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TRUNCATION OF PICTURE FUZZY SOFT GRAPH

M. VIJAYA AND D. HEMA

Abstract. The purpose of this paper is to place forward to the notion of picture fuzzy soft graph (PFSG). In this paper, we introduced picture fuzzy soft graph, lower and upper truncation of picture fuzzy soft graph. Also studied truncation of subdivision picture fuzzy soft graph and truncation of strong picture fuzzy soft graph.

1. Introduction

The origin of the graph theory started with the Konigsberg bridge problem in 1735. This problem led to the concept of the Eulerian graph. Euler studied the Konigsberg bridge problem and constructed a structure that solves the problem that is referred to as an Eulerian graph. The concept of fuzzy set theory was introduced by Zadeh [9] to solve difficulties in dealing with uncertainties. Since then the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real life problems involving ambiguous and uncertain environment.

Nagoorgani. A and Malarvizhi. J [8] described the truncation of fuzzy graph. The concept of fuzzy graph was introduced by Mordeson and Nair [7]. Muhammad A kram and Sairam Nawaz [3] introduced soft graph. Maji,Biswas and Roy [6] proposed soft set. [5] introduced graph theory. Cuong and Kreinovich [4] proposed the concept of picture fuzzy set which is a modified version of fuzzy set and Intuitionistic fuzzy set.

Cen Zuo [10] introduced picture fuzzy graph. A kram and Davvaz [1] discussed Strong intuitionistic fuzzy graphs. [2] introduced many new concepts, including soft graphs, fuzzy soft graphs and operations on fuzzy soft graphs, during this paper, we introduced picture fuzzy soft graph, lower and upper truncation of picture fuzzy soft graph. Also studied truncation of subdivision picture fuzzy soft graph and truncation of strong picture fuzzy soft graph.

2. Preliminaries

Definition 2.1. If Z is a collection of object (or element) bestowed by z. Then fuzzy set [9] A' in Z is expressed as a set of ordered pair.

$$A' = \{(Z, \lambda_{A'}(z)) : z \in Z\}$$

where, $\lambda_{A'}(z)$ is called the membership function (or characteristic function) which maps Z to the closed interval $[0, 1]$.

Key words and phrases. Picture fuzzy soft graph, lower truncation, upper truncation strong picture fuzzy soft graph, subdivision picture fuzzy soft graph.

Definition 2.2. Let D be initial universal set, Q be a set of parameters, $\wp(D)$ be the power set of D and $K \subseteq Q$. A pair (J, K) is called soft set [3] over D if and only if J is a mapping of K into the set of all subsets of the set D.

Definition 2.3. A pair (J, K) is called fuzzy soft set [2, 6] over D, where J is a mapping given by $J : K \to I^D, I^D$ denote the collection of all fuzzy subset of $D, K \subseteq Q$

Definition 2.4. Let A' be a picture fuzzy set [10, 4]. A' in Z defined by

$$A' = \{(z, \lambda_{A'}(z), \delta_{A'}(z), \varphi_{A'}(z)) : z \in Z\}$$

where, $\lambda_{A'}(z) \in [0,1], \delta_{A'}(z) \in [0,1]$ and $\varphi_{A'}(z) \in [0,1]$ follow the condition $0 \leq \lambda_{A'}(z) + \delta_{A'}(z) + \varphi_{A'}(z) \leq 1$. The $\lambda_{A'}(z)$ is used to represent the positive membership degree, $\delta_{A'}(z)$ is used to represent the neutral membership degree and $\varphi_{A'}(z)$ is used to represent the negative membership degree of the element z in the set A'. For each picture fuzzy set A' in Z, the refusal membership degree is described as

$$\pi_{A'}(z) = 1 - (\lambda_{A'}(z) + \delta_{A'}(z) + \varphi_{A'}(z)).$$

3. Picture Fuzzy Soft Graph

Definition 3.1. A pair (J, K) is called picture fuzzy soft set over D, where J is a mapping given by $J : K \to IP^D$, where IP^D denote the collection of all picture fuzzy subset of $D, K \subseteq Q$.

Definition 3.2. Let $G'^* = (W, Y)$ be a graph, $W = \{W_1, w_2, \ldots W_n\}$ be a non-empty set, $Y \subseteq W \times W, Q$ be parameter set and $K \subseteq Q$. Also let,

i. a) λ_A is a positive membership function defined on W by $\lambda_A : K \to IP^D(W) \big(IP^D(W)$ denote collection of all picture fuzzy subset in $W)$ $k \to \lambda_A(k) = \lambda_{Ak}$ (say), $k \in K$ and $\lambda_{Ak} : W \to [0,1], w_i \to \lambda_{Ak}(w_i)$ (K, λ_A) picture fuzzy soft vertex of positive membership function.
   b) δ_A is a neutral membership function defined on W by $\delta_A : K \to IP^D(W) \big(IP^D(W)$ denote collection of all picture fuzzy subset in $W)$ $k \to \delta_A(k) = \delta_{Ak}$ (say), $k \in K$ and $\delta_{Ak} : W \to [0,1], W_i \to \delta_{Ak}(W_i)$ (K, δ_A) picture fuzzy soft vertex of neutral membership function.
   c) φ_A is a negative membership function defined on W by $\varphi_A : K \to IP^D(W) \big(IP^D(W)$ denote collection of all picture fuzzy subset in $W)$ $k \to \varphi_A(k) = \varphi_{Ak}$ (say), $k \in K$ and $\varphi_{Ak} : W \to [0,1], W_i \to \varphi_{Ak}(W_i)$ (K, φ_A) picture fuzzy soft vertex of negative membership function. such that $0 \leq \lambda_{Ak}(w_i) + \delta_{Ak}(w_I) + \varphi_{Ak}(w_i) \leq 1 \forall w_i \in W, k \in K$, where A is a picture fuzzy soft set on W.

ii. a) λ_B is a positive membership function defined on Y by $\lambda_B : K \to IP^D(W \times W) \big(IP^D(W \times W)$ denote collection of all picture fuzzy subset in Y) $k \to \lambda_B(k) = \lambda_{Bk}$ (say), $k \in K$ and $\lambda_{Bk} : W \times W \to [0,1], (w_i, w_j) \to \lambda_{Bk}(w_i, w_j)$ (K, λ_B) picture fuzzy soft edge of positive membership function.
   b) δ_B is a neutral membership function defined on Y by $\delta_B : K \to IP^D(W \times W) \big(IP^D(W \times W)$ denote collection of all picture fuzzy subset in $Y)$ $k \to$

$\delta_B(k) = \delta_{Bk}$ (say), $k \in K$ and $\delta_{Bk} : W \times W \to [0,1], (w_i, w_j) \to \delta_{Bk}(w_i, w_j)$ (K, δ_B) picture fuzzy soft edge of neutral membership function.

c) φ_B is a negative membership function defined on Y by $\varphi_B : K \to IP^D(W \times W)$ $\left(IP^D(W \times W)\right.$ denote collection of all picture fuzzy subset in Y) $k \to \varphi_B(k) = \varphi_{Bk}(\text{ say }), k \in K$ and $\varphi_{Bk} : W \times W \to [0,1], (w_i, w_j) \to \varphi_{Bk}(W_i, w_j)$ (K, φ_B) picture fuzzy soft edge of negative membership function, where, B is a picture fuzzy soft set on Y.
Also satisfying the following condition,

$$\begin{aligned} \lambda_{Bk}(w_i, w_j) &\leq \min\left(\lambda_{Ak}(w_i), \lambda_{Ak}(w_j)\right), \\ \delta_{Bk}(w_i, w_j) &\leq \min\left(\delta_{Ak}(w_i), \delta_{Ak}(w_j)\right), \\ \varphi_{Bk}(w_i, w_j) &\geq \max\left(\varphi_{Ak}(w_i), \varphi_{Ak}(w_j)\right) \text{ and} \end{aligned}$$
$$0 \leq \lambda_{Bk}(w_i, w_j) + \delta_{Bk}(w_i, w_j) + \varphi_{Bk}(w_i, w_j) \leq 1$$

$\forall (W_i, W_j) \in Y, i, j = 1, 2, \ldots n$ and $k \in K$. Then

$$G'^{*} = (W, Y, (K, \lambda_A), (K, \delta_A), (K, \varphi_A), (K, \lambda_B), (K, \delta_B), (K, \varphi_B))$$

is said to be picture fuzzy soft graph and this denoted by $G'^{*}_{K,W,Y}$.

Definition 3.3. Picture fuzzy soft graph

$$G'^{*}_{K,W,Y} = (W, Y, (K, \lambda_A), (K, \delta_A), (K, \varphi_A), (K, \lambda_B), (K, \delta_B), (K, \varphi_B))$$

is said to be strong picture fuzzy soft graph, if

$$\lambda_{Bk}(w_i, w_j) = \min\left(\lambda_{Ak}(w_i), \lambda_{Ak}(w_j)\right), \quad \delta_{Bk}(w_i, w_j) = \min\left(\delta_{Ak}(w_i), \delta_{Ak}(w_j)\right)$$
$$\varphi_{Bk}(w_i, w_j) = \max\left(\varphi_{Ak}(w_i), \varphi_{Ak}(w_j)\right) \quad \forall (w_i, w_j) \in Y, k \in K.$$

Definition 3.4. The vertices and edges of $G'_{K,W,Y}$ are taken together as the vertex set of the subdivision picture fuzzy soft graph sd $\left(G'^{*}_{K,W,Y}\right)$ is defined as, $(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{sd}$ is picture fuzzy soft subset defined on $W \cup Y$.

$$(\lambda_{Ak})_{sd}(w) = \begin{cases} \lambda_{Ak}(w) & \text{if } w \in W \\ \lambda_{Bk}(w) & \text{if } w \in Y \end{cases}, \qquad (\delta_{Ak})_{sd}(w) = \begin{cases} \delta_{Ak}(w) & \text{if } w \in W \\ \delta_{Bk}(w) & \text{if } w \in Y \end{cases},$$
$$(\varphi_{Ak})_{sd}(w) = \begin{cases} \varphi_{Ak}(w) & \text{if } w \in W \\ \varphi_{Bk}(w) & \text{if } w \in Y \end{cases}$$

The fuzzy relation $(\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{sd}$ on $W \cup Y$ is,

$$(\lambda_{Bk})_{sd}(w, y) = \begin{cases} (\lambda_{Ak})_{sd}(w) \wedge (\lambda_{Ak})_{sd}(y), \text{ if } w \in W, y \in Y \text{ and } w \text{ lies on } y \\ 0, \text{ otherwise} \end{cases}$$
$$(\delta_{Bk})_{sd}(w, y) = \begin{cases} (\delta_{Ak})_{sd}(w) \wedge (\delta_{Ak})_{sd}(y), \text{ if } w \in W, y \in Y \text{ and } w \text{ lies on } y \\ 0, \text{ otherwise} \end{cases}$$
$$(\varphi_{Bk})_{sd}(w, y) = \begin{cases} (\varphi_{Ak})_{sd}(w) \wedge (\varphi_{Ak})_{sd}(y), \text{ if } w \in W, y \in Y \text{ and } w \text{ lies on } y \\ 0, \text{ otherwise} \end{cases}$$

4. Lower and Upper Truncation of Picture Fuzzy Soft Graph

Definition 4.1. Let $(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})$ be a picture fuzzy soft subset of the set W. Then lower truncation $(\lambda_{Ak}, \delta_{Ak'}\varphi_{Ak})_{(t)}$ and upper truncation $(\lambda_{Ak'}, \delta_{Ak'}, \varphi_{Ak})^{(t)}$ of $(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})$ at level 't', $0 < t \leq 1$ are defined by,

$$(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w) = \begin{cases} (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})(w), & \text{if } w \in (\lambda^t_{Ak}, \delta^t_{Ak}, \varphi^t_{Ak}) \\ 0, & \text{if } w \notin \lambda^t_{Ak}(or)W \notin \delta^t_{Ak}(or)w \notin \varphi^t_{Ak} \end{cases}$$

$$(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w) = \begin{cases} t, & \text{if } w \in (\lambda^t_{Ak}, \delta^t_{Ak}, \varphi^t_{Ak}) \\ (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})(w), & \text{if } w \notin \lambda^t_{Ak}(or)W \notin \delta^t_{Ak}(or)w \notin \varphi^t_{Ak} \end{cases}$$

where, $\lambda^t_{Ak} = \{w \in W/\lambda_{Ak} \geq t\}$, $\delta^t_{Ak} = \{w \in W/\delta_{Ak} \geq t\}$, $\varphi^t_{Ak} = \{w \in W/\varphi_{Ak} \leq t\}$ and $k \in K$.

Definition 4.2. Let $G'^*_{K,W,Y}$ be a picture fuzzy soft graph. Take $W_{(t)} = (\lambda^t_{Ak}, \delta^t_{Ak}, \varphi^t_{Ak})$, $Y_{(t)} = (\lambda^t_{Bk}, \delta^t_{Bk}, \varphi^t_{Bk})$. Then for $w \in W_{(t)}$, $(\lambda_{Ak})_{(t)}(w) = \lambda_{Ak}(w)$, $(\delta_{Ak})_{(t)}(w) = \delta_{Ak}(w)$, $(\varphi_{Ak})_{(t)}(w) = \varphi_{Ak}(w)$ and for $y \in Y_{(t)}$, $(\lambda_{Bk})_{(t)}(y) = \lambda_{Bk}(y)$, $(\delta_{Bk})_{(t)}(y) = \delta_{Bk}(y)$, $(\varphi_{Bk})_{(t)}(y) = \varphi_{Bk}(y)$.
Therefore,

$$(\lambda_{Bk})_{(t)}(w_iw_j) = \lambda_{Bk}(w_iw_j) \leq \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j) = (\lambda_{Ak})_{(t)}(w_i) \wedge (\lambda_{Ak})_{(t)}(w_j)$$
$$(\delta_{Bk})_{(t)}(w_iw_j) = \delta_{Bk}(w_iw_j) \leq \delta_{Ak}(w_i) \wedge \delta_{Ak}(w_j) = (\delta_{Ak})_{(t)}(w_i) \wedge (\delta_{Ak})_{(t)}(w_j)$$
$$(\varphi_{Bk})_{(t)}(w_iw_j) = \varphi_{Bk}(w_iw_j) \geq \varphi_{Ak}(w_i) \vee \varphi_{Ak}(w_j) = (\varphi_{Ak})_{(t)}(w_i) \vee (\varphi_{Ak})_{(t)}(w_j)$$

for all $w_i, w_j \in W_{(t)}, k \in K$.
Hence, $\left(G'^*_{K,W,Y}\right)_{(t)}$ is a lower truncation of picture fuzzy soft graph.
Take $W^{(t)} = W, Y^{(t)} = Y$.
By definition,

(i) either $(\lambda_{Bk})^{(t)}(w_iw_j) = t$ or $(\lambda_{Bk})^{(t)}(w_iw_j) = \lambda_{Bk}(w_iw_j)$,
(ii) either $(\delta_{Bk})^{(t)}(w_iw_j) = t$ or $(\delta_{Bk})^{(t)}(w_iw_j) = \delta_{Bk}(w_iw_j)$,
(iii) either $(\varphi_{Bk})^{(t)}(w_iw_j) = t$ or $(\varphi_{Bk})^{(t)}(w_iw_j) = \varphi_{Bk}(w_iw_j)$.

(i) Suppose, $(\lambda_{Bk})^{(t)}(w_iw_j) = t$ then $\lambda_{Bk}(w_iw_j) \geq t$.

$$\Rightarrow t \leq \lambda_{Bk}(w_iw_j) \leq \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j)$$
$$\Rightarrow \lambda_{Ak}(w_i) \geq t, \lambda_{Ak}(w_j) \geq t$$
$$\Rightarrow (\lambda_{Ak})^{(t)}(w_i) = t, (\lambda_{Ak})^{(t)}(w_j) = t$$
$$\Rightarrow (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = t$$

Therefore, $(\lambda_{Bk})^{(t)}(w_iw_j) = t = (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j)$
Suppose, $(\lambda_{Bk})^{(t)}(w_iW_j) = \lambda_{Bk}(w_iW_j)$ then $\lambda_{Bk}(w_iw_j) < t$
Now, we have two cases $\lambda_{Bk}(w_iW_j) < t \leq \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j)$ or $\lambda_{Bk}(W_iW_j) \leq \lambda_{Ak}(W_i) \wedge \lambda_{Ak}(w_j) < t$.
when, $t \leq \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j)$, proceeding as above $(\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = t$.

Therefore, $(\lambda_{Bk})^{(t)}(w_iw_j) = \lambda_{Bk}(w_iw_j) < t = (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j)$ when, $\lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j) < t$, we have

$\Rightarrow \lambda_{Ak}(w_i) < t, \lambda_{Ak}(w_j) \geq t (or) \lambda_{Ak}(w_i) \geq t, \lambda_{Ak}(w_j) < t (or) \lambda_{Ak}(w_i) < t \lambda_{Ak}(w_j) < t.$

$\Rightarrow (\lambda_{Ak})^{(t)}(w_i) = \lambda_{Ak}(w_i), (\lambda_{Ak})^{(t)}(w_j) = t (or) (\lambda_{Ak})^{(t)}(w_i) = t$

$(\lambda_{Ak})^{(t)}(w_j) = \lambda_{Ak}(w_j) (or) (\lambda_{Ak})^{(t)}(w_I) = \lambda_{Ak}(w_I), (\lambda_{Ak})^{(t)}(w_j) = \lambda_{Ak}(w_j)$

$\Rightarrow (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = \lambda_{Ak}(w_i) \wedge t = \lambda_{Ak}(w_i) = \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j)$ (or)

$(\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = t \wedge \lambda_{Ak}(w_j) = \lambda_{Ak}(w_j) = \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j)$ (or)

$(\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j)$

Therefore,

$(\lambda_{Bk})^{(t)}(w_iw_j) = \lambda_{Bk}(W_iW_j) \leq \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j) = (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j)$

$(\lambda_{Bk})^{(t)}(w_iw_j) \leq (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j)$

Similarly,

(ii) $(\delta_{Bk})^{(t)}(w_iw_j) \leq (\delta_{Ak})^{(t)}(w_i) \wedge (\delta_{Ak})^{(t)}(w_j)$

(iii) $(\varphi_{Bk})^{(t)}(w_iw_j) \geq (\varphi_{Ak})^{(t)}(w_i) \vee (\varphi_{Ak})^{(t)}(w_j)$ for $all w_i, w_j \in W, k \in K$.

Hence, $\left(G'^{*}_{K,W,Y}\right)^{(t)}$ is a upper truncation of picture fuzzy soft graph.

Example 4.3. Consider, picture fuzzy soft graph

$G'^{*}_{K,W,Y} = (W, Y, (K, \lambda_A), (K, \delta_A), (K, \varphi_A), (K, \lambda_B), (K, \delta_B), (K, \varphi_B))$, where

$W = \{w_1, w_2, w_3\}$ and $Y = \{(w_1, w_2), (w_2, w_3), (w_1, w_3)\}$. Let $K = \{k_1, k_2\}$ be the parameter set.

TABLE 1. Picture fuzzy soft graph $G'^{*}_{K,W,Y}$

(a)

λ_{Ak}	w_1	w_2	w_3
k_1	0.3	0.2	0.4
k_2	0.5	0.4	0.3

(b)

δ_{Ak}	w_1	w_2	w_3
k_1	0.4	0.5	0.3
k_2	0.3	0.4	0.3

(c)

φ_{Ak}	w_1	w_2	w_3
k_1	0.1	0.2	0.2
k_2	0.2	0.1	0.1

(d)

λ_{Bk}	(w_1, w_2)	(w_2, w_3)	(w_1, w_3)
k_1	0.2	0.2	0.3
k_2	0.4	0.1	0.3

(e)

δ_{Bk}	(w_1, w_2)	(w_2, w_3)	(w_1, w_3)
k_1	0.4	0.3	0.3
k_2	0.3	0.1	0.3

(f)

φ_{Bk}	(w_1, w_2)	(w_2, w_3)	(w_1, w_3)
k_1	0.2	0.2	0.2
k_2	0.2	0.1	0.2

The lower truncation $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ and upper truncation $\left(G'^{*}_{K,W,Y}\right)^{(t)}$ of picture fuzzy soft graph $G'^{*}_{K,W,Y}$ at $t = (0.2, 0.3, 0.2)$.

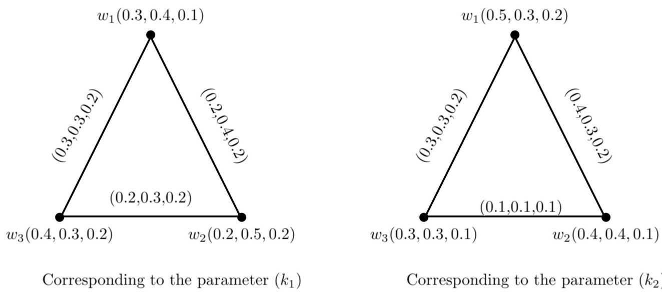

FIGURE 1. Picture fuzzy soft graph $G'^{*}_{K,W,Y}$

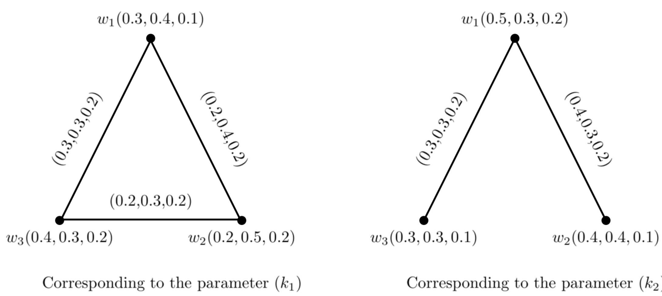

FIGURE 2. $\left(G'^{*}_{K,W,Y}\right)$

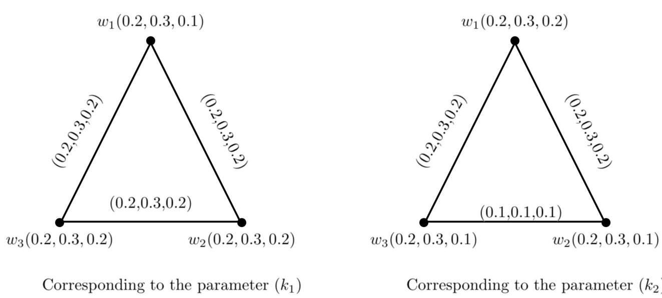

FIGURE 3. $\left(G'^{*}_{K,W,Y}\right)^{(0.2,0.3,0.2)}$

5. Truncation of the Subdivision picture Fuzzy Soft Graph

Theorem 5.1. *For any level $t, 0 < t \leq 1$, the lower truncation of the subdivision picture fuzzy soft graph $sd\left(G'^{*}_{K,W,Y}\right)$ is same as the subdivision of the lower truncation picture fuzzy soft graph $\left(G'^{*}_{K,W,Y}\right)_{(t)'}$, ie, $\left(sd\left(G'^{*}_{K,W,Y}\right)\right)_{(t)} = sd\left(\left(G'^{*}_{K,W,Y}\right)_{(t)}\right)$.*

Proof. Let $G'_{K,W,Y}$ be a picture fuzzy soft graph.
Claim: 1

We claim that vertex set $\left(sd\left(G^{**}_{K,W,Y}\right)\right)_{(t)}$ = vertex set sd $\left(\left(G^{*}_{K,W,Y}\right)_{(t)}\right)$.
Take w to be a vertex in $\left(sd\left(G'^{*}_{K,W,Y}\right)\right)_{(t)}$.
By the definition of lower truncation of $sd\left(G'^{*}_{K,W,Y}\right)$,

$$((\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{sd})_{(t)}(w) = (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{sd}(w), \text{ if } w \in \left(\lambda^t_{Ak}, \delta^t_{Ak}, \varphi^t_{Ak}\right)_{sd} \tag{5.1}$$

$$((\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{sd})_{(t)}(w) = 0, \text{ if } w \notin \left(\lambda^t_{Ak}\right)_{sd}(or) w \notin \left(\delta^t_{Ak}\right)_{sd}(or) w \notin \left(\varphi^t_{Ak}\right)_{sd} \tag{5.2}$$

for all $k \in K$.
By definition of sd $\left(G^{*}_{K,W,Y}\right)$,

$$(\lambda_{Ak})_{sd}(w) = \begin{cases} \lambda_{Ak}(w) & \text{if } w \in W \\ \lambda_{Bk}(w) & \text{if } w \in Y \end{cases} \tag{5.3}$$

$$(\delta_{Ak})_{sd}(w) = \begin{cases} \delta_{Ak}(w) & \text{if } w \in W \\ \delta_{Bk}(w) & \text{if } w \in Y \end{cases} \tag{5.4}$$

$$(\varphi_{Ak})_{sd}(w) = \begin{cases} \varphi_{Ak}(w) & \text{if } w \in W \\ \varphi_{Bk}(w) & \text{if } w \in Y \end{cases} \tag{5.5}$$

Case: 1
Let the vertex w in $\left(\text{sd}\left(G^{*}_{K,W,Y}\right)\right)_{(t)}$ be such that $w \in (\lambda^t Ak, \delta^t_{Ak}, \varphi^t_{Ak})_{sd}$

$$\text{i.e., } (\lambda_{Ak})_{sd}(w) \geq t, (\delta_{Ak})_{sd}(w) \geq t, (\varphi_{Ak})_{sd}(w) \leq t \tag{5.6}$$

Subcase: 1(a)
If $w \in W$, by (5.3),(5.4),(5.5) and (5.6) $\lambda_{Ak}(w) \geq t, \delta_{Ak}(w) \geq t$ and $\varphi_{Ak}(w) \geq t$.
Hence $w \in (\lambda^t_{Ak}, \delta^t_{Ak}, \varphi^t_{Ak})$
By definition of $\left(G'^{*}_{K,W,Y}\right)_{(t)}$,

$$(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w) = (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w) \text{ if } w \in \left(\lambda^t_{Ak}, \delta^t_{Ak}, \varphi^t_{Ak}\right)$$

ie., w is a vertex in $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ such that

$$(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w) = (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w).$$

Subcase: 1(b)
If $w \in W$, let $w = y$ (5.3), (5.4) and (5.5) $\Rightarrow (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{sd}(y) = (\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})(y)$
By (5.6), $(\lambda_{Ak})_{sd}(w) \geq t, (\delta_{Ak})_{sd}(w) \geq t, (\varphi_{Ak})_{sd}(w) \leq t$. In $\left(G'^{*}_{K,W,Y}\right)_{(t)}$

$$(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(y) = (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(y)$$

ie., $w = y \in Y$ is an edge in $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ such that $(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(y) = (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(y)$. As the vertices and edges of $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ are taken together as the vertices of $sd\left(\left(G'^{*}_{K,W,Y}\right)_{(t)}\right)$.

By Sub Case 1(a) and 1(b), w is a vertex in $sd\left(\left(G^{*}_{K,W,Y}\right)_{(t)}\right)$,

$$\left(\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{sd}\right)_{(t)}(w)=\left(\lambda_{Ak'}\delta_{Ak'}\varphi_{Ak}\right)_{sd}(w)=\left(\left(\lambda_{Ak},\delta_{Ak'}\varphi_{Ak}\right)_{(t)}\right)_{sd}(w)$$

Case:2
Let the vertex w in $\left(\mathrm{sd}\left(G'^{*}_{K,W,Y}\right)\right)_{(t)}$ be such that $w\notin\left(\lambda^{t}Ak\right)_{sd}(or)w\notin\left(\delta^{t}_{Ak}\right)_{sd}(or)w\notin\left(\varphi^{t}Ak\right)_{sd}$.

$$(5.2)\ \Rightarrow\ \left(\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{sd}\right)_{(t)}(w)=0as\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{sd}<t$$

Subcase:2(a)
If $w\in W$, by (5.3), (5.4) and (5.5) $\Rightarrow\left(\lambda_{Ak'}\delta_{Ak'}\varphi_{Ak}\right)_{sd}(w)=\left(\lambda_{Ak'},\delta_{Ak'},\varphi_{Ak}\right)(w)$. Hence $\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)(w)<t$, $w\notin\left(\lambda^{t}Ak\right)_{Sd}(or)w\notin\left(\delta^{t}_{Ak}\right)_{sd}(or)w\notin\left(\varphi^{t}_{Ak}\right)_{sd}$. By definition of $\left(G'^{*}_{K,W,Y}\right)_{(t)}$, $\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{(t)}(w)=0$. ie., w is a vertex in $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ such that $\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{(t)}(w)=0$.

Subcase: 2(b)
If $w\in W$, let $w=y$ (5.3), (5.4) and (5.5) $\Rightarrow\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{sd}(y)=\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)(y)$. Hence $\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)(w)<t$, ie., $y\notin\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)$. By definition of $\left(G'^{*}_{K,W,Y}\right)_{(t)'}$, $\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)_{(t)}(y)=0$.
ie., y is an edge in $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ such that $\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)_{(t)}(y)=0$.
Hence, by Sub Case 2(a), 2(b) and by definition of $sd\left(\left(G^{*}_{K,W,Y}\right)_{(t)}\right)$, w is a vertex in sd $\left(\left(G'^{*}_{K,W,Y}\right)_{(t)}\right)$, such that $\left(\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{(t)}\right)_{sd}(w)=0$.
By Case 1 and Case 2, $\left(\left(\lambda_{Ak},\delta_{Ak},\varphi_{Ak}\right)_{sd}\right)_{(t)}(w)=\left(\left(\lambda_{Ak},\delta_{Ak'},\varphi_{Ak}\right)_{(t)}\right)_{sd}(w)$ for all $W\in W\cup Y$.

Claim: 2
We claim that edge set $\left(\mathrm{sd}\left(G'^{*}_{K,W,Y}\right)\right)_{(t)}=$ edge set sd $\left(\left(G'^{*}_{K,W,Y}\right)_{(t)}\right)$.
Take (w,y) to be a edge in $\left(sd\left(G'^{*}_{K,W,Y}\right)\right)_{(t)}$.
By the definition of lower truncation of sd $\left(G'^{*}_{K,W,Y}\right)$,

$$\left(\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)_{sd}\right)_{(t)}(w,y)=\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)_{sd}(w,y),\ \text{if } (w,y)\in\left(\lambda^{t}Bk,\delta^{t}_{Bk},\varphi^{t}_{Bk}\right)_{sd} \tag{5.7}$$

$$\left(\left(\lambda_{Bk},\delta_{Bk},\varphi_{Bk}\right)_{sd}\right)_{(t)}(w,y)=0,\ \text{if } (w,y)\notin\left(\lambda^{t}Bk\right)_{sd}(or)(w,y)\notin\left(\delta^{t}_{Bk}\right)_{sd}(or)(w,y)\notin\left(\varphi^{t}Bk\right)_{sd} \tag{5.8}$$

for all $k \in K$.
By definition of sd $\left(G^{**}_{K,W,Y}\right)$,

$$(\lambda_{Bk})_{sd}(w,y) = \begin{cases} (\lambda_{Ak})_{sd}(w) \wedge (\lambda_{Ak})_{sd}(y), \text{ if } w \in W, y \in Y \text{ and } w \text{ lies on } y \\ 0, \text{ otherwise} \end{cases} \tag{5.9}$$

$$(\delta_{Bk})_{sd}(w,y) = \begin{cases} (\delta_{Ak})_{sd}(w) \wedge (\delta_{Ak})_{sd}(y), \text{ if } w \in W, y \in Y \text{ and } w \text{ lies on } y \\ 0, \text{ otherwise} \end{cases} \tag{5.10}$$

$$(\varphi_{Bk})_{sd}(w,y) = \begin{cases} (\varphi_{Ak})_{sd}(w) \wedge (\varphi_{Ak})_{sd}(y), \text{ if } w \in W, y \in Y \text{ and } w \text{ lies on } y \\ 0, \text{ otherwise} \end{cases} \tag{5.11}$$

Case: 3
Let $w \in W, y \in Y$ be such that $(w,y) \in (\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})^t_{sd}$

$$(5.7) \Rightarrow ((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{sd})_{(t)} = \begin{cases} (\lambda_{Bk})_{sd}(w,y) \geq t \\ (\delta_{Bk})_{sd}(w,y) \geq t \\ (\varphi_{Bk})_{sd}(w,y) \leq t \end{cases}$$

As $t > 0$,

$$\begin{aligned}
(5.9) &\Rightarrow (\lambda_{Ak})_{sd}(w) \wedge (\lambda_{Ak})_{sd}(y) \geq t \\
(5.10) &\Rightarrow (\delta_{Ak})_{sd}(w) \wedge (\delta_{Ak})_{sd}(y) \geq t \\
(5.11) &\Rightarrow (\varphi_{Ak})_{sd}(w) \vee (\varphi_{Ak})_{sd}(y) \leq t \\
&\Rightarrow (\lambda_{Ak})_{sd}(w) \wedge (\lambda_{Bk})_{sd}(y) \geq t, (\delta_{Ak})_{sd}(w) \wedge (\delta_{Bk})_{sd}(y) \geq t, \\
&\quad (\varphi_{Ak})_{sd}(w) \vee (\varphi_{Bk})_{sd}(y) \leq t. \\
&\Rightarrow (\lambda_{Ak})_{sd}(w), (\lambda_{Bk})_{sd}(y) \geq t \text{ and } w \text{ lies on } y, (\delta_{Ak})_{sd}(w), (\delta_{Bk})_{sd}(y) \geq t
\end{aligned}$$

and w lies on y,

$$(\varphi_{Ak})_{sd}(w), (\varphi_{Bk})_{sd}(y) \leq t$$

and w lies on y.

$$\Rightarrow w \in (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})^{(t)}, y \in (\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})^{(t)}$$

and w lies on y.
By definition of $\left(G^{*}_{K,w,Y}\right)_{(t)}$

$$\begin{aligned}
(\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})_{(t)}(w) &= (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})(w) asw \in (\lambda_{Ak}, \delta_{Ak}, \varphi_{Ak})^t \\
(\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{(t)}(y) &= (\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})(y) \text{ as } y \in (\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})^t
\end{aligned} \tag{5.12}$$

By (5.9), (5.10), (5.11) and (5.12) we have,

$$\begin{aligned}
(\lambda_{Bk})_{sd}(w,y) &= (\lambda_{Ak})_{(t)}(w) \wedge (\lambda_{Bk})_{(t)}(y) \text{ as } w \text{ lies on } y \\
&= \left((\lambda_{Ak})_{(t)}\right)_{sd}(w) \wedge \left((\lambda_{Bk})_{(t)}\right)_{sd}(y) \text{ as } w \text{ lies on } y \\
&= \left((\lambda_{Bk})_{(t)}\right)_{sd}(y)
\end{aligned}$$

Similarly, $(\delta_{Bk})_{sd}(w,y) = \left((\delta_{Bk})_{(t)}\right)(y), (\varphi_{Bk})_{sd}(w,y) = \left((\varphi_{Bk})_{(t)}\right)(y)$.
(w,y) to be an edge in sd $\left((G'^{*}_{K,W,Y})_{(t)}\right)$ and

$((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{sd})_{(t)}(w,y) = \left((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{(t)}\right)_{sd}(w,y)$ if $(w,y) \in (\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})^{t}_{sd}$.

Case: 4
Take (w,y) to be an edge in $\left(sd\left(G'^{*}_{K,W,Y}\right)\right)_{(t)}$ such that $((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk'})_{sd})_{(t)}(w,y) = 0$ By (5.8) $(w,y) \notin (\lambda^{t}Bk)_{sd}(or)(w,y) \notin (\delta^{t}Bk)_{sd}(or)(w,y) \notin (\varphi^{t}Bk)_{sd} \Rightarrow (\lambda_{Ak})_{sd}(w,y) < t, (\delta_{Ak})_{sd}(w,y) < t, (\varphi_{Ak})_{sd}(w,y) > t$
Subcase:4(a)
If w lies on y, (5.9), (5.10), (5.11) $\Rightarrow (\lambda_{Ak})_{sd}(w) \wedge (\lambda_{Ak})_{sd}(y) < t$

$(\delta_{Ak})_{sd}(w) \wedge (\delta_{Ak})_{sd}(y) < t, (\varphi_{Ak})_{sd}(w) \vee (\varphi_{Ak})_{sd}(y) > t$
$\Rightarrow (\lambda_{Ak})(w) \wedge (\lambda_{Bk})(y) < t, (\delta_{Ak})(w) \wedge (\delta_{Bk})(y) < t, (\varphi_{Ak})(w) \vee (\varphi_{Bk})(y) > t.$
$\Rightarrow (\lambda_{Bk})(y) < t, (\delta_{Bk})(y) < t, (\varphi_{Bk})(y) > t.$
$y \notin (\lambda_{Bk})^{t}(or) y \notin (\delta_{Bk})^{t}(or) y \notin (\varphi_{Bk})^{t}$ ie., $(\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{(t)}(y) = 0$

$$\begin{aligned}\left((\lambda_{Bk})_{(t)}\right)_{sd}(w,y) &= \left((\lambda_{Ak})_{(t)}\right)_{sd}(w) \wedge \left((\lambda_{Bk})_{(t)}\right)_{sd}(y)\\ &= (\lambda_{Ak})_{(t)}(w) \wedge (\lambda_{Bk})_{(t)}(y)\\ &= 0 \,(\text{ since, } \lambda_{Bk} = 0 \text{ and } w \text{ lies on } y)\end{aligned}$$

Similarly, $\left((\delta_{Bk})_{(t)}\right)_{sd}(w,y) = 0, \left((\varphi_{Bk})_{(t)}\right)_{sd}(w,y) = 0.$
Subcase: 4 (b)
If w does not lie on y $\left((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{(t)}\right)_{sd}(w,y) = 0.$
Hence, by Sub Case 4(a) and 4(b)
If (w,y) is an edge in $\left(sd\left(G'^{*}_{K,W,Y}\right)\right)_{(t)}$ such that $((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk},)_{sd})_{(t)}(w,y) = 0$
then $\left((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk'},)_{(t)}\right)_{sd}(W,y) = 0$
ie., $((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk},)_{sd})_{(t)}(w,y) = \left((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk},)_{(t)}\right)_{sd}(w,y)$
if $(w,y) \notin (\lambda^{t}Bk)_{sd}(or)(w,y) \notin (\delta^{t}Bk)_{sd}(or)(w,y) \notin (\varphi^{t}Bk)_{sd}$.
Hence, by Case (3) and (4)

$$((\lambda_{Bk}, \delta_{Bk}, \varphi_{Bk})_{sd})_{(t)}(w,y) = \left((\lambda_{Bk}, \delta_{Bk'}, \varphi_{Bk})_{(t)}\right)_{sd}(w,y)$$
$$\text{ie, } \left(sd\left(G^{*}_{K,W,Y}\right)\right)_{(t)} = \text{sd}\left(\left(G^{*}_{K,W,Y}\right)_{(t)}\right)$$

□

Theorem 5.2. *For any level* $t, 0 < t \leq 1$*, the upper truncation of the subdivision picture fuzzy soft graph* $sd\left(G'_{K,W,Y}\right)$ *is same as the subdivision of the upper truncation picture fuzzy soft graph* $\left(G'^{*}_{K,W,Y}\right)^{(t)}$*, ie,* $\left(sd\left(G'^{*}_{K,W,Y}\right)\right)^{(t)} = sd\left(\left(G^{*}_{K,W,Y}\right)^{(t)}\right)$.

6. Truncation of Strong Picture Fuzzy Soft Graph

Theorem 6.1. *If $G'^{*}_{K,W,Y}$ is a strong picture fuzzy soft graph, then $\left(G'^{*}_{K,W,Y}\right)_{(t)}$ is strong picture fuzzy soft graph.*

Proof. Let $G'^{*}_{K,W,Y}$ be a strong picture fuzzy soft graph. Then
$\lambda_{Bk}(w_i, w_j) = \min(\lambda_{Ak}(w_i), \lambda_{Ak}(w_j))$, $\delta_{Bk}(w_i, w_j) = \min(\delta_{Ak}(w_i), \delta_{Ak}(w_j))$
$\varphi_{Bk}(w_i, w_j) = \max(\varphi_{Ak}(w_i), \varphi_{Ak}(w_j))$ for every edge $w_iw_j$of $G'^{*}_{K,W,Y}$.

Let $(w_iw_j) \in Y_{(t)}$ be any edge of $\left(G'^{*}_{K,W,Y}\right)_{(t)}$. Then $w_i, w_j \in W_{(t)}$

$$\begin{aligned}(\lambda_{Bk})_{(t)}(w_i, w_j) &= \lambda_{Bk}(w_i, w_j), \quad (\lambda_{Ak})_{(t)}(w_i) = \lambda_{Ak}(w_i), \\ (\lambda_{Ak})_{(t)}(w_j) &= \lambda_{Ak}(w_j) \quad (\lambda_{Bk})_{(t)}(w_i, w_j) = \lambda_{Bk}(w_i, w_j) \\ &= \lambda_{Ak}(w_i) \wedge \lambda_{Ak}(w_j) \\ &= (\lambda_{Ak})_{(t)}(w_i) \wedge (\lambda_{Ak})_{(t)}(w_j)\end{aligned}$$

Similarly,

$$\begin{aligned}(\delta_{Bk})_{(t)}(w_i, w_j) &= (\delta_{Ak})_{(t)}(w_i) \wedge (\delta_{Ak})_{(t)}(w_j), \\ (\varphi_{Bk})_{(t)}(w_i, w_j) &= (\varphi_{Ak})_{(t)}(w_i) \vee (\varphi_{Ak})_{(t)}(w_j)\end{aligned}$$

Hence, $\left(G'^{*}_{K,W,Y}\right)_{(t)}$is strong picture fuzzy soft graph. □

Theorem 6.2. *If $G^{*}_{K,W,Y}$ is a strong picture fuzzy soft graph, then $\left(G^{*}_{K,W,Y}\right)^{(t)}$ is strong picture fuzzy soft graph.*

Proof. Let $G'^{*}_{K,W,Y}$ be a strong picture fuzzy soft graph. Then
$\lambda_{Bk}(w_i, w_j) = \min(\lambda_{Ak}(w_i), \lambda_{Ak}(w_j))$, $\delta_{Bk}(w_i, w_j) = \min(\delta_{Ak}(w_i), \delta_{Ak}(w_j))$
$\varphi_{Bk}(w_i, w_j) = \max(\varphi_{Ak}(w_i), \varphi_{Ak}(w_j))$ for every edge $w_iw_j$of $G'^{*}_{K,W,Y}$.

Let $(w_iw_j) \in Y_{(t)}$be any edge of $\left(G'^{*}_{K,W,Y}\right)_{(t)}$.

Case:1

(i) If $\lambda_{Ak}(w_i) \geq t$, $\lambda_{Ak}(w_j) \geq t$, $\lambda_{Bk}(w_i, w_j) \geq t$. Then

$$(\lambda_{Ak})^{(t)}(w_i) = t, \quad (\lambda_{Ak})^{(t)}(w_j) = t, \quad (\lambda_{Bk})^{(t)}(w_i, w_j) = t$$
$$\Rightarrow \quad (\lambda_{Bk})^{(t)}(w_i, w_j) = (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j).$$

(ii) Similarly, if $\delta_{Ak}(w_i) \geq t$, $\delta_{Ak}(w_j) \geq t$, $\delta_{Bk}(w_i, w_j) \geq t$. Then

$$(\delta_{Bk})^{(t)}(w_i, w_j) = (\delta_{Ak})^{(t)}(w_i) \wedge (\delta_{Ak})^{(t)}(w_j).$$

(iii) If $\varphi_{Ak}(w_i) \leq t$, $\varphi_{Ak}(w_j) \leq t$, $\varphi_{Bk}(w_i, w_j) \leq t$. Then

$$(\varphi_{Ak})^{(t)}(w_i) = t, \quad (\varphi_{Ak})^{(t)}(w_j) = t, \quad (\varphi_{Bk})^{(t)}(w_i, w_j) = t$$
$$\Rightarrow (\varphi_{Bk})^{(t)}(w_i, w_j) = (\varphi_{Ak})^{(t)}(w_i) \wedge (\varphi_{Ak})^{(t)}(w_j).$$

Case:2

(i) If $\lambda_{Ak}(w_i) \geq t$, $\lambda_{Ak}(w_j) < t$, $\lambda_{Bk}(w_i, w_j) < t$. Then

$$(\lambda_{Ak})^{(t)}(w_i) = t, \quad (\lambda_{Ak})^{(t)}(w_j) = (\lambda_{Ak})(w_j), \quad (\lambda_{Bk})^{(t)}(w_i, w_j) = (\lambda_{Bk})(w_i, w_j).$$

Also,

$$(\lambda_{Bk})(w_i, w_j) = (\lambda_{Ak})(w_i) \wedge (\lambda_{Ak})(w_j) = (\lambda_{Ak})(w_j)$$

(since, $\lambda_{Ak}(w_j) < t \leq \lambda_{Ak}(w_i)$)

$$\Rightarrow (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = t \wedge (\lambda_{Ak})(w_j)$$
$$= (\lambda_{Ak})(w_j) = (\lambda_{Bk})(w_i, w_j) = (\lambda_{Bk})^{(t)}(w_i, w_j)$$

(ii) Similarly, if $\delta_{Ak}(w_i) \geq t$, $\delta_{Ak}(w_j) < t$, $\delta_{Bk}(w_i, w_j) < t$. Then

$$(\delta_{Bk})^{(t)}(w_i, w_j) = (\delta_{Ak})^{(t)}(w_i) \wedge (\delta_{Ak})^{(t)}(w_j).$$

(iii) If $\varphi_{Ak}(w_i) \leq t$, $\varphi_{Ak}(w_j) < t$, $\varphi_{Bk}(w_i, w_j) < t$. Then

$$(\varphi_{Bk})^{(t)}(w_i, w_j) = (\varphi_{Ak})^{(t)}(w_i) \wedge (\varphi_{Ak})^{(t)}(w_j).$$

Case:3

(i) If $\lambda_{Ak}(w_i) < t$, $\lambda_{Ak}(w_j) \geq t$, $\lambda_{Bk}(w_i, w_j) < t$.
(ii) $\delta_{Ak}(w_i) < t$, $\delta_{Ak}(w_j) \geq t$, $\delta_{Bk}(w_i, w_j) < t$.
(iii) $\varphi_{Ak}(w_i) < t$, $\varphi_{Ak}(w_j) \leq t$, $\varphi_{Bk}(w_i, w_j) < t$.

Proof is similar to Case 2.

Case:4

(i) If $\lambda_{Ak}(w_i) < t$, $\lambda_{Ak}(w_j) < t$, $\lambda_{Bk}(w_i, w_j) < t$. Then

$$(\lambda_{Ak})^{(t)}(w_i) = (\lambda_{Ak})(w_i), \quad (\lambda_{Ak})^{(t)}(w_j) = (\lambda_{Ak})(w_j),$$
$$(\lambda_{Bk})^{(t)}(w_i, w_j) = (\lambda_{Bk})(w_i, w_j)$$
$$\Rightarrow (\lambda_{Ak})^{(t)}(w_i) \wedge (\lambda_{Ak})^{(t)}(w_j) = (\lambda_{Ak})(w_i) \wedge (\lambda_{Ak})(w_j)$$
$$= (\lambda_{Bk})(w_i, w_j) = (\lambda_{Bk})^{(t)}(w_i, w_j)$$

(ii) Similarly, if $\delta_{Ak}(w_i) < t$, $\delta_{Ak}(w_j) < t$, $\delta_{Bk}(w_i, w_j) < t$. Then

$$(\delta_{Bk})^{(t)}(w_i, w_j) = (\delta_{Ak})^{(t)}(w_i) \wedge (\delta_{Ak})^{(t)}(w_j).$$

(iii) If $\varphi_{Ak}(w_i) < t$, $\varphi_{Ak}(w_j) < t$, $\varphi_{Bk}(w_i, w_j) < t$. Then

$$(\varphi_{Bk})^{(t)}(w_i, w_j) = (\varphi_{Ak})^{(t)}(w_i) \wedge (\varphi_{Ak})^{(t)}(w_j).$$

Hence, $\left(G'^{*}_{K,W,Y}\right)^{(t)}$is strong picture fuzzy soft graph. □

7. Conclusion

The main objective of this paper is to introduce the thought of picture fuzzy soft graph and truncation of picture fuzzy soft graph. We tend to additionally delineated truncation of strong picture fuzzy soft graph and truncation of subdivision picture fuzzy soft graph.

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M. Vijaya:Research Advisor, PG and Research Department of Mathematics, Marudupandiyar College, Thanjavur- 613403, Tamilnadu,India, (Affiliated to Bharathidasan University,Trichirappalli).

Email address: mathvijaya23@gmail.com

D. Hema: Research Scholar, PG and Research Department of Mathematics, Marudupandiyar College, Thanjavur- 613403, Tamilnadu,India, (Affiliated to Bharathidasan University, Trichirappalli).

Email address: hema10dm@gmail.com