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AN ALGEBRAIC APPROACH TO THE FIRST AND SECOND REVAN INDICES OF STANDARD GRAPHS

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Abstract. The revan indices are important degree-based topological indices in chemical graph theory due to their applications in characterizing graph structures. The first and second revan indices of a graph G are defined using the revan degree of a vertex, where the revan degree is given by $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$. In this paper, we derive explicit formulae for the first and second revan indices of several standard graph families, namely the crown graph, gear graph, friendship graph, helm graph, and flower graph, and their corresponding line graphs. The results are obtained by determining suitable edge partitions based on the revan degrees of adjacent vertices. The derived formulae provide efficient computational expressions for these graph families and contribute to the study of Revan degree-based topological indices in graph theory.

1. Introduction

Let $G = (V, E)$ be a finite, simple, and connected graph, where $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively. The degree of a vertex $v \in V(G)$, denoted by $d_G(v)$, is defined as the number of vertices adjacent to v. The maximum and minimum vertex degrees of G are represented by $\Delta(G)$ and $\delta(G)$, respectively. Throughout this paper, all graph-theoretic terminology and notation not explicitly defined follow the standard reference [3].

One of the central concepts in chemical graph theory is the topological index, which is a numerical descriptor obtained from the structural features of a molecular graph. Topological indices are invariant under graph isomorphism and provide a compact mathematical representation of molecular structures. These descriptors have been extensively employed in Quantitative StructureProperty Relationship (QSPR) and Quantitative StructureActivity Relationship (QSAR) studies to predict various physicochemical, biological, and pharmacological properties of chemical compounds without the need for costly laboratory experiments.

Among the recently introduced graph invariants, the concept of the revan vertex degree [2] has attracted considerable attention. The revan degree of a vertex $(v \in V(G))$ is defined a $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$, where $(\Delta(G))$ and $(\delta(G))$ denote the maximum and minimum vertex degrees of (G), respectively. This definition assigns larger revan degrees to vertices having smaller ordinary degrees. If $(uv \in E(G))$, then the edge joining the revan vertices u and v is referred to as

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a revan edge, whose properties depend on the corresponding revan vertex degrees $r_G(u)$ and $r_G(v)$. Motivated by the importance of revan degree-based descriptors in mathematical chemistry and molecular structure analysis. We consider two new degree-based topological indices, namely the first revan index and the second Revan index [2], for a molecular graph G. These indices are defined as follows:

$$R_1(G) = \sum_{uv \in E(G)} [r_G(u) + r_G(v)]$$

and

$$R_2(G) = \sum_{uv \in E(G)} r_G(u)\, r_G(v),$$

where $r_G(u)$ and $r_G(v)$ denote the revan degrees of the vertices u and v, respectively.

2. $R_1(G)$ and $R_2(G)$ of Well Known Graphs and Their Line Graphs

The following well known graphs are derived from graph operations on cycle C_n.

2.1. Crown Graph. The graph $C_n \circ K_1$ is called a crown graph CW_n. The graph CW_8 and its corresponding line graph $L(CW_8)$ are shown in Figure 1.

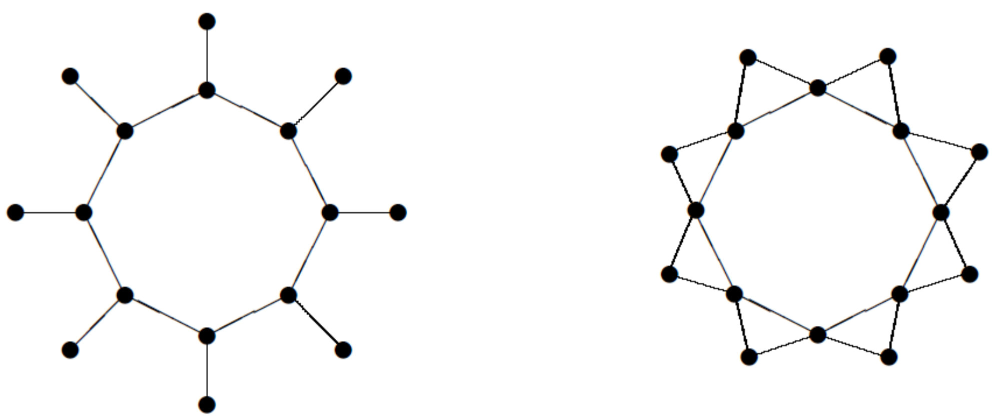

FIGURE 1. CW_8 and $L(CW_8)$.

Theorem 2.1. *For a crown graph CW_n, $(n \geq 4)$*

$$R_1(CW_n) = 6n$$

$$R_2(CW_n) = 4n.$$

Proof. A crown graph CW_n has $2n$ vertices and $2n$ edges with $\Delta(CW_n) = 3$, $\delta(CW_n) = 1$. The edge partition of the crown graph CW_n based on the revan degrees of its vertices is given in table 1. By definition of $R_1(G)$, $R_2(G)$ and from table 1, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(CW_n)$	(1, 1)	(3, 1)
No. of edges	n	n

TABLE 1. Edge partition of CW_n using revan degree of vertices

$$\begin{aligned} R_1(CW_n) &= n(1+1)+n(3+1) \\ &= 6n \end{aligned}$$

$$\begin{aligned} R_2(CW_n) &= n(1\cdot 1)+n(3\cdot 1) \\ &= 4n. \end{aligned}$$

□

Theorem 2.2. *For line graph of crown graph $L(CW_n)$, $(n \geq 4)$*

$$\begin{aligned} R_1(L(CW_n)) &= 12n \\ R_2(L(CW_n)) &= 20n. \end{aligned}$$

Proof. A line graph of crown graph $L(CW_n)$ has $2n$ vertices and $3n$ edges with $\Delta(L(CW_n)) = 4$, $\delta(L(CW_n)) = 2$. The edge partitions of line graph of crown graph $L(CW_n)$ based on the revan degrees of its vertices is given in table 2. By definition of $R_1(G)$, $R_2(G)$ and from table 2, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(L(CW_n))$	(4,2)	(2, 2)
No. of edges	2n	n

TABLE 2. Edge partition of $L(CW_n)$ using revan degree of vertices

$$\begin{aligned} R_1(L(CW_n)) &= 2n(4+2)+n(2+2) \\ &= 12n \end{aligned}$$

$$\begin{aligned} R_2(L(CW_n)) &= 2n(4\cdot 2)+n(2\cdot 2) \\ &= 20n. \end{aligned}$$

□

2.2. Gear Graph. The graph obtained by adding a vertex between every pair of adjacent vertices of the cycle C_n in the wheel W_{n+1} is called a gear graph G_n. The gear graph G_6 and its corresponding line graph $L(G_6)$ are shown in Figure 2.

Theorem 2.3. *For a gear graph G_n, $(n \geq 4)$*

$$\begin{aligned} R_1(G_n) &= 5n^2 - n \\ R_2(G_n) &= 2n^3 - 2n. \end{aligned}$$

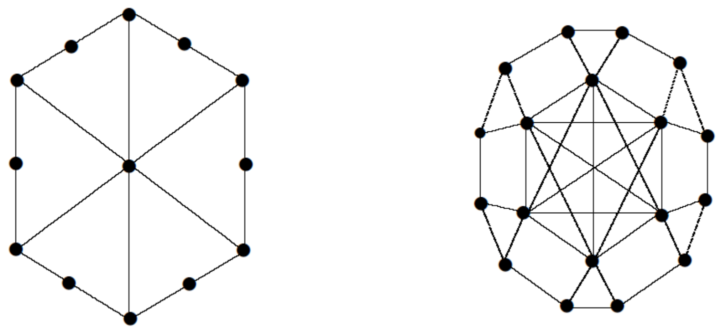

FIGURE 2. G_6 and $L(G_6)$.

Proof. A gear graph G_n has $2n+1$ vertices and $3n$ edges with $\Delta(G_n) = n$, $\delta(G_n) = 2$. The edge partitions of gear graph G_n based on the revan degrees of its vertices is given in table 3. By the definition of $R_1(G)$, $R_2(G)$ and from table 3, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(G_n)$	(n,n-1)	(n-1, 2)
No. of edges	2n	n

TABLE 3. Edge partition of (G_n) using revan degree of vertices

$$\begin{aligned} R_1(G_n) &= 2n(n + (n-1)) + n((n-1) + 2) \\ &= 2n(2n-1) + n(n+1) \\ &= 5n^2 - n \end{aligned}$$

$$\begin{aligned} R_2(G_n) &= 2n(n \cdot (n-1)) + n((n-1) \cdot 2) \\ &= 2n(n^2 - n) + n(2n-2) \\ &= 2n^3 - 2n. \end{aligned}$$

□

Theorem 2.4. *For line graph of gear graph $L(G_n), (n \geq 4)$*

$$R_1(L(G_n)) = 9n(n+1)$$

$$R_2(L(G_n)) = \frac{n(4n^2 + 29n + 7)}{2}.$$

Proof. A line graph of gear graph $L(G_n)$ has $3n$ vertices and $\frac{n^2+7n}{2}$ edges with $\Delta(L(G_n)) = n+1$, $\delta(L(G_n)) = 3$. The edge partitions of line graph of gear graph $L(G_n)$ based on the revan degrees of its vertices is given in table 4. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of $L(G_n)$, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(L(G_n))$	(n+1,n+1)	(n+1, 3)	(3,3)
No. of edges	2n	2n	$\frac{n(n-1)}{2}$

TABLE 4. Edge partition of $(L(G_n))$ using revan degree of vertices

$$\begin{aligned} R_1(L(G_n)) &= 2n((n+1)+(n+1)) + 2n((n+1)+3) + \frac{n(n-1)}{2}(3+3) \\ &= 2n(2n+2) + 2n(n+4) + \frac{n(n-1)}{2}(6) \\ &= 9n(n+1) \end{aligned}$$

$$\begin{aligned} R_2(L(G_n)) &= 2n((n+1)\cdot(n+1)) + 2n((n+1)\cdot 3) + \frac{n(n-1)}{2}(3\cdot 3) \\ &= 2n(n+2)^2 + 2n(3n+3) + \frac{n(n-1)}{2}(9) \\ &= \frac{n(4n^2+29n+7)}{2}. \end{aligned}$$

□

2.3. Friendship Graph. A friendship graph C_3^n [1], is a graph defined as a collection of n copies cycle graph C_3 with a common central vertex. The friendship graph C_3^4 and its corresponding line graph $L(C_3^4)$ are shown in Figure 3.

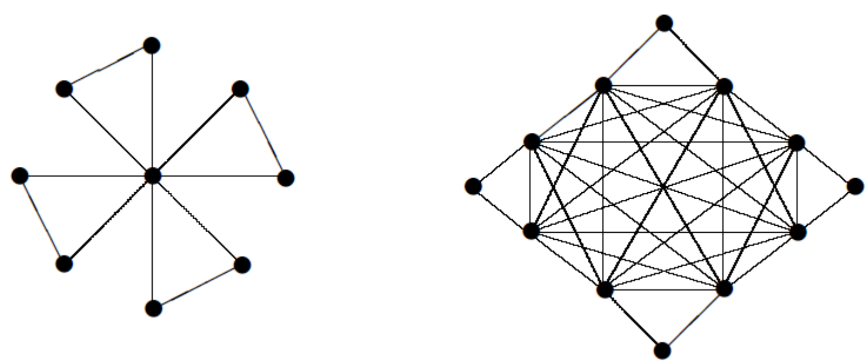

FIGURE 3. C_3^4 and $L(C_3^4)$.

Theorem 2.5. *For a friendship graph C_3^n, $n \geq 2$*

$$\begin{aligned} R_1(C_3^n) &= 10n^2 + 2n \\ R_2(C_3^n) &= 8n^3 + 4n^2. \end{aligned}$$

Proof. A friendship graph C_3^n has $2n+1$ vertices and $3n$ edges with $\Delta(C_3^n) = 2n$, $\delta(C_3^n) = 2$. The edge partitions of friendship graph C_3^n based on the revan degrees of its vertices is given in table 5. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of C_3^n, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(C_3^n)$	(2n,2n)	(2n, 2)
No. of edges	2n	n

TABLE 5. Edge partition of (C_3^n) using revan degree of vertices

$$\begin{aligned} R_1(C_3^n) &= 2n(2n+2n) + n(2n+2) \\ &= 2n(4n) + n(2n+2) \\ &= 10n^2 + 2n \end{aligned}$$

$$\begin{aligned} R_1(C_3^n) &= 2n(2n \cdot 2n) + n(2n \cdot 2) \\ &= 2n(4n^2) + n(4n) \\ &= 8n^3 + 4n^2. \end{aligned}$$

□

Theorem 2.6. *For line graph of friendship graph $L(C_3^n)$, $n \geq 2$*

$$R_1(L(C_3^n)) = 12n^2$$

$$R_2(L(C_3^n)) = 16n^2 - 4n.$$

Proof. A line graph of friendship graph $L(C_3^n)$ has $3n$ vertices and $n(2n+1)$ edges with $\Delta(L(C_3^n)) = 2n$, $\delta(L(C_3^n)) = 2$. The edge partitions of line graph of friendship graph $L(C_3^n)$ based on the revan degrees of its vertices is given in table 6. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of $L(C_3^n)$, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(L(C_3^n))$	(2n,2)	(2, 2)
No. of edges	2n	n(2n-1)

TABLE 6. Edge partition of $(L(C_3^n))$ using revan degree of vertices

$$\begin{aligned} R_1(L(C_3^n)) &= 2n(2n+2) + n(2n-1)(2+2) \\ &= 12n^2 \end{aligned}$$

$$\begin{aligned} R_2(L(C_3^n)) &= 2n(2n \cdot 2) + n(2n-1)(2 \cdot 2) \\ &= 16n^2 - 4n. \end{aligned}$$

□

2.4. Helm Graph. The helm graph H_n [5], is the graph constructed by adjoining a pendant edge at each vertex of the cycle C_n in a wheel graph W_{n+1}.The helm graph H_6 and its corresponding line graph $L(H_6)$ are shown in Figure 4.

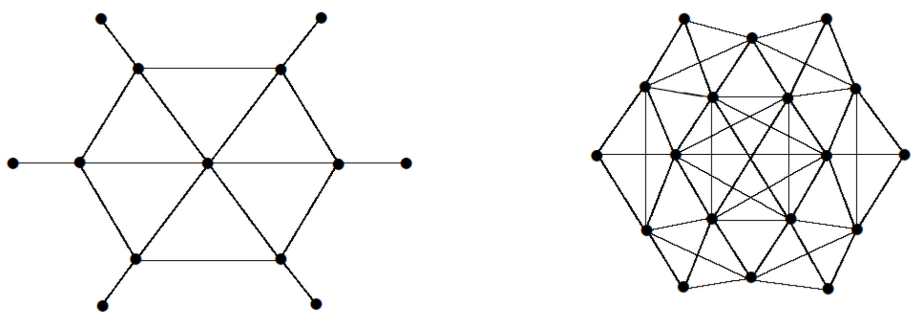

FIGURE 4. H_6 and $L(H_6)$.

Theorem 2.7. *For helm graph H_n, $n \geq 4$*

$$R_1(H_n) = 5n^2 - 11n$$

$$R_2(H_n) = 2n^3 - 8n^2 + 6n.$$

Proof. A helm graph H_n has $2n+1$ vertices and $3n$ edges with $\Delta(H_n) = n$, $\delta(H_n) = 1$. The edge partitions of helm graph $L(H_n)$ based on the revan degrees of its vertices is given in table 7. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of (H_n), for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(H_n)$	(n,n-3)	(n-3, n-3)	(n-3, 1)
No. of edges	n	n	n

TABLE 7. Edge partition of (H_n) using revan degree of vertices

$$\begin{aligned} R_1(H_n) &= n(n+(n-3)) + n((n-3)+(n-3)) + n((n-3)+1) \\ &= n \cdot (2n-3)) + n \cdot ((2n-6)) + n \cdot (n-2) \\ &= 5n^2 - 11n \end{aligned}$$

$$\begin{aligned} R_2(H_n) &= n(n \cdot (n-3)) + n((n-3) \cdot (n-3)) + n((n-3) \cdot 1) \\ &= n(n^2 - 3n)) + n(n-3)^2 + n(n-3) \\ &= 2n^3 - 8n^2 + 6n. \end{aligned}$$

□

Theorem 2.8. *For line graph of helm graph $L(H_n)$, $n \geq 4$*

$$R_1(L(H_n)) = 12n^2 + 6n$$

$$R_1(L(H_n)) = 3n^3 + \frac{27n^2}{2} - \frac{15n}{2}$$

Proof. A line graph of helm graph $L(H_n)$ has $3n$ vertices and $\frac{n(n+11)}{2}$ edges with $\Delta(L(H_n)) = n+2$, $\delta(L(H_n)) = 3$. The edge partitions of line graph of helm graph $L(H_n)$ based on the revan degrees of its vertices is given in table 8. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of $L(H_n)$, for $(n \geq 4)$, we get

$(r_G(u), r_G(v))$ where $uv \in E(L(H_n))$	(3,3)	$(3,\ n-1)$	$(n-1, n+2)$	$(3, n+2)$	$(n-1, n-1)$
No. of edges	$\frac{n(n-1)}{2}$	$2n$	$2n$	n	n

TABLE 8. Edge partition of $(L(H_n))$ using revan degree of vertices

$$\begin{aligned} R_1(L(H_n)) &= \frac{n(n-1)}{2}(3+3) + 2n(3+(n-1)) + 2n((n-1)+(n+2)) + n(3+(n+2)) \\ &\quad + n((n-1)+(n-1)) \\ &= \frac{n(n-1)}{2}(6) + 2n(n+2)) + 2n(2n+1) + n(n+5) + n(2n-2) \\ &= 12n^2 + 6n \\ R_2(L(H_n)) &= \frac{n(n-1)}{2}(3 \cdot 3) + 2n(3 \cdot (n-1)) + 2n((n-1) \cdot (n+2)) + n(3 \cdot (n+2)) \\ &\quad + n((n-1) \cdot (n-1)) \\ &= \frac{n(n-1)}{2}(9) + 2n(3n-3)) + 2n(n^2+n-2) + n(3n+6) + n(n-1)^2 \\ &= 3n^3 + \frac{27n^2}{2} - \frac{15n}{2}. \end{aligned}$$

□

2.5. Flower Graph. A flower graph F_n [4] is generated by joining each pendant vertex to the central vertex of the helm graph. The flower graph F_6 and its corresponding line graph $L(F_6)$ are shown in Figure 5.

Theorem 2.9. *For a flower graph F_n, $n \geq 4$*

$$R_1(F_n) = 12n^2 - 4n$$

$$R_2(F_n) = 4n^2(2n-1).$$

Proof. A flower graph F_n has $2n+1$ vertices and $4n$ edges with $\Delta(F_n) = 2n$, $\delta(F_n) = 2$. The edge partitions of flower graph (F_n) based on the revan degrees of its vertices is given in table 9. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of $L(H_n)$, for $(n \geq 4)$, we get

$$R_1(F_n) = n((2n-2)+(2n-2)) + n(2+(2n-2)) + n((2n-2)+2n) + n(2+2n)$$

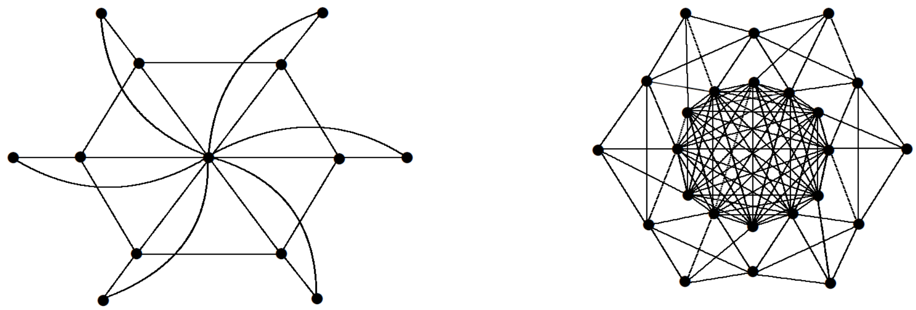

FIGURE 5. F_6 and $L(F_6)$.

$(r_G(u), r_G(v))$ where $uv \in E(F_n)$	(2n-2,2n-2)	(2, 2n-2)	(2n-2, 2n)	(2, 2n)
No. of edges	n	n	n	n

TABLE 9. Edge partition of (F_n) using revan degree of vertices

$$\begin{aligned} &= n(4n-4)+2n^2+n(4n-2)+n(2+2n) \\ &= 12n^2-4n. \end{aligned}$$

$$\begin{aligned} R_2(F_n) &= n((2n-2)\cdot(2n-2))+n(2\cdot(2n-2))+n((2n-2)\cdot 2n)+n(2\cdot 2n) \\ &= n(2n-2)^2+n(4n-4)+2n^2(2n-2)+4n^2 \\ &= 4n^2(2n-1). \end{aligned}$$

□

Theorem 2.10. *For line graph of flower graph $L(F_n)$, $n \geq 4$*

$$R_1(L(F_n)) = 40n^2+16n.$$

$$R_2(L(F_n)) = 12n^3+94n^2-6n.$$

Proof. A line graph of flower graph $L(F_n)$ has $4n$ vertices and $2n(n+3)$ edges with $\Delta(L(F_n)) = 2n+2$, $\delta(L(F_n)) = 4$. The edge partitions of line graph of flower graph $(L(F_n))$ based on the revan degrees of its vertices is given in table 10. By definition of $R_1(G)$, $R_2(G)$ and the edge partitions of $L(F_n)$, for $(n \geq 4)$, we get

$$\begin{aligned} R_1(L(F_n)) &= \frac{n(n-1)}{2}(6+6)+\frac{n(n-1)}{2}(4+4)+n^2(6+4)+n(4+(2n+2))+2n(4+2n) \\ &\quad +n(6+(2n+2))+2n((2n+2)+2n)+n(2n+2n) \\ &= \frac{n(n-1)}{2}(12)+\frac{n(n-1)}{2}(8)+n^2(6+4)+n(2n+6)+2n(4+2n) \\ &\quad +n(2n+8)+2n((4n+2n)+n(4n) \\ &= 40n^2+16n \end{aligned}$$

$$R_2(L(F_n)) = \frac{n(n-1)}{2}(6\cdot 6)+\frac{n(n-1)}{2}(4\cdot 4)+n^2(6\cdot 4)+n(4\cdot(2n+2))+2n(4\cdot 2n)$$

Table 10. Edge partition

(d_u, d_v) where $uv \in E(L(F_n))$	No. of edges
(2n, 2n)	$\frac{n(n-1)}{2}$
(2n+2, 2n+2)	$\frac{n(n-1)}{2}$
(2n, 2n+2)	n^2
(2n+2, 4)	n
(2n+2, 6)	2n
(2n, 4)	n
(4, 6)	2n
(6, 6)	n

$$\begin{aligned} & +n(6 \cdot (2n+2)) + 2n((2n+2) \cdot 2n) + n(2n \cdot 2n) \\ = \quad & \frac{n(n-1)}{2}(36) + \frac{n(n-1)}{2}(16) + n^2(24) + n(8n+8) + 2n(8n) \\ & +n(12n+12) + 2n((4n^2+4n) + n(4n^2) \\ = \quad & 12n^3 + 94n^2 - 6n. \end{aligned}$$

□

3. Conclusion

In this paper, we derived explicit formulae for the first and second Revan indices of the crown graph, gear graph, friendship graph, helm graph, flower graph, and their corresponding line graphs. The results were obtained using edge partitions based on the Revan degrees of adjacent vertices. The derived formulae provide an efficient method for computing these indices and contribute to the study of Revan degree-based topological indices in graph theory and mathematical chemistry. Future work may extend these results to other graph families and graph operations.

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