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MODELING FINANCIAL MARKET INTERPOLATIONS USING MARTINGALE DEFLATORS

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Abstract. In this work a new types of deflators are considered. We call these deflators interpolating deflators. They may be applied to construct interpolations of financial markets, including arbitrage ones. In the arbitrage-free case the interpolations obtained with the help of strictly positive deflators coincide with interpolations obtained with the help of equivalent martingale measures. In the case of static market, when the stock under consideration admits in the terminal time 3 values, we prove the criterion on the coincidence of all admissible deflators with all interpolating deflators. This result is a generalization of the correspondent result about interpolating martingale measures.

1. Introduction

The problem of interpolating stochastic systems in order to improve their properties is very important for carrying out various kinds of calculations within these systems, as well as for making optimal decisions. One of such problems, namely the task of transforming incomplete financial markets into complete ones, was considered in 1987 in the work of M. Takku and W. Willinger [1]. In this work the transition from incomplete markets to complete markets was carried out by replacing the original martingale measure with a nonequivalent martingale measure. Another technique was proposed by A.V. Melnikov and K.M. Feoktistov [2]. They completed the financial market by adding additional risky assets to the shares of this original market, functionally dependent on the original ones. In the works of I.V. Pavlov and M.N. Bogacheva [3]–[4] the foundation was laid for a principally different method of transition from incomplete markets to complete ones. To solve the problem of transforming incomplete arbitrage-free markets into complete arbitrage-free financial markets, the method of interpolation of financial markets was used, associated with the use of Haar filtrations and martingale interpolation. Further, in this direction, research continued in the articles of I.V. Pavlov, V.V. Gorgorova, A.G. Danekyants, T.A. Volosatova, V.V. Shamraeva, I.V. Tsvetkova, N.V. Neumerzhitskaia [5]–[16]. The following definition of deflators is well-known.

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Definition 1.1. Let $\big(\Omega, F = (\mathcal{F}_k)_{k=0}^K, P\big)$ be a filtered space, where Ω is a set $K < \infty$, $F = (\mathcal{F}_k)_{k=0}^K$ be a strictly increasing family of σ-fields on Ω (filtration) and P be a probability on $\mathcal{F}_K$. Let $Z = (Z_k, \mathcal{F}_k)_{k=0}^K$ be an adabted process (stock price). A process $D = (D_k, \mathcal{F}_k, P)_{k=0}^K$ is said deflator of the stock price Z if

(1) $D_0 = 1$ (normalization condition);
(2) $D = (D_k, \mathcal{F}_k, P)_{k=0}^K$ is a martingale;
(3) $DZ = (D_k Z_k, \mathcal{F}_k, P)_{k=0}^K$ is a martingale.

In this work we begin to investigate so-called interpolating deflators. The definition of interpolating deflators is a new one. This definition allows the deflator to accept non-positive values, which makes it possible to study both arbitrage and arbitrage-free financial markets. The main result of this article is Theorem 1, in which a very important case is considered, when all admissible deflators are interpolating deflators.

2. Interpolation of stock prices with the help of deflators and deflator uniqueness properties

In this section and below, we assume that $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ is a finite set, $F = (\mathcal{F}_k)_{k=0}^1$ is a one-step filtration on Ω, where $\mathcal{F}_0 = \{\Omega, \emptyset\}$ and $\mathcal{F}_1 = \sigma\{\omega_1, \omega_2, \dots, \omega_n\}$. Fix on $\mathcal{F}_1$ a non-degenerated probability measure P (physical probability). Denote $p_k = P(\omega_k) > 0$ $(k = 1, 2, \dots, n)$. Let $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$ be an adapted process. Denote $a = Z_0$, $b_k = Z_1(\omega_k)$ $(k = 1, 2, \dots, n)$. Despite the fact that a and b_k can take negative and zero values, we will call the process Z a stock price. Consider another adapted process $D = (D_k, \mathcal{F}_k)_{k=0}^1$. Then, for $K = 1$, the conditions of Definition 1.1 take the form (E^P is the expectation with respect to P):

$$D_0 = 1; \tag{2.1}$$

$$E^P[D_1] = D_0; \tag{2.2}$$

$$E^P[D_1 Z_1] = D_0 Z_0. \tag{2.3}$$

Definition 2.1. A deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$ is said admissible if for all nonempty subset $I \subset \{1, 2, \dots, n\}$ the following inequality is true:

$$\sum_{k \in I} p_k d_k \neq 0. \tag{2.4}$$

Remark 2.2. If we take $I = \{1, 2, \dots, n\}$, the inequality (2.4) is satisfied automatically. For such I that $|I| = 1$ the inequalities (2.4) are equivalent to $d_k \neq 0$ $(k = 1, 2, \dots, n)$. For such I that $|I| = n - 1$ the inequalities (2.4) are equivalent to $d_k \neq \frac{1}{p_k}$ $(k = 1, 2, \dots, n)$.

It is trivial that strictly positive deflators satisfy the condition (2.4).

The following idea of the interpolation of a stock price using a deflator is a new one. Consider a sequence of moments of time $0 = t_0 < t_1 < \dots < t_m = 1$ and a filtration $H = (\mathcal{H}_{t_i})_{i=0}^m$ satisfying the conditions: $\mathcal{H}_{t_0} = \mathcal{F}_0$ and $\mathcal{H}_{t_m} = \mathcal{F}_1$.

We call such filtration interpolating filtration of $F = (\mathcal{F}_k)_{k=0}^1$. Construct two martingales:

$$X = (X_{t_i}, \mathcal{H}_{t_i})_{i=0}^m, \; where \; X_{t_i} = E^P \left[D_1 Z_1 | \mathcal{H}_{t_i} \right]$$

and

$$Y = (Y_{t_i}, \mathcal{H}_{t_i})_{i=0}^m, \; where \; Y_{t_i} = E^P \left[D_1 | \mathcal{H}_{t_i} \right].$$

It is obvious that since $\{D_1 = 0\} \subset \{D_1 Z_1 = 0\}$ P-a.s., we have $\{Y_{t_i} = 0\} \subset \{Y_{t_i} = 0\}$ P-a.s.

Definition 2.3. We say that the process $Z^{int} = \left(Z_{t_i}^{int}, \mathcal{H}_{t_i}\right)_{i=0}^m$, where

$$Z_{t_i}^{int} = \begin{cases} \frac{X_{t_i}}{Y_{t_i}}, & if \; Y_{t_i} \neq 0, \\ 1, & if \; Y_{t_i} = 0, \end{cases} \tag{2.5}$$

is H-interpolation of stock price Z with the help of deflator D.

It is trivial, that $Z_0^{int} = \frac{X_0}{Y_0} = Z_0$ and $Z_1^{int} = \frac{X_1}{Y_1} = Z_1$ (cf. Remark 2.2).

Example 2.4. Let us consider arbitrage-free situation, i.e. the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$ admits a martingale measure Q , equivalent to the physical measure P. Denote $D_0 = 1$ and $D_1 = \frac{dQ}{dP}$. It is well-known that the process $D = (D_k, \mathcal{F}_k)_{k=0}^1$ is a strictly positive deflator of the stock price Z. Using Bayes formula, we get

$$E^Q \left[Z_1 | \mathcal{H}_{t_i} \right] = \frac{E^P \left[D_1 Z_1 | \mathcal{H}_{t_i} \right]}{E^P \left[D_1 Z_1 | \mathcal{H}_{t_i} \right]} = \frac{X_{t_i}}{Y_{t_i}} = Z_{t_i}^{int}, \tag{2.6}$$

and H-interpolation of stock price Z with the help of deflator D coincides with the martingale interpolation of Z with respect to the martingale measure Q (c.f. [3]–[4]. Conversely, if we have a strictly positive deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ and define $dQ = D_1 dP$, we obtain a martingale measure Q and the same effect.

The following new definition of interpolating deflators is especially important for further considerations.

Definition 2.5. Let $\mathbf{H}$ =$\{H_\alpha\}_{\alpha \in A}$ be a family of interpolating filtrations of $F = (\mathcal{F}_k)_{k=0}^1$. We say that a deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ is $\mathbf{H}$-interpolating deflator if for all $H = H_\alpha \in \mathbf{H}$ the stock price interpolation $Z^{int} = \left(Z_{t_i}^{int}, \mathcal{H}_{t_i}\right)_{i=0}^m$ admits only one deflator, namely, the deflator $Y = (Y_{t_i}, \mathcal{H}_{t_i})_{i=0}^m$ (c.f. general definition 1.1 of a deflator).

Example 2.6. Consider the arbitrage-free situation again. We say that a martingale measure $Q \sim P$ is $\mathbf{H}$-interpolating if for all $H = H_\alpha \in \mathbf{H}$ the stock price interpolation $\left(E^Q \left[Z_1 | \mathcal{H}_{t_i} \right], \mathcal{H}_{t_i}\right)_{i=0}^m$ admits only one martingale measure, namely, the martingale measure $Q \sim P$. From formula (2.6) and from the one-to-one correspondence between martingale measures and strictly positive deflators, it follows that a martingale measure $Q \sim P$ is $\mathbf{H}$-interpolating iff the corresponding deflator is $\mathbf{H}$-interpolating. The theory of interpolating martingale measures in the case when $\mathbf{H}$ is a set of interpolating Haar filtrations (or a set of special interpolating Haar filtrations) was developed in the articles [3]–[16].

3. Some remarks on deflators of stock price $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$

Consider first the case $n = 1$, i.e. $\Omega = \{\omega_1\}$. This is a deterministic situation. We have: $Z_0 = a$, $Z_1 = b_1$, $D_0 = D_1 = 1$. If $a \neq b_1$, there are not any deflators. If $a = b_1$, there is a single deflator $D_0 = D_1 = 1$.

Now consider the case $n = 2$, i.e. $\Omega = \{\omega_1, \omega_2\}$. It is easy to see that if $b_1 \neq b_2$, a deflator exists and is unique. If $b_1 = b_2 = a$, there are infinitely many deflators. However, none of these deflators are interpolating, because there are no strictly intermediate σ-fields between $\mathcal{F}_0$ and $\mathcal{F}_1$. If $b_1 = b_2 \neq a$, there are not any deflators.

The case $n = 3$ is more interesting, so we will consider it in more detail. Denote $d_1 = D_1(\omega_1)$, $d_2 = D_1(\omega_2)$, $d_3 = D_1(\omega_3)$. The system for finding deflators (i.e. the numbers d_1, d_2, d_3) has the form:

$$\begin{cases} p_1 d_1 + p_2 d_2 + p_3 d_3 = 1 \\ p_1 b_1 d_1 + p_2 b_2 d_2 + p_3 b_3 d_3 = a. \end{cases} \tag{3.1}$$

If $b_1 = b_2 = b_3 \neq a$, there are not any deflators. If $b_1 = b_2 = b_3 = a$, there are infinitely many deflators. It is easy to show that in this case there are also no interpolation deflators.

Now let at least two of the numbers b_1, b_2, b_3 be different. Suppose $b_1 \neq b_2$ (other cases are similar). Solving system (3.1) with respect to d_1 and d_2, we obtain infinitely many deflators:

$$\begin{cases} d_1 = \frac{(b_2-a)+p_3(b_3-b_2)d_3}{p_1(b_2-b_1)} \\ d_2 = \frac{(a-b_1)+p_3(b_1-b_3)d_3}{p_2(b_2-b_1)}. \end{cases} \tag{3.2}$$

In case $n > 3$ we get the same as in the case $n = 3$.

4. Interpolating deflators in the case $n = 3$

Let $D = (D_k, \mathcal{F}_k)_{k=0}^1$ be an admissible deflator of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$. In the case $n = 3$ the admissibility is equivalent to the fulfillment of the inequalities $d_k \neq 0$ and $d_k \neq \frac{1}{p_k}$ $k = 1, 2, 3$. Let $A = \{1, 2, 3\}$. Define the interpolating filtrations $H^{(\alpha)} = \left(\mathcal{H}_{t_i}^{(\alpha)}\right)_{i=0}^2$ $(\alpha \in A)$ of the filtration $F = (\mathcal{F}_k)_{k=0}^1$ in the following way: $\mathcal{H}_{t_1}^{(1)} = \sigma\{\omega_1\}$, $\mathcal{H}_{t_1}^{(2)} = \sigma\{\omega_2\}$, $\mathcal{H}_{t_1}^{(3)} = \sigma\{\omega_3\}$. It is obvious that, apart from $H^{(1)}$, $H^{(2)}$ and $H^{(3)}$ there are no strictly intermediate σ-fields between $\mathcal{F}_0$ and $\mathcal{F}_1$. Therefore, apart from $H^{(1)}$, $H^{(2)}$ and $H^{(3)}$ there are no interpolating filtrations of $F = (\mathcal{F}_k)_{k=0}^1$. Define $\mathbf{H} = \{H^{(1)}, H^{(2)}, H^{(3)}\}$.

Theorem 4.1. *1) If the numbers a, b_1, b_2, b_3 are different, then any admissible deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$ is the $\mathbf{H}$-interpolating one.*

2) If there exists an admissible $\mathbf{H}$-interpolating deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$, then the numbers a, b_1, b_2, b_3 are different.

Proof. For any $\alpha \in A$ construct the process $Z^{int} = \left(Z_{t_i}^{int}, \mathcal{H}_{t_i}\right)_{i=0}^2$ ($\mathbf{H}$-interpolation of stock price Z with the help of admissible deflator D) and prove that this deflator is interpolating one. Let $\alpha = 1$. We have (c.f. 2.3):

$$X_0 = a,\ X_{t_1} = E^P\left[D_1 Z_1 | \mathcal{H}_{t_1}^{(1)}\right] = b_1 d_1 I_{\{\omega_1\}} + \frac{b_2 d_2 p_2 + b_3 d_3 p_3}{p_2 + p_3} I_{\{\omega_2,\omega_3\}},$$
$$X_1 = b_1 d_1 I_{\{\omega_1\}} + b_2 d_2 I_{\{\omega_2\}} + b_3 d_3 I_{\{\omega_3\}};$$
$$Y_0 = 1,\ Y_{t_1} = E^P\left[D_1 | \mathcal{H}_{t_1}^{(1)}\right] = d_1 I_{\{\omega_1\}} + \frac{d_2 p_2 + d_3 p_3}{p_2 + p_3} I_{\{\omega_2,\omega_3\}},$$
$$Y_1 = d_1 I_{\{\omega_1\}} + d_2 I_{\{\omega_2\}} + d_3 I_{\{\omega_3\}};$$
$$Z_0^{int} = a,\ Z_{t_1}^{int} = \frac{X_{t_1}}{Y_{t_1}} = b_1 I_{\{\omega_1\}} + \frac{b_2 d_2 p_2 + b_3 d_3 p_3}{b_2 p_2 + b_3 p_3} I_{\{\omega_2,\omega_3\}},$$
$$Z_1^{int} = b_1 I_{\{\omega_1\}} + b_2 I_{\{\omega_2\}} + b_3 I_{\{\omega_3\}}.$$

Let now $\hat{D}$ be arbitrary deflator of the process Z^{int}. Since $\hat{D}$ is a martingale, it is of the form:

$$\hat{D}_0 = 1, \hat{D}_{t_1} = \hat{d}_1 I_{\{\omega_1\}} + \frac{\hat{d}_2 p_2 + \hat{d}_3 p_3}{p_2 + p_3} I_{\{\omega_2,\omega_2\}}, \hat{D}_1 = \hat{d}_1 I_{\{\omega_1\}} + \hat{d}_2 I_{\{\omega_2\}} + \hat{d}_3 I_{\{\omega_3\}}.$$

Prove that $\hat{D} = Y$. Consider the process $\hat{D} Z^{int}$:

$$\hat{D}_0 Z_0^{int} = a,\ \hat{D}_{t_1} Z_{t_1}^{int} = b_1 \hat{d}_1 I_{\{\omega_1\}} + \frac{b_2 d_2 p_2 + b_3 d_3 p_3}{b_2 p_2 + b_3 p_3} \cdot \frac{\hat{d}_2 p_2 + \hat{d}_3 p_3}{p_2 + p_3} I_{\{\omega_2,\omega_3\}},$$
$$\hat{D}_1 = b_1 \hat{d}_1 I_{\{\omega_1\}} + b_2 \hat{d}_2 I_{\{\omega_2\}} + b_3 \hat{d}_3 I_{\{\omega_3\}}.$$

The martingale property of this process implies the equality:

$$\frac{b_2 \hat{d}_2 p_2 + b_3 \hat{d}_3 p_3}{p_2 + p_3} = \frac{b_2 d_2 p_2 + b_3 d_3 p_3}{b_2 p_2 + b_3 p_3} \cdot \frac{\hat{d}_2 p_2 + \hat{d}_3 p_3}{p_2 + p_3}.$$

Denoting $c = \frac{b_2 d_2 p_2 + b_3 d_3 p_3}{b_2 p_2 + b_3 p_3}$, we obtain the following equation for finding $\hat{d}_1, \hat{d}_2$ and $\hat{d}_3$:

$$p_2(b_2 - c)\hat{d}_2 + p_3(b_3 - c)\hat{d}_3 = 0.$$

Adding the equations of the system (3.1) written for $\hat{d}_1, \hat{d}_2$ and $\hat{d}_3$, we get:

$$\begin{cases} p_1 \hat{d}_1 + p_2 \hat{d}_2 + p_3 \hat{d}_3 = 1 \\ p_1 b_1 \hat{d}_1 + p_2 b_2 \hat{d}_2 + p_3 b_3 \hat{d}_3 = a \\ p_2(b_2 - c)\hat{d}_2 + p_3(b_3 - c)\hat{d}_3 = 0. \end{cases} \tag{4.1}$$

It is clear that $\hat{d}_1 = d_1, \hat{d}_2 = d_2, \hat{d}_3 = d_3$ is a solution of the system (4.1). Prove that this solution is unique. The main determinant of system (4.1) is

$$\Delta = p_1 p_2 p_3 (b_1 - c)(b_2 - c).$$

But it follows from (2.6): $b_1 - c = b_1 - \frac{a - b_1 p_1 d_1}{1 - p_1 d_1} = \frac{b_1 - a}{1 - p_1 d_1}$. Hence $\Delta = \frac{p_1 p_2 p_3}{1 - p_1 d_1}(b_1 - a)(b_2 - b_3) \neq 0$ and the equation (4.1) has only one solution.

We get the same for $\alpha = 2$ and $\alpha = 3$. Therefore the deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$ is the **H**-interpolating one.

3) Let there exist an admissible **H**-interpolating deflator $D = (D_k, \mathcal{F}_k)_{k=0}^1$ of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$. Let $\alpha = 1$. Repeating the reasoning in part 1) of the proof, we find that $\Delta = \frac{p_1 p_2 p_3}{1 - p_1 d_1}(b_1 - a)(b_2 - b_3) \neq 0$. Hence $a \neq b_1$ and $b_2 \neq b_3$

Performing the same actions for $\alpha = 2$ and $\alpha = 3$, we obtain, respectively,$a \neq b_2$, $b_1 \neq b_3$ and $a \neq b_3$, $b_1 \neq b_2$. As a result, we get that the numbers a, b_1, b_2, b_3 are different. The proof is terminated. $\square$

Remark 4.2. The case $n > 3$ is much more complicated. The second part of the Theorem 4.1 remains true in this case as well. As for the first part, it can be proved that for $n > 3$ there always exist admissible deflators that are not interpolating ones.

5. Conclusion

The most interesting question consists in the following: find the sufficient conditions on parameters of the market, under which interpolating deflators exist (for interpolating martingale measures c.f., for example, [14], [15]).

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