Global and Stochastic Analysis
Vol. 4 No. 1, January (2017), 63-69

MUK PUBLICATIONS
Open Access Publisher

AN INVENTORY MODEL FOR WEIBULL DETERIORATING ITEMS WITH EXPONENTIAL DEMAND AND TIME-VARYING HOLDING COST

CHAITANYAKUMAR RAPOLU AND DEEPA KANDPAL

Abstract. This paper develops a deterministic inventory model for those inventory systems where stored inventory deteriorates continuously with weibull distribution. It is assumed that the demand rate is exponential with time and the time-varying holding cost is linear. This model is developed without allowing shortages. To check the effect of different parameters the sensitivity analysis is carried out with an example.

1. Introduction

Generally, deterioration is considered as spoilage, damage, devaluation, evaporation etc. Deterioration of physical goods in stock is a real characteristic. Fruits, vegetables, meat, foodstuffs, gasoline, fashion and seasonal goods, blood, chemicals, medicines, radioactive substances, photographic films, etc., deteriorate during their usual storage period. So it is necessary to develop optimal inventory policies to control and maintain different kind of deteriorating items.

From time to time many researchers have extensively studied deteriorating inventory systems. Whitin [13] for the first time he studied fashion items deteriorating at the end of prescribed storage period. Ghare and Schrader [4] first proposed exponential deteriorating inventory model with constant demand. Shah and Jaiswal [12] considered uniform deterioration rate. Covert and Philip [1] established a model for items with two parameter weibull deterioration. Dave and Patel [3] considered linear demand and constant deterioration rate. Dash et al. [2] considered constant deterioration rate with exponential declining demand. Goel and Aggarwal [6] formulated an inventory model considering power pattern demand. Mandal and Phaujdar [7] derived an EPQ model for deteriorating items considering linear stock-dependent demand and constant production rate. Pal et al. [10], Padmanabhan and Vrat [9], Giri et al. [5], Ray et al. [11] considered stock dependent demand rate. Mishra [8] developed an inventory model for time dependent deterioration with salvage value.

In present paper a deterministic economic order quantity model is derived considering two parameter weibull deterioration rate with exponential demand function and linear holding cost.

2000 *Mathematics Subject Classification.* 90B05.
Key words and phrases. Inventory, Deteriorating item, Weibull, Exponential.

2. Assumptions

- Two parameter weibull deterioration rate.
- Demand rate is exponential function of time.
- Holding cost is a linear function of time.
- Shortages are not permitted.
- Lead time is negligible.
- Instant and infinite replenishment rate.

3. Notations

- $D(t) = ke^{\gamma t}$ is demand function; $0 \leq \gamma \ll 1$.
- $I(t)$ = Inventory level at any instant of time t; $0 \leq t \leq T$.
- A = Ordering cost per order.
- C_d = Deterioration cost per unit (purchase price minus salvage value).
- $h(t) = x + yt$, inventory variable holding cost.
- $\alpha\beta t^{\beta-1}$ = The two parameter weibull deterioration rate; $0 \leq \alpha \leq 1$, $\beta > 0$

4. Model development

The inventory level will continuously decrease with time due to demand and deterioration. The change of inventory level can be described by the below differential equation.

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -ke^{\gamma t}; 0 \leq t \leq T \tag{4.1}$$

Solution of equation (4.1) using boundary condition $I(T) = 0$ is

$$I(t) = k[(T-t) + \frac{\gamma(T^2-t^2)}{2} + \frac{\alpha(T^{\beta+1}-t^{\beta+1})}{\beta+1} + \frac{\alpha\gamma(T^{\beta+2}-t^{\beta+2})}{\beta+2} + \alpha t^{\beta}(t-T)] \tag{4.2}$$

Initial order quantity at $t = 0$ is

$$I_0 = I(0) = k[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}] \tag{4.3}$$

Ordering cost per unit time is

$$OC = \frac{A}{T} \tag{4.4}$$

Total demand during the cycle period $[0, T]$ is

$$\int_0^T D(t)dt = \int_0^T ke^{\gamma t} dt = \frac{k}{\gamma}[e^{\gamma T} - 1] \tag{4.5}$$

The number of deteriorated units in $[0, T]$ is

$$I_0 - \int_0^T D(t)dt = k[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}] - \frac{k}{\gamma}[e^{\gamma T} - 1] \tag{4.6}$$

Cost due to deterioration per unit time is

$$DC = \frac{kC_d}{T}[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}] - \frac{kC_d}{\gamma T}[e^{\gamma T} - 1] \tag{4.7}$$

Inventory variable holding cost per unit time is

$$HC = \frac{1}{T}\int_0^T (x+yt)I(t)dt$$
$$= \frac{xk}{T}[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3}]$$
$$+\frac{yk}{T}[\frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha\beta T^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha\gamma T^{\beta+4}}{2(\beta+4)}] \tag{4.8}$$

Total variable inventory cost per unit time is

$$TC(T) = OC + DC + HC$$
$$= \frac{A}{T} + \frac{kC_d}{T}[T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{\alpha\gamma T^{\beta+2}}{\beta+2}] - \frac{kC_d}{\gamma T}[e^{\gamma T} - 1]$$
$$+\frac{xk}{T}[\frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\gamma T^{\beta+3}}{\beta+3}]$$
$$+\frac{yk}{T}[\frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha\beta T^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{\alpha\gamma T^{\beta+4}}{2(\beta+4)}] \tag{4.9}$$

Our aim is to minimize the total variable inventory cost per unit time, the necessary and sufficient conditions to minimize total cost $TC(T)$ for a given value T are

$$\frac{\partial TC(T)}{\partial T} = -\frac{A}{T^2} + kC_d[\frac{\gamma}{2} + \frac{\alpha\beta T^{\beta-1}}{\beta+1} + \frac{\alpha\gamma(\beta+1)T^{\beta}}{\beta+2}] - \frac{kC_d}{\gamma T^2}[\gamma T e^{\gamma T} - e^{\gamma T} + 1]$$
$$+xk[\frac{1}{2} + \frac{2\gamma T}{3} + \frac{\alpha\beta T^{\beta}}{\beta+2} + \frac{\alpha\gamma(\beta+2)T^{\beta+1}}{\beta+3}]$$
$$+yk[\frac{T}{3} + \frac{3\gamma T^2}{8} + \frac{\alpha\beta T^{\beta+1}}{2(\beta+3)} + \frac{\alpha\gamma(\beta+3)T^{\beta+2}}{2(\beta+4)}] = 0. \tag{4.10}$$

$$\frac{\partial^2 TC(T)}{\partial^2 T} = \frac{A}{T^3} + kC_d[\frac{\alpha\beta(\beta-1)T^{\beta-2}}{\beta+1} + \frac{\alpha\gamma(\beta+1)T^{\beta-1}}{\beta+2}]$$
$$-\frac{kC_d}{\gamma}[(\frac{T\gamma^2 e^{\gamma T} - \gamma e^{\gamma T}}{T^2}) - (\frac{T\gamma^2 e^{\gamma T} - 2T\gamma e^{\gamma T}}{T^4}) - \frac{2}{T^3}]$$
$$+xk[\frac{2\gamma}{3} + \frac{\alpha\beta^2 T^{\beta-1}}{\beta+2} + \frac{\alpha\gamma(\beta+1)(\beta+2)T^{\beta}}{\beta+3}]$$
$$+yk[\frac{1}{3} + \frac{3\gamma T}{4} + \frac{\alpha\beta(\beta+1)T^{\beta}}{2(\beta+3)} + \frac{\alpha\gamma(\beta+2)(\beta+3)T^{\beta+1}}{2(\beta+4)}] > 0. \tag{4.11}$$

5. Numerical Example and Sensitivity Analysis

5.1. Example-1. In the above developed model, consider the following values of different parameters in proper units.

$A = 500$, $C_d = 15$, $\alpha = 0.04$, $\beta = 2$, $k = 250$, $\gamma = -0.02$, $x = 5$, $y = 0.05$.
Solving equation (4.9) in R programming with the above values of parameters, we obtain the optimal cycle length $T^* = 0.837232$ and from equation (4.3) and (4.9), the optimal order quantity $Q^* = 209.487252$ and the optimal total inventory cost $TC^* = 1155.314$.

5.2. Sensitivity Analysis. Now we study the effect of changes in values of parameters $A, \alpha, \beta, k, \gamma, x$ and y with Example-1.

TABLE 1

Parameter		% Change	T^*	Q^*	TC^*
A	750	+50%	1.009976	253.33466	1425.798
	600	+20%	0.911251	228.23018	1269.683
	400	-20%	0.754205	188.54582	1029.665
	250	-50%	0.603840	150.77671	808.9180
α	0.06	+50%	0.812102	204.02205	1174.388
	0.048	+20%	0.826713	207.20167	1163.092
	0.032	-20%	0.848469	211.92560	1147.321
	0.02	-50%	0.866897	215.91716	1134.884
β	3	+50%	0.839592	209.36129	1141.236
	2.4	+20%	0.837651	209.24816	1148.644
	1.6	-20%	0.838150	210.18216	1164.078
	1	-50%	0.844641	212.90362	1183.654
γ	-0.01	+50%	0.832727	211.58570	1158.625
	-0.016	+20%	0.835402	210.31658	1156.653
	-0.024	-20%	0.839101	208.67100	1153.955
	-0.03	-50%	0.841979	207.47011	1151.877
k	375	+50%	0.692127	259.39180	1406.233
	300	+20%	0.768798	230.66288	1261.862
	200	-20%	0.928626	186.10622	1037.487
	125	-50%	1.151517	144.78302	828.5111
x	7.5	+50%	0.703641	175.82148	1393.743
	6	+20%	0.775622	193.93878	1255.751
	4	-20%	0.915131	229.20865	1046.177
	2.5	-50%	1.081919	271.70633	860.2226
y	0.075	+50%	0.836283	209.24744	1156.040
	0.06	+20%	0.836852	209.39123	1155.605
	0.04	-20%	0.837613	209.58353	1155.023
	0.025	-50%	0.838170	209.72430	1154.586

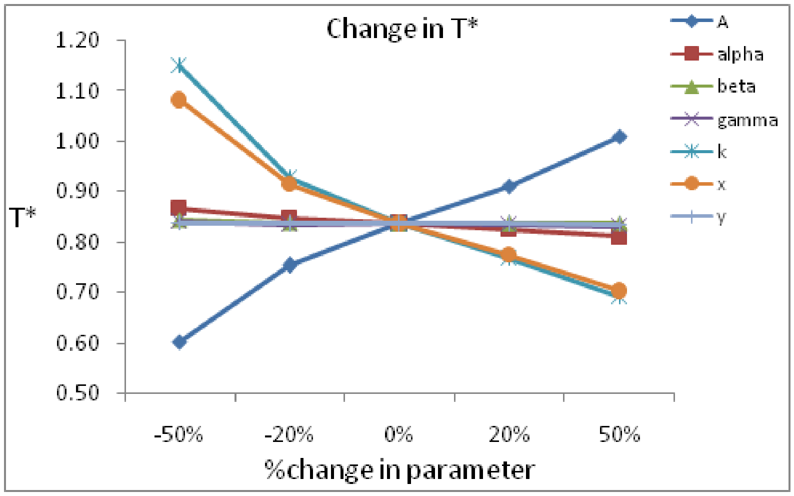

FIGURE 1

From above table and chart it is clear that as ordering cost A increases, the optimal cycle length T^* increases. As demand parameters, deterioration parameters and carrying cost parameters increases, the optimal cycle length T^* decreases. That is as $k, \gamma, \alpha, \beta, x$ and y increases, T^* decreases.

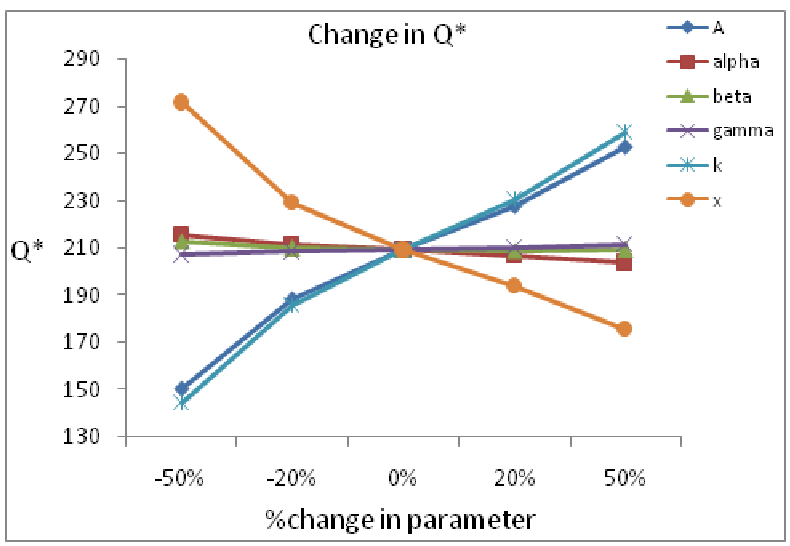

FIGURE 2

In above chart we can observe that as ordering cost A and demand parameters k and γ increases, the optimal order quantity Q^* increases. Also as deterioration and carrying cost parameters α, β and x increases, Q^* decreases.

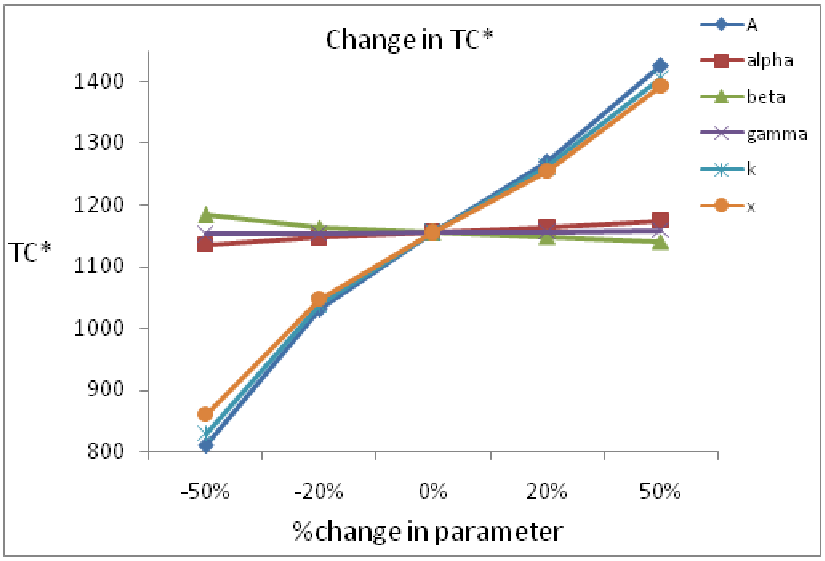

FIGURE 3

From table-1 and above chart we can observe that as A increases, that is as ordering cost increases the corresponding total cost increases. Increase in parameter α results in increase in total cost, however increase in shape parameter (β) of weibull distribution results in decrease in the ordering quantity Q^* and hence total cost also decreases. We can also observe that as x and y, the parameters associated with the carrying cost increases, the corresponding total cost increases. Increase in k and γ, the parameters associated with demand also results in increase in total cost.

5.3. Example-2. $A = 500$, $C_d = 20$, $\alpha = 0.08$, $\beta = 4$, $k = 400$, $\gamma = 0.01$, $x = 10$, $y = 0.05$. Solving equation (4.9) in R programming with the above values of parameters, we obtain the optimal cycle length $T^* = 0.4779953$ and from equation (4.3) and (4.9), the optimum order quantity $Q^* = 195.933769$ and the optimal total variable inventory cost $TC^* = 2038.264$.

6. Conclusion

In present paper we approximated optimal inventory ordering policies for the items having Weibull deterioration rate and exponential demand. It is supposed that the carrying cost is linear function of time. The sensitivity analysis clearly shows increase/decrease in the value of cost with the corresponding increase/decrease in the parameter values.

References

1. Covert, Richard P., and George C. Philip. "An EOQ model for items with Weibull distribution deterioration." AIIE transactions 5.4 (1973): 323-326.
2. Dash, Bhanu Priya, Trailokyanath Singh, and Hadibandhu Pattnayak. "An inventory model for deteriorating items with exponential declining demand and time-varying holding cost." American Journal of Operations Research 4.01 (2014): 1.
3. Dave, Upendra, and L. K. Patel. "(T, S i) policy inventory model for deteriorating items with time proportional demand." Journal of the Operational Research Society 32.2 (1981): 137-142.
4. Ghare, P. M., and G. F. Schrader. "A model for exponentially decaying inventory." Journal of industrial Engineering 14.5 (1963): 238-243.
5. Giri, B. C., et al. "An inventory model for deteriorating items with stock-dependent demand rate." European Journal of Operational Research 95.3 (1996): 604-610.
6. Goel VP, Aggarwal SP. Order level inventory system with power demand pattern for deteriorating items. InProceedings of the All India Seminar on Operational Research and Decision Making 1981 Mar (pp. 19-34). New Delhi: University of Delhi.
7. Mandal, B. N., and S. Phaujdar. "An inventory model for deteriorating items and stock-dependent consumption rate." Journal of the operational Research Society 40.5 (1989): 483-488.
8. Mishra, Vinod Kumar. "Inventory model for time dependent holding cost and deterioration with salvage value and shortages." The Journal of Mathematics and Computer Science 4.1 (2012): 37-47.
9. Padmanabhan, G., and Prem Vrat. "EOQ models for perishable items under stock dependent selling rate." European Journal of Operational Research 86.2 (1995): 281-292.
10. Pal, S., A. Goswami, and K. S. Chaudhuri. "A deterministic inventory model for deteriorating items with stock-dependent demand rate." International Journal of Production Economics 32.3 (1993): 291-299.
11. Ray, J., A. Goswami, and K. S. Chaudhuri. "On an inventory model with two levels of storage and stock-dependent demand rate." International Journal of Systems Science 29.3 (1998): 249-254.
12. Shah, Y. K., and M. C. Jaiswal. "An order-level inventory model for a system with constant rate of deterioration." Opsearch 14.3 (1977): 174-184.
13. Whitin, Thomson M. Theory of inventory management. Princeton University Press, 1957.

Department of statistics, The Maharaja Sayajirao University of Baroda, Vadodara, India

E-mail address: rapolu123@gmail.com

Department of statistics, The Maharaja Sayajirao University of Baroda, Vadodara, India

E-mail address: dhk052000@yahoo.com