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RADIATION EFFECT TO MHD OSCILLATORY FLOW IN A CHANNEL FILLED THROUGH A POROUS MEDIUM WITH HEAT GENERATION

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Abstract. The present investigation is the combined effect of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature with heat source. The governing partial differential equations are derived for the fluid and solved by analytical using perturbation method. The various parameter involving governing equations are discussed through graphically.

KEYWORDS: Radiation, Oscillatory flow, Magnetic field, Porous medium and heat generation

1. INTRODUCTION

Magnetohydrodynamics also called magneto-fluid dynamics or hydromagnetic is the study of the magnetic properties and behaviour of electrically conducting fluids, examples of such magneto fluids include plasmas, liquid metals, salt water and electrolytes. The study of magnetohydrodynamics (MHD) draws from two well-known branches of physics, electrodynamics and hydrodynamics, along with a provision to include their coupling. The basic laws of electrodynamics described in the form of Maxwell's Equations supplemented by the generalized Ohm's law are sufficient for the purpose. The hydrodynamics of a fluid is expressed in the form of conservation laws of mass, momentum and energy. These laws treat the fluid as a continuum. The continuum descriptioll is valid if the mean free path of the constituent particles is much shorter than the spatial scales on which the flow is visualized. Thus, according to this criterion, any substance can be treated as a continuum at some spatial scale. The magnetohydrodynamic phenomena are a consequence of the mutual interaction of the fluid flow and the magnetic field. In view of the above consideration, Finite difference solution for an MHD free convective rotating flow past an accelerated vertical plate studied by Yeddala et. al. [1], M. Rajaiah et. al. [2] communicated on chemical and Soret effect on MHD free convective flow past an accelerated vertical plate in presence of inclined magnetic field through porous medium, Ch Kesavaiah et. al. [3] shows that the radiation and mass transfer effects on moving vertical plate with variable temperature and viscous dissipation, Rajaiah et. al. [4] expressed their view on unsteady MHD free convective fluid flow past a vertical porous plate with Ohmic heating in the Presence of suction or injection, Chenna Kesavaiah and Satyanarayana [5] explained in detailed on MHD and Diffusion Thermo effects on flow accelerated vertical plate with chemical reaction, Chenna Kesavaiah et. al. [6] has been studied natural convection heat transfer oscillatory flow of an elastico-viscous fluid from vertical plate,

Fluid flows though porous media has been an important research topic for decades especially in the area of ground water movement, petroleum engineering, geology and geophysics. In recent years, the flow of fluids through porous media has become an

important topic because of the recovery of crude oil from the pores of the reservoir rocks; in this case, Darcy's law represents the gross effect. Meanwhile, there has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous media because of the effect on magnetic fields on the performance of many systems. In view of the above some of the researchers accounted, Haranth and Sudhakaraiah [7] has been considered the viscosity and Soret effects on unsteady hydromagnetic gas flow along an inclined plane, Chenna Kesavaiah et. al. [8] carried out radiation and thermo - diffusion effects on mixed convective heat and mass transfer flow of a viscous dissipated fluid over a vertical surface in the presence of chemical reaction with heat source, Rajaiah and Sudhakaraiah [9] depicted the radiation and Soret effect on Unsteady MHD flow past a parabolic started vertical plate in the presence of chemical reaction with magnetic dissipation through a porous medium, Srinathuni Lavanya and Chenna Kesavaiah [10] demonstrated on radiation and Soret effects to MHD flow in vertical surface with chemical reaction and heat generation through a porous medium, Mallikarjuna Reddy et. al. [11] revealed the effects of radiation and thermal diffusion on MHD heat transfer flow of a dusty viscoelastic fluid between two moving parallel plates, Srinathuni Lavanya and Chenna Kesavaiah [12] exhibited on heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction.
The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Many authors carried their research work, Rajaiah and Sudhakaraiah [13] shows the effect of unsteady MHD free convection flow past an accelerated vertical plate with chemical reaction and Ohmic heating, Bhavana [14] conveyed on the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source, Ch Kesavaiah et. al. [15] communicated on radiation absorption, chemical reaction and magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux, Satyanarayana et. al. [16] explained on viscous dissipation and thermal radiation effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving porous plate, Srinathuni Lavanya et. al. [17] expressed the radiation, heat and mass transfer effects on magnetohydrodynamic unsteady free convective Walter's memory flow past a vertical plate with chemical reaction through a porous medium, Ch Kesavaiah et. al. [18] has been show that the radiation and mass transfer effects on MHD mixed convection flow from a vertical surface with Ohmic heating in the presence of chemical reaction, Chenna Kesavaiah et. al. [19] has been studied the radiation effect on unsteady flow past an accelerated isothermal infinite vertical plate with chemical reaction and heat source.

In the present paper, we investigate the combined effects of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature. In the following sections, the problem is formulated, solved and the pertinent results are discussed. The shear stress and Nusselt number has been discussed analytically.

2. FORMULATION OF THE PROBLEM

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer as shown in Fig. 1. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system (*x*, *y*) where *x* lies along the centre of the channel, *y* is the distance measured in the normal section. Then, assuming a Boussinesq's incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K}u - \frac{\sigma B_0^2 u}{\rho} - g\left(T - T_0\right) \tag{2.1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\left(\frac{\partial q}{\partial y}\right) - \frac{Q_0}{\rho C_p}\left(T - T_0\right) \tag{2.2}$$

Boundary conditions

$$\begin{aligned} u = 0, T = T_w \quad & y = 1 \\ u = 0, T = T_0 \quad & y = 0 \end{aligned} \tag{2.3}$$

where *u* is the axial velocity, *t* the time, *T* the fluid temperature, *P* the pressure, *g* the gravitational force, *q* the radiative heat flux, β the coefficient of volume expansion due to temperature, C_p the specific heat at constant pressure, *k* the thermal conductivity, *K* the porous medium permeability coefficient, $B_0 = \left(\mu_e H_0\right)$ the electromagnetic induction, μ_e the magnetic permeability, H_0 the intensity of magnetic field, σ_e the conductivity of the fluid, ρ the fluid density and ν is the kinematic viscosity coefficient. It is assumed that both walls temperature T_0 , T_w are high enough to induce radiative heat transfer. Following Cogley et. al. [20], it is assumed that the fluid is optically thin with a relatively.

where $$\frac{\partial q}{\partial y} = 4\alpha^2\left(T_0 - T_w\right) \tag{2.4}$$

where α is the mean radiation absorption coefficient. The following dimensionless variables and parameters are introduced: Introducing the following non-dimensional quantities

$$\bar{y} = \frac{y}{b}, \bar{u} = \frac{u}{U}, \mathrm{Re} = \frac{Ua}{\nu}, \theta = \frac{T^* - T_s^*}{T_n^* - T_s^*}, \bar{t} = \frac{tU}{a}, H^2 = \frac{a^2\sigma_e B_0{}^2}{\rho\nu}, P_e = \frac{Ua\rho Cp}{k}$$

$$Gr = \frac{g\beta a^2\left(T_w - T_0\right)}{\nu U}, \quad N^2 = \frac{4\alpha^2 a^2}{k}, \quad \bar{\mathrm{P}} = \frac{aP}{\rho\nu U}, \quad \phi = \frac{Q_0\, a}{U\rho Cp}, \omega = \frac{\omega^* b^2}{\nu} \tag{2.5}$$

where *U* is the flow mean velocity. The dimensionless governing equations together with the appropriate boundary conditions, (neglecting the bars for clarity) can be written as

where Gr - Grashoff number, $N-$ Radiation parameter, $\mathrm{P}e-$ Prandtl number, $H-$ Hartman number, $\phi-$ Heat source parameter

Introducing the radiative heat flux in equation (2.2), then the equations (2.2) and (2.5) become

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(s^2 + H^2\right)u - + Gr\theta \tag{2.6}$$

$$Pe\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta-\phi\,\theta \tag{2.7}$$

The corresponding boundary conditions

$$\begin{aligned}&u=0,\theta=0 \quad y=0\\ &u=0,\theta=1 \quad y=1\end{aligned} \tag{2.8}$$

3. SOLUTION OF THE PROBLEM

In order to solve equations (2.6) - (2.8) for purely oscillatory flow, let

$$\begin{aligned}&-\frac{\partial P}{\partial t}=\lambda e^{i\omega t}\\ &u(y,t)=u_0(y)e^{i\omega t}\\ &\theta(y,t)=\theta_0(y)e^{i\omega t}\end{aligned} \tag{2.9}$$

where λ is a constant and ω is the frequency of the oscillation. Substituting the above expressions in equation (2.9) into Equations 2.(6) to (2.7), we obtain:

$$\frac{d^2\theta_0}{\partial y^2}+m_1^2\theta_0=0 \tag{2.10}$$

$$\frac{d^2\theta}{dy^2}+m_2{}^2u_0=-\lambda-Gr\theta_0 \tag{2.11}$$

The corresponding boundary conditions:

$$\begin{aligned}&u_0=0,\theta_0=0 \qquad y=0\\ &u_0=0,\theta_0=1 \qquad y=1\end{aligned} \tag{2.12}$$

where $m_1=\sqrt{N^2-i\omega Pe-\phi Pe}$ and $m_2=\sqrt{s^2+H^2+i\omega Pe}$ equations (2.10) to (2.11) are solved and the solution for fluid velocity and temperature are given as follows:

$$\theta(y,t)=\frac{\sin(m_1y)}{\sin m_1}e^{i\omega t}$$

$$u(y,t)=\left[\frac{Gr}{\left(m_1{}^2+m_2{}^2\right)}\left(\frac{\sin(m_1y)}{\sin m_1}-\frac{\sin(m_2y)}{\sin m_2}\right)\right.$$
$$\left.+\frac{\lambda\sinh(m_1y)}{m_2{}^2\sinh(m_2)}\cosh((m_2)-1)+\frac{\lambda}{m_2{}^2}(1-\cosh(m_2y))\right]e^{i\omega t}$$

The shear stress at the upper wall of the channel is given by

$$\tau=\mu\frac{\partial u}{\partial y}=-\left[\frac{Gr}{\left(m_1{}^2+m_2{}^2\right)}\left(\frac{m_1\cosh(m_1y)}{\sinh m_1}-\frac{\cosh(m_2y)}{\sinh m_2}\right)\right.$$
$$\left.+\frac{\lambda\cosh(m_1y)}{m_2\sinh(m_2)}\cosh((m_2)-1)-\frac{\lambda}{m_2}(\sinh(m_2y))\right]e^{i\omega t}$$

The rate of heat transfer across the channel's wall is given as

$$Nu = -\frac{\partial \theta}{\partial y} = -\frac{m_1 \cos(m)_1}{\sin(m_1)} e^{i\omega t}$$

3. RESULTS AND DISSCUSSION

For numerical validation of the our analytical results, we have taken the real part of the results obtained in equations (15) – (19) and made use of the following parameter values: *Pe* = 0.71, *Gr* = 1, *Re* = 1, *H* = 1 *N* = 1, λ = 1, *t* = 0, ω = 1, *s* = 1. The velocity profile is plotted in Figures (2) and (3). It can be observed that the fluid velocity profile is parabolic with maximum magnitude along the channel centreline and minimum at the walls. However, it is interesting to note that the magnitude of fluid velocity increases with an increase in radiation parameter and decreases with an increase in Hartmann number i.e. magnetic field intensity. Figure 4 show that the velocity distribution against y for different values of Gr, we noticed that the velocity distribution increases with increase in Gr. The effect of heat generation, Prandtl number and Reynold's number show in figures (5), (6), (7). From these figure we observe that the velocity distribution decreases with increases ϕ, Pe, Re. In Figures (8), (9), (10) and (11) we observed that the fluid temperature increases transversely with maximum value at the lower wall and maximum value at the upper wall. However, a general increase in the fluid temperature is observed with an increase in radiation parameter through absorption of heat, Prandtl number and frequency parameter; while the fluid temperature decreases with increases in heat generation.

4. CONCLUSION

This paper investigates the heat transfer to MHD oscillatory flow in a channel filled with porous medium. The velocity and temperature profiles are obtained analytically and used to compute the wall shear stress and rate of heat transfer at the channel walls. Generally, our results show that increasing magnetic field intensity reduces wall shear stress while increasing radiation parameter through heat absorption causes an increase in the magnitude of wall shear stress.

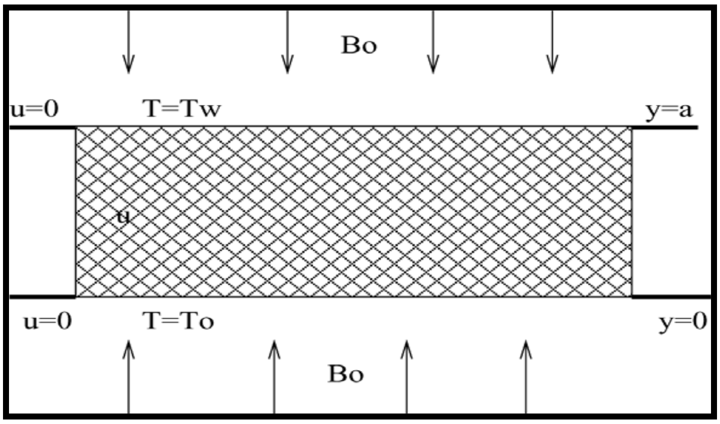

Figure (1): Geometry of the problem.

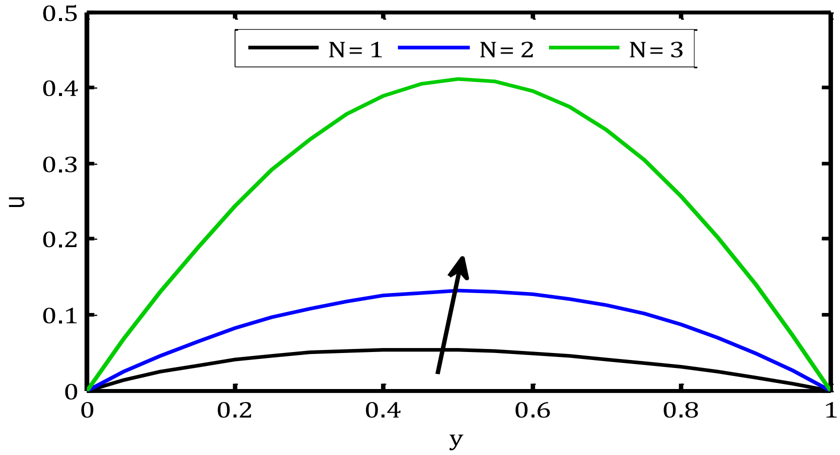

Fig. (2). Velocity distribution for different values of N

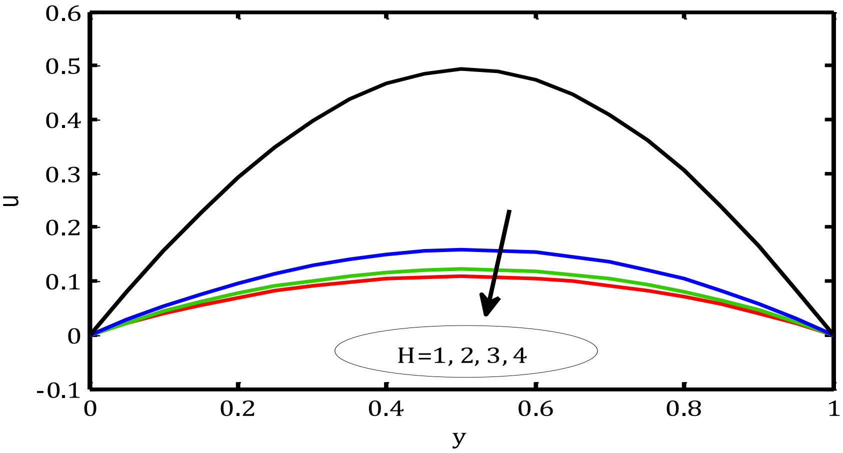

Fig. (3). Velocity distribution for different values of H

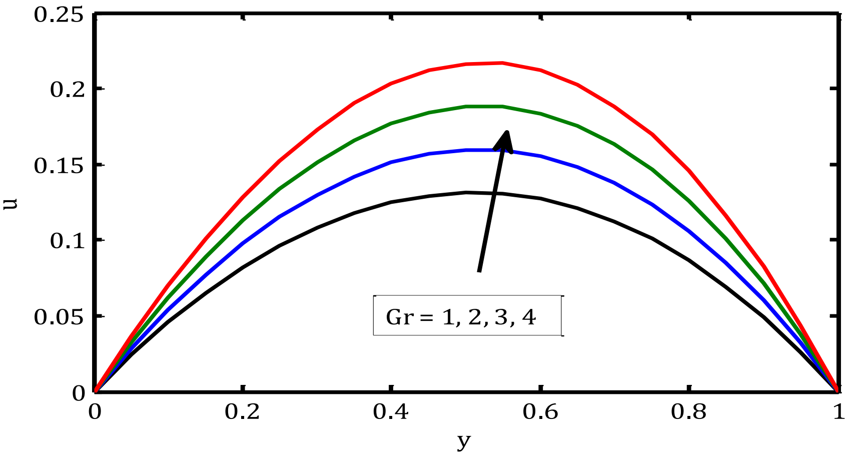

Fig. (4). Velocity distribution for different values of Gr

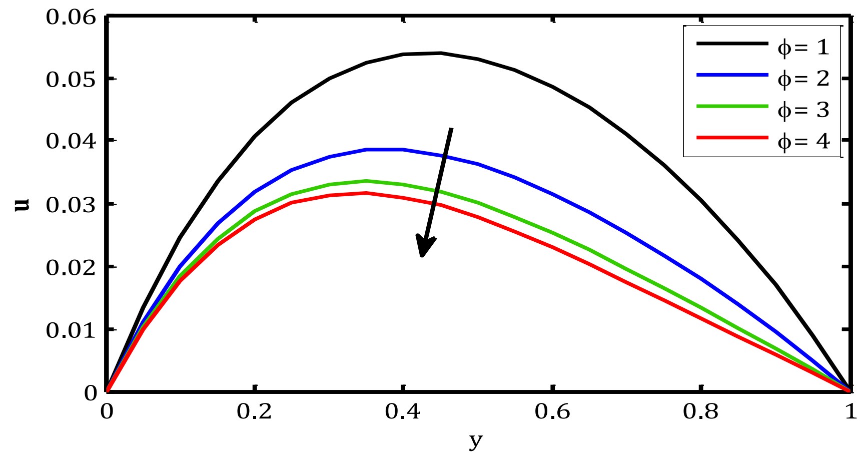

Fig. (5). Velocity distribution for different values of φ

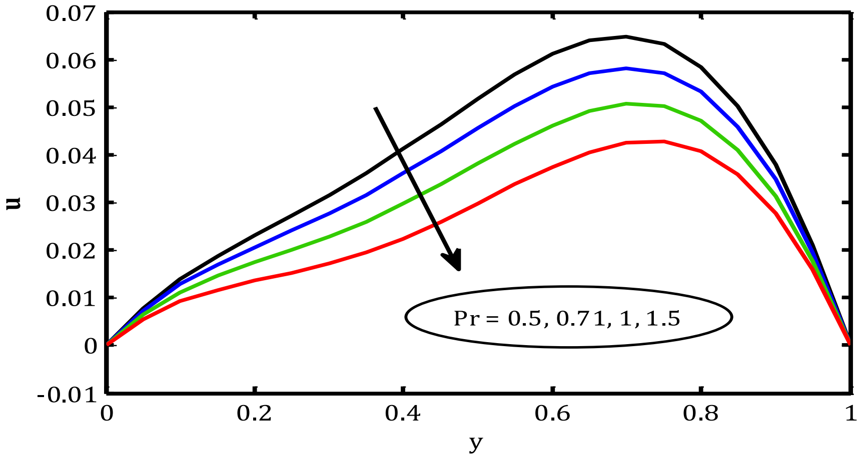

Fig. (6). Velocity distribution for different values of Pr

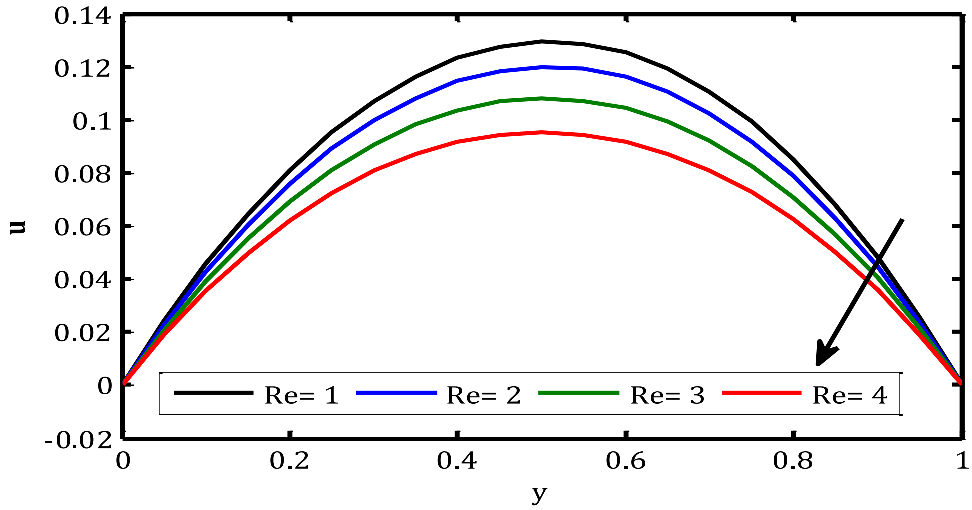

Fig. (7). Velocity distribution for different values of Re

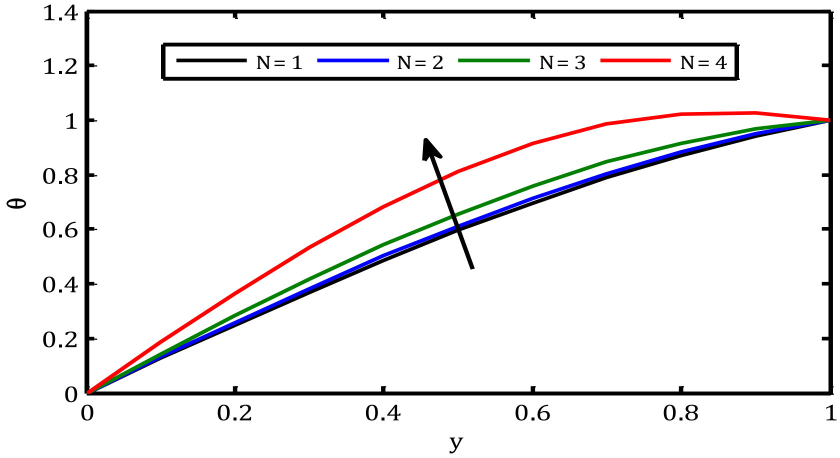

Fig. (8). Temperature distribution for different values of N

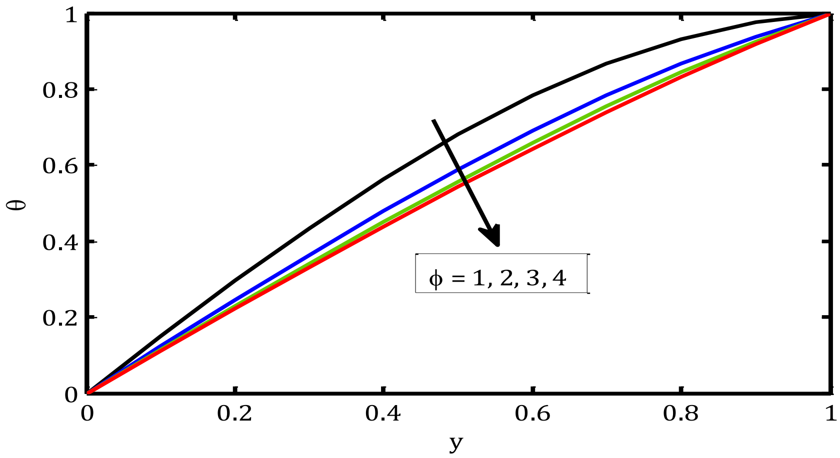

Fig. (9). Temperature distribution for different values of ϕ

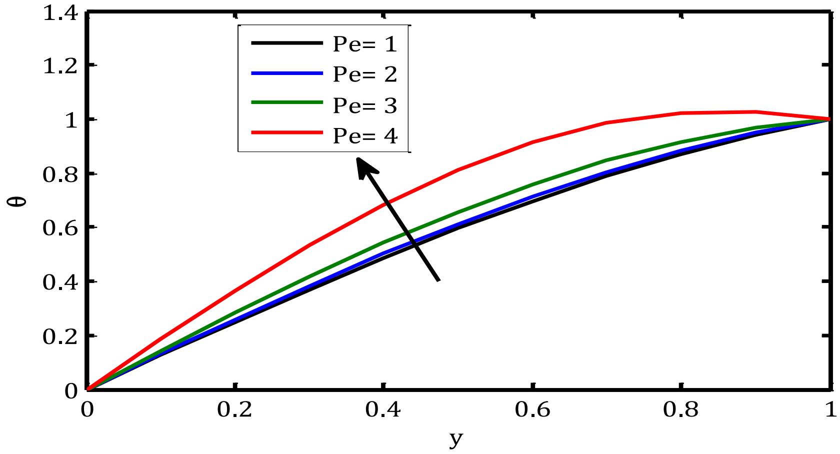

Fig. (10). Temperature distribution for different values of Pe

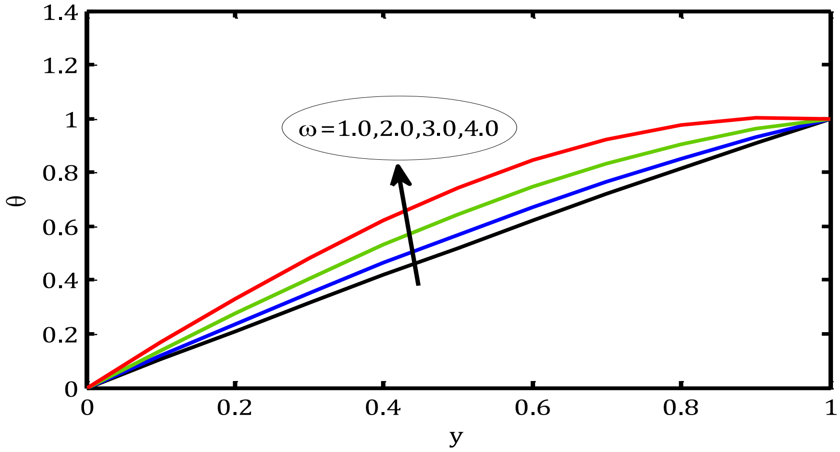

Fig. (11). Temperature distribution for differnt values of ω

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