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WORKING VACATION IN QUEUEING - STOCK SYSTEM WITH DELUSIVE SERVER

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Abstract. In this proposal, we contemplate a uninterrupted review queueing-stock system and magnitude of waiting space is N with volume stock of S units. We speculate that purchasers check in as per the Poisson process accompanied by parameter $\lambda(>0)$ and claim absolutely one quantity of item. The server furnishes a couple of service. An order for $Q(=\tilde{S}-\tilde{s}>\tilde{s}+1)$ element is emplaced at any time the stock level plunges to $\tilde{s}$ and the things are acquired only later a arbitrary duration which is dispersed as exponential with parameter$\beta(>0)$. The delusive server is otherwise known as breakdown server which means normal operation has been stopped for an while due to the various causes. The server commences a working vacation each time, when waiting space be reduced to empty or stock turn into zero or both cases happen, which is also exponentially distributed. The server may delusive at any instance whether busy or working vacation period, i.e., server is declining for a precised interval of time, delusive time is exponentially distributed with parameter α_b for busy period as well as for α_v for working vacation period. The server instantly started for repair action and repair time follows an exponentially distributed with parameter η_b for busy period as well as η_v for working vacation period. The sensitive analysis is also accomplished to procure stationary measures of system performance together with we computed predicted cost rate. The outcomes are exemplified with numerical pattern in order to discover the convexity of the integral form of anticipated cost rate.

1. Introduction

Servi and Finn [26] have imported a flourish semi vacation policy tagged working vacation (WV) policy, where the server execute at a lesser rate alternately a complete termination whole in the vacation span in the M/M/1 multi-queue system, they wielded the results to assess the realization depth of gateway router in fiber networks. Sigman and Simchi-Levi[27] were suggested stock design with affirmative service time and pretended about the processing of stock require an arbitrarily distributed positive amount of time, thus leading to the formation of queue. Since the plentiful survey on stock typical with positive service times are noted.

Maike Schwarz et al.[21] has proposed the model M/M/1 Queueing systems carry stock. They derived M/M/1-systems includes stock concealed by

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uninterrupted review and diverse stock management policies, and lost sales. Here the requirement is Poisson and exponentially distributed service and lead times. Anbazhagan et al.[1] considered an stock system with service provision . They presumed that the appeal realizing according to a poission process and the thing will be received by the customer after a arbitary time of service. Rajukumar et al.[23] considered an stock system accompany an infinite size of orbit in which the purchasers who may retry and renege exponentially. He gathered the stipulation for ergodicity and some performance actions of the system. Rajkumar [24] has produced the model for uninterrupted review stock system with dual types of purchasers reported corresponding to a poission process with exponential lead time among ordering policy(s,S).

Sivakumar[28] investigated about uninterrupted review stock system along with Markovian demand. The server took a rest which is distributed exponentially whenever the stock level get into nil and he has been determined about long run absolute awaited cost rate. Saravanarajan and Chandrasekaran[29] were described M/G/1 feedback queueing system equipped twin kind of service facility with working vacations with rest time and the system can breakdowns at any moment of time where random and repair time which are distributed as exponential. Rajadurai et al.[25] inspected a individual server feedback in retrial queueing system possessing several working vacations and vacation interruption. Here the server functioning at a decreased assistance rate throughout the working vacation (WV) period. Berman and Kim [3] perused a queueing stock system having Poisson arrivals, augmented service times and nullity lead times. Berman and Sapna [5] deliberated queueing stock systems with Poisson arrivals, arbitrarily distributed work span and aught lead times. Berman, Kaplan plus Shimshak [2] explored about several of stock management which is used at service facilities.

Berman and Sapna[6] debated about the dispute of stock regulation of service sections at a service set-up which is having less waiting area for purchasers. Elango [10] considered a uninterrupted review decaying (s, S) stock system with a service facility made up of determinate waiting room and a unique server. The soul time of every component and reorder's lead time are postulated to have autonomous exponential distributions. Paul Manuel et al.[22] probed a uninterrupted review destructible stock system with mixed server service facility. Choi and park [9] derived insensitive bounds for various performance measures of a unitary server retrial queue with generally distributed inter retrial times with Bernoulli schedule, under the special assumption that only the consumer at the apogee of the orbit queue. Bo Keun Kim [7] analysed about the substitute server which will come into service while the main sever encountered breakdown. Gautam Choudhury [11] investigated about second optional service with delayed repair.

Gautam Choudhury [12] deals with a scrutiny M/G/1 type queueing system with dual phases of service under Bernoulli vacation schedule below different vacation policies. Goyal. S.K. [13] has developed a model about the recent development in deteriorating stock. Jau-Chuan Ke [14] analysed distribution of the system scale and sojourn time within M/G/1 queueing system with an undependable server and generalized vacations. Jinting Wang [16] studied about the essential and optional service.Jinting Wang [17] inspected about searching process of purchasers instead of finding the following purchasers to be served. Krishnamoorthy et al.[18] analyzes two diversified servers in M/M/2 queueing system in which one server commences vacation whenever the waiting purchasers level get into null. Koroliuk et al.[19] presented an effective assessment of peculiarity of queueing-stock systems with a perishable stocks and vacations of servers having some features. Madhu Jain [20] examined about the arrival rate differs as per the servers position and the server employs at a alternate rate in place of being fully idle during the vacation. Charan Jeet Singh et al.[8] developed about non-Markovian model using the general distributions of the needed/optional service, delay to restore and restore times for the bulk arrival to eveluate the queue size and waiting period distributions .

Symbols :

$[A]_{xy}$: The component/auxiliary matrix at $(x,y)^{th}$ location of A
0 : Null matrix
I_N : Unit matrix of order N
e : Pertinent dimension for $1's$ column vector

$$Y(t) : \begin{cases} 0 & \text{if server operating at delusive in working vacation period} \\ 1 & \text{if server operating at normal in working vacation period} \\ 2 & \text{if server performing on delusive in hectic period} \\ 3 & \text{if server performing on normal in hectic period} \end{cases}$$

2. Mathematical Model Description and Analysis

2.1. Mathematical Model Description. We contemplate a uninterrupted review queueing stock system along with Poisson arrival process. Magnitude of waiting space is N with maximum stock of S units. An entering purchasers establishes a single waiting line respective to their form of arrivals and gives out proportionately. Moreover we postulate that the check in purchasers follow a Poisson process besides $\lambda > 0$ and claim absolutely one unit of item. The server furnishes couple of service. Upon the termination of type-1 service induces single purchaser exit the entity with probability p_1 and type-2 service induces a unique purchaser exit the entity and diminish stock by single element with probability p_2. The service times follow exponential distribution in both cases. The type-1 service accomplished with parameter $p_1\mu_b$ and type-2 service accomplished with parameter $p_2\mu_b$. The purchaser may choose either types of service which is stated above. An order $Q(= \tilde{S} - \tilde{s} > \tilde{s} + 1)$ item is emplaced on any occasion the stock level plunges to $\tilde{s}$ and the elements are collected after a arbitrary duration which is disposed as augmented with parameter $\beta(> 0)$. The server commences a working vacation every time, when waiting space be reduced to empty or stock turn into zero or both cases happen, which is exponentially distributed. Whether busy or working vacation period, (i.e.) server is declining for a precised interlude time and delusive time is exponentially distributed with parameter α_b for busy period and α_v for working vacation period. The server immediately started for repair and repair time follows an exponentially distributed with parameter η_b for busy period as well as η_v for working vacation period. Whenever the server come back from vacation, if it is find atleast a single consumer in the waiting zone and stock on hand with server active, then the server starts to serve the waiting purchaser. Service completion on working vacation period with parameter μ_v. The lead time is presumed to follow an exponential distribution.

2.2. Analysis. Let $I(t)$, $X(t)$, $Y(t)$ stands for the stock on hand, amount of purchasers (stand by and being served) in the system and server condition at time t. From the above surmise we produced the input and output processes, it may be exhibited that,
$(I,\ X,\ Y) = (I(t),\ X(t),\ Y(t))$; t ≥ 0 on the state space E = $E_1 \cup E_2$, where
$E_1 = \{(x, y, z)/x = 0, 1, 2, 3...., \tilde{S},\ y = 0, 1, 2, ...N,\ z = 0, 1\}$
$E_2 = \{(x, y, z)/x = 1, 2, 3...., \tilde{S},\ y = 1, 2, 3.....N,\ z = 2, 3\}$

To express the infinitesimal generator,

$$A = (a((x, y, z)(x', y', z'))), (x, y, z)(x', y', z') \in E \tag{2.1}$$

of this action we use the consecutive arguments :

- The arrival of purchaser build a change over with intensity λ from (x, y, z) to (x, y+1, z), x = 0, 1, 2, ..., $\tilde{S}$, y = 0, 1, 2, ..., N, z = 0, 1 (or) from (x, y, z) to (x, y+1, z), x = 1, 2, ..., $\tilde{S}$, y = 1, 2, ..., N, z = 2, 3.
- Type 1 service completion causes a change over with intensity $p_1\mu_b$ from (x, y, z) to (x, y-1, z), x = 1, 2, ...,$\tilde{S}$, y = 2, 3, ..., N, z = 3 (or) from (x, y, 3) to (x, y-1, 1), x = 1, 2, ..., $\tilde{S}$, y = 1.
- Type 2 service completion causes a change over with intensity $p_2\mu_b$ from (x, y, z) to (x-1, y-1, z), x = 2, 3, ..., $\tilde{S}$, y = 2, 3, ..., N, z = 3 (or) from (x, y, 3) to (x-1, y-1, 1), x = 1, 2, ..., $\tilde{S}$, y = 1 , x = 1, y = 2, 3, ..., N.
- The termination of service from primary demands create a change over with intensity μ_v from (x, y, 1) to (x, y, 1), x = 0, y = 1, 2, ..., N
- The intensity change over β takes place whenever replenishment occur from (x, y, z) to (x+Q, y, z), x = 0, 1, 2, ..., $\tilde{s}$, y = 0, 1, 2, ..., N, z = 0, 1 (or) from (x, y, z) to (x+Q, y, z) x = 1, 2, 3, ..., $\tilde{s}$, y = 1, 2, ..., N, z = 2, 3.
- Server changes from delusive at working vacation period to normal at working vaction period with intensity η_v from (x, y, 0) to (x, y, 1), x = 0, 1, 2, ..., $\tilde{S}$, y = 0, 1, 2, ..., N
- Server changes from delusive at busy period to normal at busy period with intensity η_b from (x, y, 2) to (x, y, 3), x = 1, 2, 3, ..., $\tilde{S}$, y = 1, 2, ..., N.

- Server changes from normal at working vacation period to delusive at working vacation period with intensity α_v from
(x, y, 1) to (x, y, 0), x = 0, 1, 2, ..., $\tilde{S}$, y = 0, 1, 2, ..., N.
- Server changes from normal at busy period to delusive at busy period with intensity α_b from
(x, y, 3) to (x, y, 2), x = 1, 2, ..., $\tilde{S}$, y = 1, 2, 3, ..., N.
- Server changes from normal at working vacation period to normal at busy period with intensity θ from
(x, y, 1) to (x, y, 3), x = 0, 1, 2, ..., $\tilde{S}$, y = 0, 1, 2, ..., N.

For other change over from (x, y, z) to (x$'$, y$'$, z$'$) where (x, y, z) $\neq$ (x$'$, y$'$, z$'$) the rate is null. Finally note that,

$$a((x,y,z),(x^{'},y^{'},z^{'})) = -\sum_x \sum_y \sum_{z_{(x,y,z)\neq(x^{'},y^{'},z^{'})}} a((x,y,z),(x^{'},y^{'},z^{'}))$$

Hence a((x, y, z)(x$'$, y$'$, z$'$)) can be written as,

$$\begin{cases}
\lambda, & \begin{array}{l} x' = x, x = 0,1,2...,\tilde{S} \\ y' = y+1, y = 0,1,2....N-1 \\ z' = z, z = 0,1 \\ or \\ ix = x, = 1,2,3...,\tilde{S} \\ y' = y+1, y = 1,2,3....N-1 \\ z' = z, z = 2,3 \end{array} \\
\\
p_1\mu_b, & \begin{array}{l} x' = x, x = 1,2,3...\tilde{S} \\ y' = y-1, y = 2,3,4...N \\ z' = z, z = 3 \\ or \\ x' = x, x = 1,2,3...\tilde{S} \\ y' = y-1, y = 1 \\ z' = z-2, z = 3 \end{array} \\
\\
p_2\mu_b, & \begin{array}{l} x' = x-1, x = 2,3,4,...\tilde{S} \\ y' = y-1, y = 2,3,4,...N \\ z' = z, z = 3 \\ or \\ x' = x-1, x = 1 \\ y' = y-1, y = 1 \\ z' = z-2, z = 3 \end{array}
\end{cases}$$

$$\begin{cases}
\mu_v, & x' = x, x = 0 \\
& y' = y, y = 1, 2, 3, ...N \\
& z' = z, z = 1 \\
\\
\beta, & x' = x + Q, = 0, 1, 2, ...\tilde{s} \\
& y = y, y = 0, 1, 2, ...N \\
& z' = z, z = 0, 1 \\
& or \\
& x' = x + Q, x = 1, 2, ...\tilde{s} \\
& y = y, y = 1, 2, ...N \\
& z' = z, z = 2, 3 \\
\\
\eta_v, & x' = x, x = 0, 1, 2, ...\tilde{S} \\
& y' = y, y = 0, 1, 2, ...N \\
& z = z + 1, z = 0 \\
\eta_b, & x' = x, x = 1, 2, 3, ...\tilde{S} \\
& y' = y, y = 1, 2, 3....N \\
& z' = z - 1, z = 3 \\
& or \\
\alpha_v, & x' = x, x = 0, 1, 2, ...\tilde{S} \\
& y' = y, y = 0, 1, 2, ...N \\
& z' = z - 1, z = 1 \\
\\
\alpha_b, & x' = x, x = 1, 2, 3, ...\tilde{S} \\
& y' = y, y = 1, 2, 3, ...N \\
& z' = z - 1, z = 3 \\
\\
\theta, & x' = x, x = 1, 2, 3, ...\tilde{S} \\
& y' = y, y = 1, 2, 3, ...N \\
& z' = z + 2, z = 1 \\
\\
\varepsilon_1 = -(\lambda + \eta_v), & x' = x, x = \tilde{S}, \tilde{S} - 1..., \tilde{s} + 1 \\
& y' = y, y = 0, 1, 2....N - 1 \\
& z' = z, z = 0 \\
\\
\varepsilon_2 = -(\alpha_v + \lambda), & x' = x, x = \tilde{S}, \tilde{S} - 1..., \tilde{s} + 1 \\
& y' = y, y = 0 \\
& z' = z, z = 1 \\
\\
\varepsilon_3 = -(\alpha_v + \lambda + \theta), & x' = x, x = \tilde{S}, \tilde{S} - 1..., \tilde{s} + 1 \\
& y' = y, y = 1, 2, ..., N - 1 \\
& z' = z, z = 1 \\
\\
\varepsilon_4 = -(\lambda + \eta_b), & x' = x, x = \tilde{S}, \tilde{S} - 1..., \tilde{s} + 1 \\
& y' = y, y = 0, 1, 2....N - 1 \\
& z' = z, z = 2
\end{cases}$$

$$
\begin{cases}
\varepsilon_5 = -(\alpha_b + \lambda + p_1\mu_b + p_2\mu_b), & x' = x, x = \tilde{S}, \tilde{S}-1..., \tilde{s}+1 \\
& y' = y, y = 1,2,3,4...N-1 \\
& z' = z, z = 3 \\
\varepsilon_6 = -(\eta_v), & x' = x, x = \tilde{S}, \tilde{S}-1..., \tilde{s}+1 \\
& y' = y, y = N \\
& z' = z, z = 0 \\
\varepsilon_7 = -(\alpha_v + \theta), & x' = x, x = \tilde{S}, \tilde{S}-1..., \tilde{s}+1 \\
& y' = y, y = N \\
& z' = z, z = 1 \\
\varepsilon_8 = -(\eta_b), & x' = x, x = \tilde{S}, \tilde{S}-1..., \tilde{s}+1 \\
& y' = y, y = N \\
& z' = z, z = 2 \\
\varepsilon_9 = -(\alpha_b + p_1\mu_b + p_2\mu_b), & x' = x, x = \tilde{S}, \tilde{S}-1..., \tilde{s}+1 \\
& y' = y, y = N \\
& z' = z, z = 3 \\
\varepsilon_{10} = -(\lambda + \eta_v + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
& y' = y, y = 0,1,2....N-1 \\
& z' = z, z = 0 \\
& or \\
& x' = x, x = 0 \\
& y' = y, y = 0,1,2,3....N-1 \\
& z' = z, z = 0 \\
\varepsilon_{11} = -(\alpha_v + \lambda + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
& y' = y, y = 0 \\
& z' = z, z = 1 \\
& or \\
& x' = x, x = 0 \\
& y' = y, y = 0 \\
& z' = z, z = 1 \\
\varepsilon_{12} = -(\alpha_v + \lambda + \theta + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
& y' = y, y = 1,2,...,N-1 \\
& z' = z, z = 1 \\
\varepsilon_{13} = -(\lambda + \eta_b + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
& y' = y, y = 0,1,2....N-1 \\
& z' = z, z = 2
\end{cases}
$$

$$
\begin{cases}
\varepsilon_{14} = -(\alpha_b + \lambda + p_1\mu_b + p_2\mu_b + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
 & y' = y, y = 2, 3, 4...N-1 \\
 & z' = z, z = 3 \\
\varepsilon_{15} = -(\eta_v + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
 & y' = y, y = N \\
 & z' = z, z = 0 \\
 & or \\
 & x' = x, x = 0 \\
 & y' = y, y = N \\
 & z' = z, z = 0 \\
\varepsilon_{16} = -(\alpha_v + \theta + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
 & y' = y, y = N \\
 & z' = z, z = 1 \\
\varepsilon_{17} = -(\eta_b + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
 & y' = y, y = N \\
 & z' = z, z = 2 \\
\varepsilon_{18} = -(\alpha_b + p_1\mu_b + p_2\mu_b + \beta), & x' = x, x = \tilde{s}, \tilde{s}-1, ...1 \\
 & y' = y, y = N \\
 & z' = z, z = 3 \\
\varepsilon_{19} = -(\alpha_v + \lambda + \beta + \mu_v), & x' = x, x = 0 \\
 & y' = y, y = 1, 2, ..., N-1 \\
 & z' = z, z = 1 \\
\varepsilon_{20} = -(\alpha_v + \beta + \mu_v), & x' = x, x = 0 \\
 & y' = y, y = N \\
 & z' = z, z = 1
\end{cases}
$$

The more general form is

$$A = \begin{array}{c} \tilde{S} \\ \tilde{S}-1 \\ \tilde{S}-2 \\ \vdots \\ \tilde{s} \\ \tilde{s}-1 \\ \vdots \\ 1 \\ 0 \end{array} \overset{\begin{array}{ccccccccc} \tilde{S} & \tilde{S}-1 & \tilde{S}-2 & \dots & \tilde{s} & \tilde{s}-1 & \dots & 1 & 0 \end{array}}{\begin{pmatrix} L & M & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & L & M & \dots & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & L & \dots & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots \\ R & \mathbf{0} & \mathbf{0} & \dots & L_1 & M & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & R & \mathbf{0} & \dots & \mathbf{0} & L_1 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & R & \dots & \mathbf{0} & \mathbf{0} & \dots & L_1 & M_1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & R_1 & \mathbf{0} & \dots & \mathbf{0} & L_2 \end{pmatrix}}$$

where,

$$L = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \overset{\begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array}}{\begin{pmatrix} A_0 & B_0 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ C_0 & A & B & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C & A & B & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C & A & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & A & B \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & C & A_1 \end{pmatrix}}$$

with,

$$A = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \overset{\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}}{\begin{pmatrix} \varepsilon_1 & \eta_v & 0 & 0 \\ \alpha_v & \varepsilon_3 & 0 & \theta \\ 0 & 0 & \varepsilon_4 & \eta_b \\ 0 & 0 & \alpha_b & \varepsilon_5 \end{pmatrix}} \quad A_1 = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \overset{\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}}{\begin{pmatrix} \varepsilon_6 & \eta_v & 0 & 0 \\ \alpha_v & \varepsilon_7 & 0 & \theta \\ 0 & 0 & \varepsilon_8 & \eta_b \\ 0 & 0 & \alpha_b & \varepsilon_9 \end{pmatrix}}$$

$$A_0 = \begin{array}{c} 0 \\ 1 \end{array} \overset{\begin{array}{cc} 0 & 1 \end{array}}{\begin{pmatrix} \varepsilon_1 & \eta_v \\ \alpha_v & \varepsilon_2 \end{pmatrix}} \quad C_0 = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \overset{\begin{array}{cc} 0 & 1 \end{array}}{\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & p_1\mu_b \end{pmatrix}} \quad B_0 = \begin{array}{c} 0 \\ 1 \end{array} \overset{\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}}{\begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \end{pmatrix}}$$

$$B = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \overset{\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}}{\begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix}} \quad C = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \overset{\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}}{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_1\mu_b \end{pmatrix}}$$

$$L_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array} \\ \begin{pmatrix} D_0 & B_0 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ C_0 & D & B & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C & D & B & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C & D & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & D & B \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & C & D_1 \end{pmatrix} \end{array}$$

with,

$$D_0 = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} \varepsilon_{10} & \eta_v \\ \alpha_v & \varepsilon_{11} \end{pmatrix} \end{array} \quad D = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \varepsilon_{10} & \eta_v & 0 & 0 \\ \alpha_v & \varepsilon_{12} & 0 & \theta \\ 0 & 0 & \varepsilon_{13} & \eta_b \\ 0 & 0 & \alpha_b & \varepsilon_{14} \end{pmatrix} \end{array}$$

$$D_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \varepsilon_{15} & \eta_v & 0 & 0 \\ \alpha_v & \varepsilon_{16} & 0 & \theta \\ 0 & 0 & \varepsilon_{17} & \eta_b \\ 0 & 0 & \alpha_b & \varepsilon_{18} \end{pmatrix} \end{array}$$

$$L_2 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array} \\ \begin{pmatrix} D_0 & B_0 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ C_1 & E_0 & B & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_1 & E_0 & B & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C_1 & E_0 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & E_0 & B_0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & C_1 & E_0 \end{pmatrix} \end{array}$$

with,

$$E_0 = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \varepsilon_{10} & \eta_v & 0 & 0 \\ \alpha_v & \varepsilon_{19} & 0 & 0 \end{pmatrix} \end{array} \quad E_1 = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \varepsilon_{15} & \eta_v & 0 & 0 \\ \alpha_v & \varepsilon_{20} & 0 & 0 \end{pmatrix} \end{array} \quad C_1 = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & \mu_v \end{pmatrix} \end{array}$$

$$R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array} \\ \begin{pmatrix} F_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_1 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_1 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & F_1 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & F_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & F_1 \end{pmatrix} \end{array}$$

with,

$$F_0 = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix} \end{array} \quad F_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix} \end{array}$$

$$R_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array} \\ \begin{pmatrix} F_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_2 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & F_2 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & F_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & F_2 \end{pmatrix} \end{array}$$

with,

$$F_2 = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \end{pmatrix} \end{array}$$

$$M = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ G_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G_1 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G_1 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & G_1 & \mathbf{0} \end{pmatrix} \end{array}$$

with,

$$G_0 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & p_2\mu_b \end{pmatrix} \end{array} \quad G_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_2\mu_b \end{pmatrix} \end{array}$$

$$M_1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ N-1 \\ N \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \dots & N-1 & N \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ G_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G_0 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G_0 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & G_0 & \mathbf{0} \end{pmatrix} \end{array}$$

2.2.1. *Steady State Results.* It might be noticed from the system of the infinitesimal generator grid A that the homogeneous Markov process (I(t),X(t),Y(t), t ≥ 0) is undeductible on the fixed state space E . Hence, the limiting distribution of the Markov process abound. Let $\boldsymbol{\psi}$, disperse as $\boldsymbol{\psi} = (\boldsymbol{\psi}^{(\tilde{S})}, \boldsymbol{\psi}^{(\tilde{S}-1)}, ..., \boldsymbol{\psi}^{(1)}, \boldsymbol{\psi}^{(0)})$, indicate the steady state probability vector of A. That is, $\boldsymbol{\psi}$ might be fulfil

$$\boldsymbol{\psi} A = 0 \ \text{ and } \ \sum_{x}\sum_{y}\sum_{z}\boldsymbol{\psi}^{(x,y,z)} = 0 \quad {\scriptstyle (x,y,z)\in E}$$

$$a((x,y,z),(x^{'},y^{'},z^{'})) = -\sum_{x}\sum_{y}\sum_{z} a((x,y,z),(x^{'},y^{'},z^{'})) \quad {\scriptstyle (x,y,z)\neq(x^{'},y^{'},z^{'})}$$

The initial mathematical statement of the above harvest the underneath batch of equations:

$$\begin{aligned} \boldsymbol{\psi}^{(x)}L + \boldsymbol{\psi}^{(x-Q)}R &= 0, \ x = \tilde{S} \\ \boldsymbol{\psi}^{(x+1)}M + \boldsymbol{\psi}^{(x)}L + \boldsymbol{\psi}^{(x-Q)}R &= 0, \ x = \tilde{S}-1, \tilde{S}-2, ..., Q+1 \\ \boldsymbol{\psi}^{(x+1)}M + \boldsymbol{\psi}^{(x)}L + \boldsymbol{\psi}^{(x-Q)}R_1 &= 0, \ x = Q \\ \boldsymbol{\psi}^{(x+1)}M + \boldsymbol{\psi}^{(x)}L &= 0, \ x = Q-1, Q-2, ..., \tilde{s}+1 \\ \boldsymbol{\psi}^{(x+1)}M + \boldsymbol{\psi}^{(x)}L_1 &= 0, \ x = \tilde{s}, \tilde{s}-1, ...1 \\ \boldsymbol{\psi}^{(x+1)}M_1 + \boldsymbol{\psi}^{(x)}L_2 &= 0, \ x = 0 \end{aligned}$$

Behind prolonged demonstration, the preceding equations, exclude that third one

$$
\begin{aligned}
\boldsymbol{\psi}^{x)} &= (-1)^{Q}\boldsymbol{\psi}^{(Q)}M^{Q-1}M_1\left(\frac{1}{L_2}\right)\left(\frac{1}{L_1}\right)^{\tilde{s}}\left(\frac{1}{L}\right)^{Q-\tilde{s}-1}, \quad x=0 \\
\boldsymbol{\psi}^{(x)} &= (-M)^{Q-x}\boldsymbol{\psi}^{(Q)}\left(\frac{1}{L_1}\right)^{\tilde{s}+1-x}\left(\frac{1}{L}\right)^{Q-\tilde{s}-1}, \quad x=1,2,...\tilde{s} \\
\boldsymbol{\psi}^{(x)} &= (-M)^{Q-x}\boldsymbol{\psi}^{(Q)}\left(\frac{1}{L}\right)^{Q-x}, \quad x=\tilde{s}+1,\tilde{s}+2,...,Q-1 \\
\boldsymbol{\psi}^{(x)} &= (-1)^{x-1}\boldsymbol{\psi}^{(Q)}(M)^{2Q-x}R\left(\sum_{n=0}^{\tilde{S}-x}\left(\frac{1}{L_1}\right)^{n+1}\left(\frac{1}{L}\right)^{2Q-(x+n)}\right), \quad x=Q+1,Q+2,...,\tilde{S}
\end{aligned}
$$

Where $\boldsymbol{\psi}^{(Q)}$ has been acquired by solving,

$$\boldsymbol{\psi}^{(Q+1)}M+\boldsymbol{\psi}^{(Q)}L+\boldsymbol{\psi}^{(0)}R_1 = 0$$

and

$$\sum_{i=0}^{\tilde{S}}\boldsymbol{\psi}^{(x)}e = 1,$$

that is

$$\boldsymbol{\psi}^{(Q)}\ [(-1)^{Q}\ M^{Q}\sum_{n=0}^{\tilde{S}-Q-1}\left(\tfrac{1}{L_1}\right)^{n+1}\left(\tfrac{1}{L}\right)^{Q-(1+n)} + L + (-1)^{Q}\ M^{Q-1}\ M_1\ R_1 \left(\tfrac{1}{L_2}\right)\left(\tfrac{1}{L_1}\right)^{\tilde{s}}\left(\tfrac{1}{L}\right)^{Q-\tilde{s}-1}]=0$$

and

$$\boldsymbol{\psi}^{(Q)}\ [\ (-1)^{Q}\ M^{Q-1}\ M_1\left(\tfrac{1}{L_2}\right)\left(\tfrac{1}{L_1}\right)^{\tilde{s}}\left(\tfrac{1}{L}\right)^{Q-\tilde{s}-1}+\sum_{x=1}^{\tilde{s}}(-M)^{Q-x}\left(\tfrac{1}{L_1}\right)^{\tilde{s}+1-x}\left(\tfrac{1}{L}\right)^{Q-\tilde{s}-1}+\sum_{x=\tilde{s}+1}^{Q-1}(-M)^{Q-x}\left(\tfrac{1}{L}\right)^{Q-x}+\mathrm{I}+\sum_{x=Q+1}^{\tilde{S}}(-1)^{x-1}(M)^{2Q-x}R\ (\sum_{n=0}^{\tilde{S}-x}\left(\tfrac{1}{L_1}\right)^{n+1}\left(\tfrac{1}{L}\right)^{2Q-(x+n)})]\ e=1.$$

3. System perfomance measures

In this segment, we determine a few system behaviour computations with respect to the steady state analysis.

3.1. Mean Stock Level. Here ε_I express the mean stock numbers in the steady state, which is accustomed by

$$\varepsilon_I = \sum_{x=1}^{\tilde{S}} \sum_{y=1}^{N} \sum_{z=0}^{3} x\boldsymbol{\psi}^{(x,y,z)} + \sum_{x=1}^{\tilde{S}} \sum_{z=0}^{1} x\boldsymbol{\psi}^{(x,0,z)}$$

3.2. Mean Reorder Rate. Here ε_R express the mean reorder rate, which is accustomed by

$$\varepsilon_R = \sum_{y=1}^{N} p_2\mu_b\boldsymbol{\psi}^{(\tilde{s}+1,y,3)}$$

3.3. Mean Repair Rate. Here ε_{REP} express the mean repair rate, which is accustomed by

$$\varepsilon_{REP} = \sum_{x=0}^{\tilde{S}} \sum_{y=0}^{N} \sum_{z=0}^{1} \alpha_v\boldsymbol{\psi}^{(x,y,z)} + \sum_{x=1}^{\tilde{S}} \sum_{y=1}^{N} \sum_{z=1}^{2} \alpha_b\boldsymbol{\psi}^{(x,y,z)}$$

3.4. Mean Number of Purchasers Waiting while Server is on Vacation and Delusive Period. Here ε_W express the average amount of consumers in the waiting line while server is in active working vacation and delusive working vacation and delusive busy cycle in the steady state and is accustomed by

$$\varepsilon_W = \sum_{x=1}^{\tilde{S}} \sum_{y=1}^{N} \sum_{z=0}^{2} y\boldsymbol{\psi}^{(x,y,z)} + \sum_{y=1}^{N} \sum_{z=0}^{1} y\boldsymbol{\psi}^{(0,y,z)}$$

3.5. Mean Count of Purchasers in the System When the Server is Delivering Service. Here ε_B express the amount of purchasers in the waiting area, when the server is unavailable (hectic) in the steady state and is accustomed by

$$\varepsilon_B = \sum_{x=1}^{\tilde{S}} \sum_{y=1}^{N} y\boldsymbol{\psi}^{(x,y,3)}$$

3.6. Cost Analysis. For the sake of enumerate the overall anticipated price per unit time, we have used the underneath representations:
C_h : The stock holding price per unit element per unit time.
$C_{\tilde{s}}$: The setup price for each order.
C_w : Waiting price of a purchaser per unit time.
C_R : Repair price per unit element per unit time.

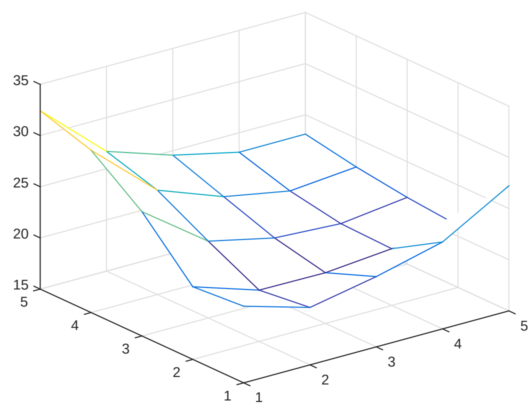

FIGURE 1. A emblematic three-dimensional structure layout for total expected cost estimate of N=5, λ = 3.2, θ= 1.4, η_v=0.5, η_b=0.2, α_b= 0.99, α_v=0.1, μ_v=0.58, β = 1.99, $p1\mu_b$=0.5, $p2\mu_b$=0.5, c_h=0.66, c_s=65, c_w=1.6, c_R=0.15

Then the long-run expected cost rate accustomed by,

$$T(\tilde{S},\, \tilde{s},\, N) = C_h\ \varepsilon_I + C_{\tilde{s}}\ \varepsilon_R + C_w\ \varepsilon_W + C_R\ \varepsilon_{REP}$$

4. Numerical analysis

We exhibited the systematic numerical outcomes as succeed of the equation T($\tilde{S}$, $\tilde{s}$, N).We can scrutinized the nature of this equation by taking into account of ($\tilde{S}$,$\tilde{s}$) in order to shown the convexity through numerical illustration. The given table indicates about the sum predicted cost estimate for distinct sequence of $\tilde{S}$ and N when the integrity for auxiliary parameters and costs are speculated.

$\tilde{S}$			$\tilde{s}$		
	5	6	7	8	9
16	22.4880218498	20.6089672107	21.8665666020	23.4936862272	27.2220239784
17	**22.0965022297**	**20.0111580628**	**19.9546612790**	**20.5433108277**	**21.0517547390**
18	27.1472053965	22.5031348248	21.0608730097	20.7051724409	21.5095904277
19	30.8601475684	25.1972034258	22.8052747815	21.6072118147	22.1889708014
20	32.4270041077	26.6977894066	24.5688499417	23.0992690485	23.1138556039

• Significance of fluctuating setup costs and holding cost on optimal values,

c_s / c_h	63		64		65		66		67	
0.64	17	6	17	7	17	7	17	7	17	7
	19.72147		19.83749		19.93166		20.02584		20.12001	
0.65	17	6	17	7	17	7	17	7	17	7
	19.73124		19.84899		19.94316		20.03733		20.13151	
0.66	17	6	17	7	17	7	17	7	17	7
	19.74102		19.86049		19.95466		20.04883		20.14300	
0.67	17	6	17	7	17	7	17	7	17	7
	19.75080		19.87199		19.96616		20.06033		20.15450	
0.68	17	6	17	7	17	7	17	7	17	7
	19.76058		19.88349		19.97766		20.07183		20.16600	

• Impact on differing setup cost and waiting cost on optimal values,

c_s / c_w	63		64		65		66		67	
0.8	17	8	17	7	17	7	17	6	17	6
	15.26209		17.73050		19.93166		21.70050		23.40940	
1.2	17	8	17	7	17	7	17	6	17	6
	15.27435		17.74200		19.94316		21.71028		23.41918	
2.0	17	8	17	7	17	7	17	6	17	6
	15.28660		17.75350		19.95466		21.72006		23.42896	
1.6	17	8	17	7	17	7	17	6	17	6
	15.29886		17.76500		19.96616		21.72984		23.43873	
2.4	17	8	17	7	17	7	17	6	17	6
	15.31112		17.77650		19.97766		21.73961		23.44851	

• Impression of changing waiting cost and holding cost on optimal values,

c_w / c_h	1.4		1.5		1.6		1.7		1.8	
0.64	17	7	17	7	17	7	17	7	17	7
	18.66574		18.75991		18.85408		18.94825		19.04242	
0.65	17	7	17	7	17	7	17	7	17	7
	19.21603		19.31020		19.40437		19.49854		19.59271	
0.66	17	6	17	7	17	7	17	7	17	7
	19.74102		19.86049		19.95466		20.04883		20.14300	
0.67	17	6	17	6	17	6	17	7	17	7
	20.16825		20.30331		20.43838		20.59112		20.69329	
0.68	17	6	17	6	17	6	17	6	17	6
	20.59547		20.73054		20.86561		21.00068		21.13574	

• Effect of deviating setup cost and repair cost on optimal values,

c_s / c_r	63		64		65		66		67	
0.13	17	6	17	7	17	7	17	7	17	7
	19.37336		19.65251		19.93166		20.21082		20.47501	
0.14	17	6	17	7	17	7	17	7	17	7
	19.38486		19.66401		19.94316		20.22231		20.48478	
0.15	17	6	17	7	17	7	17	7	17	7
	19.39636		19.67551		19.95466		20.23381		20.49456	
0.16	17	6	17	7	17	7	17	7	17	7
	19.40786		19.68701		19.96616		20.24531		20.50434	
0.17	17	6	17	7	17	7	17	7	17	7
	19.41935		19.69851		19.97766		20.25681		20.51412	

- Implement of wavering repair cost and waiting cost on optimal values,

c_r / c_w	0.13		0.14		0.15		0.16		0.17	
0.8	17	8	17	7	17	7	17	6	17	6
	14.66550		17.19520		19.39636		21.23665		22.94555	
1.2	17	8	17	7	17	7	17	6	17	6
	14.97605		17.47435		19.67551		21.47836		23.18726	
1.6	17	8	17	7	17	7	17	6	17	6
	15.28660		17.75350		19.95466		21.72006		23.42896	
2.0	17	8	17	7	17	7	17	6	17	6
	15.59715		18.03265		20.23381		21.96176		23.67066	
2.4	17	8	17	7	17	7	17	6	17	6
	15.90770		18.31181		20.49456		22.20346		23.91236	

- Influence of vacillating repair cost and holding cost on optimal values

c_r / c_w	0.13		0.14		0.15		0.16		0.17	
0.64	17	7	17	7	17	7	17	7	17	7
	19.20801		19.30219		19.39636		19.49053		19.58470	
0.65	17	7	17	7	17	7	17	7	17	7
	19.48717		19.58134		19.67551		19.76968		19.86385	
0.66	17	6	17	7	17	7	17	7	17	7
	19.74102		19.86049		19.95466		20.04883		20.14300	
0.67	17	6	17	6	17	7	17	7	17	7
	19.98272		20.11779		20.23381		20.32799		20.42216	
0.68	17	6	17	6	17	6	17	7	17	7
	20.22443		20.35949		20.49456		20.60714		20.70131	

5. Conclusion

Within this proposal, we have analysed a uninterrupted review queueing stock system($\tilde{S}$, $\tilde{s}$) with Poisson arrival process together with the magnitude of waiting room is N. We investigated about couple of service with working vacation along with server delusive and repair. We have also obtained various stationary measures. The outcomes are exemplified with numerical pattern in order to find out the convexity of the total expected cost rate.

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