Global and Stochastic Analysis
Vol. 8 No. 3 (December, 2021)
Special Issue: Modern Stochastic Models and Problems of Actuarial Mathematics

METAHEURISTICS IN PARAMETRIC PROCESS DESIGN: LASER CUTTING STUDY ANALYSIS

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Abstract. This paper aims at highlighting and addressing the major issues in applying metaheuristic algorithms (MAs) in parametric design of manufacturing processes. Findings from a comprehensive literature review on MAs application in optimising manufacturing processes are presented, including their comparisons and major concerns. The soft computing-based methodology is proposed and applied on the laser cutting process, to depict its benefits and performance of the four tested MAs: genetic algorithm (GA), simulated annealing (SA), particle swarm optimisation (PSO) and teaching-learning based optimisation (TLBO). These algorithms were benchmarked in terms of solution accuracy, speed of convergence and sensitivity of the algorithm to its own hyper- parameters tuning. The concluding remarks were drawn, followed by recommendations for future activities and applications

1. Introduction

An engineering process is affected by three major types of parameters [1]: (i) control parameters used to manage a process execution; (ii) signal factors have a very high and direct effect on a process response, but they cannot be easily identified for a vast majority of processes, (iii) noise factors negatively affect a process performance, causing a process response deviation from the target value that leads to a quality loss. The parametric process design aims at finding an optimal set of process control factors that produces the desired response mean value and minimise the response variation, for all responses of a process, subjected to several constraints. Due to an increased dynamicity, modern processes have become very complex, involving a large number of parameters and responses, with highly non-linearity and unknown interdependencies. To tackle such complexity, the soft computing techniques play a major role in modelling and parametric optimisation of contemporary processes. The machine learning techniques, such as regression modelling and artificial neural networks (ANNs) are typically used to model the input-output process interdependences. Knowing the desired values of process responses, a process need to be set in such a way to produce the desired outputs with minimal variation, i.e. an optimal set of process control parameters needs to be found. MAs are efficient in addressing this this task. Since novel

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2000 *Mathematics Subject Classification.* Primary 68T20; Secondary 68W50.
Key words and phrases. Teaching-Learning Based Optimisation (TLBO); Particle Swarm Optimisation (PSO); Genetic Algorithm (GA); Simulated Annealing (SA); Artificial Neural Networks (ANNs); Parametric Process Design.

processes in Industry 4.0 environment are controlled by multiple parameters, affected by noise factors and producing multiple correlated responses, the parametric process design nowadays is a very demanding task

2. Metaheuristic Algorithms

Metaheuristic algorithms (MAs) could be divided into two major group: (i) point-based algorithms, such as SA algorithm, and (ii) evolutionary algorithms (EAs) that are population-based metaheuristics motivated by natural evolutionary mechanisms, such as GA, PSO, ant colony optimisation (ACO), artificial bee colony (ABC), etc.

In GA, an initial population is formed from n chromosomes whose objective values are evaluated to find the best ones for the next iteration. A new population is made using genetic operators: scaling, parent chromosome selection, crossover of two parents, mutation and migration that defines how exactly offspring are produced. Objectives of new chromosomes are evaluated, and procedure continues until a halting condition is satisfied, e.g. a predefined number of iteration [2].

SA is a point-based MA, mimicking a natural phenomenon of thermodynamics in a metal annealing. Starting with an initial point and temperature, SA arbitrary generates a new point whose distance from the previous one is determined by an annealing function and current temperature. After objective value assessment, a point is adopted according to a probability of acceptance function, based on a discrepancy between new and old objective, initial temperature and current temperature (controlled by the temperature function). Once a specified number of points is adopted, reannealing is applied, controlled by a reannealing interval. The procedure continues until a stopping condition is achieved [3].

Swarm intelligence is a subset of EAs, and PSO is its main representative. PSO is based on a social behaviour in a swarm, where each individual particle is presented as a vector with position and velocity. After initialisation with an initial swarm of n particles, particle positions and velocities are altered during a movement, based on the best location of a particle (pbest) and of a swarm (gbest) reached so far. A new velocity is determined based on the previous one, pbest and gbest, and three major parameters: inertia weight (w), self-adjustment learning factor (c_1) and social learning factor (c_2). A new particle position is computed using the previous one and a new velocity. Once the objective function is evaluated for all particles, the swarm is renewed and the procedure repeats until a completion condition is met [4].

The main criticism of MAs is the necessity of a proper hyper-parameters selection to avoid a premature convergence and to reach a global optimum in a reasonable time [5]. This especially refers to the algorithm specific parameters. Besides, point-based MAs, such as SA, could be very slow and might not embrace a whole search space. In overall, since MAs do not reproduce identical results even with the same settings, the algorithm hyper-parameters tuning is of pivotal importance.

This issue has been resolved in some of the recently developed EAs, such as the TLBO algorithm mimicking the teaching-learning process in a classroom. After population (learners) and variables (subjects) initialisation, students are learning from the teacher. The learners grades (objective function) are assessed; the new solutions, i.e. improved students, are adopted if they are better than the previous ones. Then, students are interacting with each other to further improve their knowledge (grades), and then their grades are evaluated. Students with the highest grades are kept in a population. This iterative process continues until the termination condition is met. It is important to notice that there are no specific parameters to be tuned, except for two parameters common for all MAs: population size and number of iterations [6].

3. Metaheuristic Algorithms in Parametric Process Design

To tackle the complexity of modern processes, in recent years the parametric design has been frequently addressed by the soft computing techniques, and, in particular, the MAs that relay on the stochastic optimisation principles. This section presents findings of the recent comprehensive literature reviews; a few hundred of selected studies from highly reputable journals in soft computing, industrial and manufacturing domains have been studied, focusing on the papers that reported a benchmark of different algorithms applied to the same problem [7-8].

The analysis was organised into two main groups: (i) known analytical process model; (ii) unknown process model. Optimisation of the modern, emerging processes belongs to the latter case, where feed forward (FF) ANNs with error back propagation (BP) were the most regularly utilised to map dependence between process parameters and outputs, followed by regression modelling (mainly quadratic models were employed).

MA performances were assessed in terms of solution accuracy and convergence speed, since these two criteria were mainly reported in the reviewed papers. The following MAs were included since their benchmarks were noticed in three or more studies: GA, SA PSO, ACO, ABC, Hoopoe Heuristic (HH), harmony search (HS), scatter search (SS), TLBO and cuckoo search (CS). Certain conclusions could be drawn from a larger sample of studies (more than ten studies) that reported comparisons: PSO scored better than SA, GA and ABC for both criteria; SA scored better than GA in terms of the solution quality. Some observations could be listed based on a smaller number of comparisons (three to ten studies): TLBO and CS seem to be the most successful algorithms in terms of solution accuracy and, partly, convergence speed, followed by PSO that showed beneficial results in comparison to the others (ACO, HH, HS, SS).

In overall, EAs were efficient in parametric design of manufacturing processes. However, the following issues have been identified: (i) an incomplete understanding of the parametric process design since majority of studies focused only on a response mean value ignoring a response variability, which is a major concern since variability is a main cause of real industrial problems; (ii) for multi-response processes, development of a single objective function was mainly performed by subjectively assigning weights to individual objectives; (iii) the Pareto front-based algorithms are effective in dealing with two or maximum three objectives, but their application on larger problems requires subjective dealing with trade-offs among multiple fronts; (iv) effects of the MA own settings on the algorithm results have not been assessed, which is problematic viz. repeatability and scalability of the algorithm results [8].

4. Intelligent, Soft Computing-Based Methodology for Parametric Process Design

An intelligent methodology for parametric process design has been developed, based on ANNs and MAs, used to map and optimise processes, respectively. The designed experimentation is used to collect input (process parameters) output (process responses) data, that are pre-processed as follows [9]. The responses are expressed via the Taguchis quality loss function (QL) that adequately encloses both the response mean and variability, and then normalised (NQL). Details of the Taguchi parametric design could be found in [1]. The correlated NQLs are transformed into independent components by means of principal component analysis (PCA). Using grey relational analysis (GRA), all independent components are combined into a single measure based on their contributions from PCA. Details of PCA could be found in [10], and of GRA in [11]. Hence, an integrated process performance measure (PPM [0,1]) is developed in an entirely objective fashion (the higher PPM, the better is the process). The process parameter effects on PPM are calculated, and the parameter values that maximise PMP are adopted as a potentially good solution.

The ANNs are employed to establish the relationship between the process parameters and PPM. To obtain the best model, ANNs activation function, learning rate and momentum are carefully tuned, and diverse topologies (number of neurones in a hidden layer) are tested. The best ANN is the one with minimal mean square error (MSE) and maximal correlation between the network predictions and the original data (R). The selected model is used as the objective function for MAs to find the optimal process parameters that maximise PPM.

The four MAs are studied with diverse algorithm-specific settings. In GA, the following parameters are used: initialisation with a potentially favourable option (result of the pre-processing method); rank scaling; adaptive feasible mutation; forward migration with fraction 0.2. Three types of selection (stochastic uniform, roulette wheel, tournament) and crossover functions (single point, two points, arithmetic) are tested. In total, nine GAs are generated for each problem. SA algorithms are initialised with a possibly good solution; the other parameters are varied: (i) initial temperature: $10, 100$ and 500; (ii) temperature function: exponential, fast and Boltzmann; (iii) annealing function: fast and Boltzmann; (iv) reannealing interval: 10 and 100. Therefore, 36 SA algorithms are developed for each use case. For PSO, both types of initialisation are analysed: random and initialisation with a favourable solution; the other parameters are varied: (i) inertia weight range: $[0.1; 1.1], [0.4; 0.9], [0.5; 2.5], [1.0.; 5.0]$; (ii) learning factors: $c_1 = c_2 = 0.1; c_1 = c_2 = 0.5; c_1 = c_2 = 2.0; c_1 = c_2 = 5$, and $c_1 = 0.7, c_2 = 1.5$. In total, 16 randomly initialised PSOs and 16 PSOs with a potentially good initialisation are generated for each case. The TLBO algorithm does not require any specific settings.

For all algorithms, the population size equals $5n$, where n is the number of process control parameters. As stopping conditions, 2000 iterations or change in the objective function less than 10^{-9} over 100 iterations are adopted. For each MA, the best algorithm is selected according to the maximal PPM value, and its result is adopted as the optimal process parameters set. The algorithms are benchmarked according to: (i) solution accuracy (PPM value and corresponding process parameters set); (ii) speed of convergence (number of iterations needed to reach optimum); (iii) sensitivity to the algorithm-specific settings (range of PPM and of optimal process parameter values).

5. Case Study: Laser Cutting Process

An experiment was conducted to estimate the influence of control parameters of the Nd:YAG laser cutting in processing Nimonic 263 sheets. The four process parameters were considered as controllable variables, and they were studied at three levels. Therefore, the experiment was designed using orthogonal array L9 containing nine trials, which were repeated twice. At the output, seven characteristics were considered as responses: six responses need to be minimised, and the remaining one is to be maximised. For each trail, responses were measured three times (the sample size equals three). Details could be found in [12].

For each response, the response data presented in Table 1 are converted into the respective QL values that were normalised (NQLs). The PCA was applied on the NQLs to produce a set of uncorrelated components (Table 1):

$$\begin{gathered}
\mathrm{Y}_1(k) = 0.459 \cdot \ \mathrm{NQL}_{\mathrm{Kd}}(k) + 0.396 \cdot \ \mathrm{NQL}_{\mathrm{Kt}}(k) - \\
-0.202 \cdot \ \mathrm{NQL}_{\mathrm{HV}}(k) + 0.315 \cdot \mathrm{NQL}_{\mathrm{G}}(k) + 0.481 \cdot \ \mathrm{NQL}_{\mathrm{Ra}}(k) + \\
+0.469 \cdot \ \mathrm{NQL}_{Rms}(k) + 0.200 \cdot \ \mathrm{NQL}_{\mathrm{PV}}(k) \\
\cdots\cdots\cdots\cdots\cdots \\
\mathrm{Y}_7(k) = -0.194 \cdot \ \mathrm{NQL}_{\mathrm{Kd}}(k) - 0.228 \cdot \ \mathrm{NQL}_{\mathrm{Kt}}(k) - \\
-0.103 \cdot \ \mathrm{NQL}_{\mathrm{HV}}(k) + 0.020 \cdot \mathrm{NQL}_{\mathrm{G}}(k) + 0.818 \cdot \ \mathrm{NQL}_{\mathrm{Ra}}(k) - \\
-0.443 \cdot \ \mathrm{NQL}_{Rms}(k) - 0.046 \cdot \ \mathrm{NQL}_{\mathrm{PV}}(k)
\end{gathered} \tag{1}$$

The GRA was applied over the above components to integrate them into the process performance measure (PPM), i.e. the grey relational grade (Table 1), in respect to their proportions of contribution from PCA: 0.561; 0.188; 0.112; 0.089; 0.03; 0.017; 0.003. The effects of process control parameters on the PPM were calculated, and the parameter values that maximise PPM were suggested as a potential solution from the space of discrete solutions considered in the experiment: $Np = 12; f = 3; P = 2100; v = 4000$. In the optimisation stage, this set is used for initialisation of GA, SA and PSO.

The feed-forward back-propagation ANNs were utilised to model the relation between the process control parameters, at the input, and the PPM, at the output. Among several topologies, the network with 14 neurones in a hidden layer demonstrated the best performance: MSE equals $1.5 \cdot 10^{-6}$; R equals 0.99 [12].

Such an excellent model served as the objective function for four MAs whose results were benchmarked (Table 2). GA, SA and PSO were run with different own hyper-parameter settings. Since TLBO does not have specific hyper-parameters, it was run five times to assess its repeatability. The following conclusions could be drawn:

Trial no.	Process control parameters				Process responses							Principal components $Y_j(k)$ ($j = 1, \ldots 7; k = 1, \ldots 18$)							PPM_k ($k =$
	Np	f	P	v	Kd	Kt	HV	G	Ra	Rms	PV	$Y_1(k)$	$Y_2(k)$	$Y_3(k)$	$Y_4(k)$	$Y_5(k)$	$Y_6(k)$	$Y_7(k)$	$1, \ldots 18$
1	4	1	1400	4000	0.063	1.13	235.2	0	8.5	10.7	104.9	0.51	−0.57	−0.31	0.04	0.24	0.06	−0.11	0.6974
2	4	2	2100	4500	0.057	0.97	209.7	0	8.6	10.7	106.3	0.25	−0.80	−0.65	−0.40	0.22	−0.20	−0.09	0.6932
3	4	3	2800	5000	0.077	1.20	219.4	0	9.5	11.4	190.6	0.75	−0.38	−0.82	−0.46	0.61	0.09	−0.15	0.5799
4	8	1	2100	5000	0.047	0.90	236.3	1	5.9	7.4	159.6	0.52	−0.11	0.38	−0.76	0.57	−0.18	−0.10	0.7042
…																			
18	12	3	2100	4000	0.058	0.91	231.0	0	5.3	6.3	107.0	0.09	−0.40	−0.22	−0.15	0.18	0.10	−0.14	0.8672

Table 1. A part of experimental design with controllable process parameters and measured responses, and results of PCA and GRA [12]

- The maximal PPM value found by GA and SA algorithms is lower than the one found by PSO and TLBO. Therefore, PSO and TLBO overperformed the remaining two algorithms in terms of the solution accuracy.
- The GA, run with different hyper-parameters, showed the largest dispersion of PPM and optimal set, followed by SA. The PSO algorithm, both randomly initialised and initialised with a potentially good solution, showed very good robustness due to very narrow ranges of PPMs and the optimal process parameter values. The TLBO algorithm is designed as a robust one. Here, TLBO showed excellent repeatability, since it generated identical results in five runs.
- The SA algorithm showed the slowest convergence, which could have been expected since it performs a point-to-point search (i.e. it is not a population-based algorithm), contrary to the remaining three algorithms. The PSO initialised with a potentially good solution needed minimal number of iteration to find the optimum, followed by GA and randomly initialised PSO. The TLBO convergence rate was slightly lower than the GA and PSO rate.

In overall, it could be seen that PSO demonstrated better accuracy and significantly better robustness and convergence rate than GA and SA. Seeding initial swarm with a potentially favourable solution did not significantly affect the overall algorithm performance, except of a small improvement in the convergence rate. This proves that PSO is less affected by its own settings than GA and SA. The TLBO algorithm showed remarkable repeatability, along with an implied robustness. Its convergence is slightly slower but comparable to PSO. Figure 1 shows the convergence along iterations for both versions of the PSO algorithm and for the TLBO algorithm

6. Concluding Remarks

In a contemporary industrial environment, the soft computing techniques have emerged as effective tools to address complexity of the parametric design of modern processes. The ANNs are typically applied to map the process whose empirical model is unknown, based on which the MAs are applied to find the optimal process parameters setting that delivers the required process responses. An extensive literature analysis on the MAs effectiveness in optimising industrial processes indicated that GA was the most frequently used MA, followed by SA and PSO, and

Algorithm performance	GA	SA	PSO initialised randomly	PSO initialised by good solution	TLBO
Range of PPM values	0.8754÷0.9007	0.8909÷0.9007	0.8940÷0.9008	0.8937÷0.9008	0.9008
Range of optimal process parameter values	[12÷14; 2.4÷3; 1920÷2095; 4000÷4047]	[13.7÷14; 2.5÷3; 1932÷2063; 4000÷4025]	[14; 2.5÷3; 1953÷2100; 4000]	[14; 2.6÷3; 2034÷2101; 4000]	[14; 3; 2034; 4000]
Maximal PPM	0.9007	0.9007	0.9008	0.9008	0.9008
Optimal process parameters corresponding to max. PPM	[14; 3; 2039; 4000]	[14; 3; 2039; 4000]	[14; 3; 2034; 4000]	[14; 3; 2034; 4000]	[14; 3; 2034; 4000]
Range of number of iterations needed to find optimum	19÷750	30÷1500	23÷70	6÷30	47
Number of iterations needed to find max. PPM	22	400	23	6	47

Table 2. Summary of GA, SA and PSO results with diverse settings and TLBO results

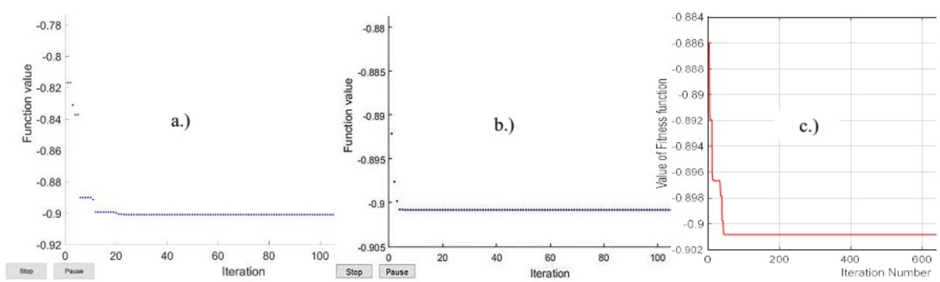

Fig.1. The algorithm convergence vs. iterations: a.) PSO with a random initialisation; b.) PSO initialised with a potentially good solution; c.) TLBO

that PSO showed better performance that the other MAs except for the TLBO and CS algorithms. The literature review also revealed several issues that were presented in the paper.

The above issues are effectively resolved in the suggested soft computing-based methodology. Comparison of the four MAs in the observed study indicated superior performance of the TLBO, followed by PSO. These results are aligned with the findings from a comprehensive literature review. Based on these findings and taking into account the embedded algorithm robustness and excellent repeatability, the TLBO algorithm appears as a favourable toll, in terms of applicability and effectiveness, for tackling similar optimisation problems. Besides, it has to be noted that a shallow ANN performed an excellent process modelling using very small data set, showing remarkable mapping accuracy. This is mainly due to preprocessing method, implying data normalisation and standardisation prior to their integration into a single measure in a fully objective manner.

Future research will include comparison with the CS algorithm and a few recently developed EAs that show beneficial characteristics in terms of robustness.

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