_2 = 0$.

There are zero remaining decision variables. The outcomes showed that goal 1's aspiration level was exactly reached by transportation time costs, with a zero positive deviation and aspersion level of 130 hours. Goal 2's aspiration level was precisely reached by transportation costs, with a zero positive deviation and aspiration level of 3500 units.

5. Conclusion:

This paper investigates an issue with stochastic supply and demand parameters that express an extreme value distribution. I have employed three distinct strategies, including the chance-constrained programming approach, to convert random variable problem into an equivalent deterministic problem to determine individual solution. Secondly, applying binary variables that reduce the MCMOTP into a MOTP, for every goal, choose one aspiration level from a range of levels. Each choice's binary variables are determined by the function $\frac{\ln(p)}{\ln(2)}$, where p indicate total number of choices. Following the introduction, a mixed integer programming problem is created by converting the auxiliary and additional constraints into terms of binary variables with multi-choice parameters, and finally, to get an optimal solution, we have used the goal programming approach. We can reduce the delivery time, the risk of sending the items, and so on compared to minimising the cost coefficient for the transformation problem. At the end of the I have come to the conclusion that the developed model is very applicable to TP in real life, and that by solving the model, the decision maker has added more information to help them make the best choice. Further investigation is required to compare the performance of the models using multi-objective strategies like the fuzzy programming method, weighting method, and ϵ -constant method.

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