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MODELING AND OPTIMIZATION OF WATER SUPPLY PROCESSES AT LARGE PUMPING STATIONS

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Abstract. In the work, on the basis of the algebraic model of water supply processes at large pumping stations, issues of modeling and optimization of water supply processes are considered, as well as the results of their calculations are presented.

1. Introduction

Currently, in connection with the increase in the cost of electricity and electrical equipment, as well as increased requirements for saving water resources, the issue of modeling and optimization of water supply processes at large pumping stations, which are one of the main energy consumers in the republic, is particularly urgent. Most pumping stations are operated in modes that underutilize the significant capabilities inherent in pumping units. An increase in non-productive losses of electricity and a decrease in the efficiency of the pumping station are observed. Therefore, there is a need to create such methods of controlling water supply regimes that allow maximum use of the capabilities of the pumping station and optimize water supply modes according to the selected quality criterion.

2. Methods and models

Consider a pumping station where N pumping units with axial rotary vane pumps are installed. The performance of large pumping stations is regulated by changing the number and composition of working pumping units and the turning angles of the axial pump vanes.

The main problems of mathematical modeling of water supply modes of large pumping stations is to determine such mathematical relationships and operators that accurately describe the water supply process and are simple to implement on a modern computer. One of the approaches for modeling the operating modes of complex systems at present is algebraic, which on the basis of set theory and logical operators determines the sequence of the modeling process.

The main characteristic of the i-th pump unit for the development of a mathematical model is its universal operational characteristic [1, 2] which consists of a variety of flow and energy characteristics

$$\Omega_e^i = \Omega_{H,Q,\phi}^i \cup \Omega_{H,\eta,\phi}^i \tag{2.1}$$

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where $\Omega^i_{H,Q,\phi}$, $\Omega^i_{H,\eta,\phi}$ - sets defining the flow characteristic of the pumping unit, i.e.

$$\Omega^i_{,Q,\phi} = \left[Q^i_k \ \left(H^i , \phi^i_k\right), \ ^i \in D^i_H , \ \ \phi^i_k \in D^i_\phi\right] \tag{2.2}$$

$$\Omega^i_{H,\eta,\phi} = \left[\eta^i_k \ \left(H^i , \phi^i_k\right), \ ^i \in D^i_H , \ \ \phi^i_k \in D^i_\phi\right] \tag{2.3}$$

where $D^i_H = \left[H_{\min}, H^i_{\max}\right]$, $D^i_\phi = \left\{\phi_{\min} , \ \phi^i_{\max}\right\}$ - a plurality of ranges for changing the height of the lift and the angle of rotation of the blades of the i-th pump unit; $Q^i_k\left(H^i , \ \phi^i_k\right)$ and $\eta^i_k\left(H^i , \ \phi^i_k\right)$ - the dependence of water flow and efficiency of the pump unit on the lift height at different angles of rotation of the pump blades.

The characteristic of the pressure loss of the pipeline of the i-th pump unit can be represented in the form of the following set

$$\Omega^i_T = \left\{Q^i_T \ \left(H^i + \Delta H^i\right), \ H^i \in D^i_H \ , \ \Delta H^i \in D^i_T\right\}, \tag{2.4}$$

where Q^i_T, $\left(H^i + \Delta H^i\right)$ - pressure characteristic of the pipeline of a working pump unit; $H^i = H^i_{US} - H^i_{DS}$ - pumping station geometric height; $D^i_T = \left[\Delta H^i_{\min}, \Delta H^i_{\max}\right]$ and $D^i_H = \left[H^i_{\min}, H^i_{\max}\right]$ ranges of changes in the geometric height of the pump station and pressure loss in the pipeline.

The values of the components in sets (2.1), (2.2) and (2.3) are determined by the catalog of pumping units and pipelines produced by the respective manufacturers and are specified during the operation of pumping stations.

Valid area D in the coordinates $H - Q$ are determined with the following borders

$$D = \begin{cases} D^i_{1\max} = \Omega^{i\max}_T \cap \Omega^i_{H,Q,\phi}, \\ D^i_{1\min} = \Omega^{i\min}_T \cap \Omega^i_{H,Q,\phi}, \\ D^i_{2\max} = \Omega_{H,Q,\phi_{\max}}; \\ D^i_{2\min} = \Omega_{H,Q,\phi_{\min}}, \end{cases} \tag{2.5}$$

where $\Omega^{i\max}_T$ and $\Omega^{i\min}_T$ - pipeline characteristics at maximum and minimum geometric lift heights.

If given Q^i and H^i are located inside the area D, i.e the point $A\left(Q^i, H^i\right)$ with borders (2.5), it is believed that the given operating mode of the pumping station can be implemented in practice, otherwise this cannot be done.

The condition of the pumping station units at each moment of time is determined by three parameters N, N^p, Ψ^p here N - number of working pumping units, N^p - many capacities of working pumping units; Ψ^p - many turning angles of the blades of the working pumping units.

Knowing the universal operational characteristics of the units and pipelines of the pumping station, the water levels in the upper and lower downstream, as well as the state of the units of the pumping station, you can determine all the main parameters of the pumping station water supply modes at any time.

The algorithm for modeling the water supply modes of large pumping stations, based on the algebraic approach, can be represented in the form of the following sequence:

(1) three $\{N, N^p, \psi^p\}$ are determined the number and angle of rotation of the blades of the i-th working pump unit, i.e n^i and ϕ^i;

(2) from the many universal characteristics of the pumps, the characteristic corresponding to the pump is determined n^i, i.e is chosen $\Omega_e^{n^i}$;
(3) from these characteristics Ω_{H,Q,ϕ^i}, the corresponding flow characteristic is determined by the angle $\phi^i_{n^i}$.
(4) the appropriate operating characteristic of the piping of the pumping unit is selected $\Omega_T^{n^i}$;
(5) the operating point of the pump unit is determined, corresponding to the number n^i , how the intersection of the operating characteristics of the pipeline with the characteristics of the pump, i.e

$$R^{n^i}\left(Q_R^{n^i}, H_R^{n^i}\right) = \Omega^{n^i}_{H,Q,\phi^i} \cap \Omega_T^{n^i}$$

where $Q_R^{n^i}$ and $H_R^{n^i}$ - the coordinates of the working point in the plane $H-Q$;
(6) the coordinates of the operating point and the corresponding energy characteristics determine the efficiency of the pump

$$\eta_R^{n^i} = \Omega^{n^i}_{H_R^{n^i}}, \eta, \phi^i$$

(7) the power consumption of the pump unit with the number n^i is determined

$$N_R^{n^i} = g\frac{Q_R^{n^i} \cdot H_R^{n^i}}{r_R^{n^i}}$$

(8) where $\eta_R = \eta_R^{n^i} \cdot \eta_g$ - efficiency of pumping units, η_g - Motor efficiency;
(9) it is determined the productivity of the pumping station and the power consumption in the form of

$$Q_{DS} = \sum_{i\in n^R} Q_R^i$$

$$N_{DS} = \sum_{i\in n^R} N_R^i$$

.

The task of optimizing water supply modes at large pumping stations is to determine the number and numbers of working pumping units, as well as the rotation angles of their blades, ensuring a minimum power consumption of the pumping station to implement a water supply schedule with a given accuracy.

Mathematically, this problem can be formulated as follows

$$I = \sum_{i\in n^R} N_i \to \min$$

$$\left|\sum_{i\in n^R} Q_i - Q_n\right| \leq \varepsilon$$

where Q_n - set water supply of the pumping station, ε - water accuracy.

The controlling actions are the three parameters

$$D_N = \left\{N, N^R, \phi^P\right\}, \qquad M \leq N$$

The sequence of solving the optimization problem is as follows:

Based on the number of pumping units in operation, acceptable areas are determined , which cover a given water supply mode

$$\forall N, \exists\ (D_N : Q_N \in D_N, D_N \in D, N \leq M)$$

Possible composition of working pumping units capable of creating this productivity is determined

$$\forall N, \forall N^P, \exists \left(D_N^P : D_N^P \in D, Q \in D_N^P, N \leq M\right)$$

Possible conditions of the pumping station are determined that provide water supply with a given accuracy from the multiple turning angles of the blades of the pumping units

$$\forall j, \forall N, \forall N^P, \forall \phi_j^P, \exists \left(\left| Q_n - Q_j^{DS}\right| \leq \varepsilon, Q_j^{DS} \in D_N^P, N \leq M\right)$$

We calculate the power of the pumping station for all variants of its conditions

$$\forall j : N_j^{DS} = \sum_{j \in N_j^P} N_i^J$$

We select from all the options for the composition of the operating pumping units and the sets of blade turns in which the power consumption of the pumping station is minimal

$$\forall_j : \exists \left(j_* : N_{j*}^{DS} = \min N_j^{DS}, U_{j*} = \left\{ N_{j*}, N_{j*}^P, \psi_{j*}^P \right\}\right)$$

The above modeling and optimization algorithms consist of operations that are easily implemented in programming languages and databases.

Using the developed algebraic approach, modeling and optimization of water supply processes were carried out on the example of a large pumping station - pumping station No. 1 of the Karshi main canal. The results of optimization of water supply modes at pump station No. I are shown in table 1. The best option,

Table 1. THE RESULTS OF THE CALCULATION OF OPERATING MODES DS-I

Option	Set of operating pumping units	Turning angles of blade						Expense	Criteria
		HA-1	HA-2	HA-3	HA-4	HA-5	HA-6		
1	1,2,3,4,5	0,0	0,0	-2,0	-6,0	- 6,0	-	190,10	40,471
2	1,2,3,4,6	0,0	-4,0	-4,0	-4,0	-	-2,0	190,00	40,330
3	1,2,3,5,6	0,0	-6,0	0,0	-	-2,0	-6,0	190,20	40,719
4	1,2,4,5,6	0,0	-4,0	-	-4,0	- 4,0	-2,0	190,00	40,336
5	1,3,4,5,6	0,0	-	0,0	-6,0	-2,0	-6,0	190,10	40,343
6	2,3,4,5,6	-	0,0	-4,0	-4,0	-4,0	-2,0	190,10	40,343
7	1,2,3,4,5,6	-2,0	-6,0	-8,0	-8,0	-8,0	-8,0	192,20	42,464

ensuring the flow rate of the pumping station $190 \frac{m^3}{s}$, is option 2.

Conclusion

The developed algebraic approach to modeling and optimization of water supply processes at large pumping stations is easily implemented on a modern computer due to the compactness of its algorithms and the use of the existing database.

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