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Rank Based TOPSIS Approach for Evaluating the Performance of Metaheuristics

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Abstract - **Multiple-criteria decision-making (MCDM) is a sub-discipline of operations research that explicitly evaluates multiple conflicting criteria in decision making. The application of MCDM is not limited to daily life; it also plays a vital part in a wide range of fields including business, governance, medicine, engineering and others. The current study introduces a Rank Based Technique for MCDM, in already existing 'Technique for Order Preference by Similarity to Ideal Solution' (TOPSIS) for the selection of best optimization algorithms. Most of the statistical tests replace the actual data values with signs (either positive or negative) rendering the exact data values useless and leading to high computational costs of statistical tests. To overcome computational cost, an approach has been developed based on TOPSIS named Rank Based TOPSIS (RB-TOPSIS). Since the TOPSIS was not able to handle directly such kind of data so to construct the decision matrix for TOPSIS, a new approach has been introduced. After data conversion; algorithms into alternatives, test functions into decision makers and measures into criteria, RB-TOPSIS was implemented. IEEE Congress on Evolutionary Computation 2017 (CEC's 2017) competition has been taken to evaluate the performance of RB-TOPSIS. The study compares RB-TOPSIS with the 12 algorithms selected in IEEE CEC 2017 competition and considers their statistics in terms of best, worst, median, mean and standard deviations. The experimental results demonstrate that RB-TOPSIS not only overcomes the computational cost but also has better segmentation accuracy.**

Index Terms - MCDM, TOPSIS, CEC 2017, Optimization Algorithms, Ranking.

Mathematics Subject Classification - 90C15, 90C30, 90C59, 90C90.

INTRODUCTION

Decision making is an inevitable part of everyday life. It is done in the times of various occasions 'when people do not have time for reflection and decisions are taken as automatically and subconsciously. When we do not have sufficient time to take decisions, decisions are made consciously but quite automatically. Some decisions are the result of detailed and time-spanned examination. The making of decisions is a realm of dynamic and active research as in society, behavioral sciences, psychology, military, neurology, management and economics [1-6]. If there is no need to decide instantaneously and the background is complicated, it also develops into a division of computer sciences and applied mathematics and has been flourishing in the last decades [7, 8].

Still there have been complications to make decisions. Though the advancements in the field of brain research have been spectacular, though there are found inadequate yet well-founded clues as how we can mimic analytical capacity of brain. The Responses to easy and simple stimuli are foreseen correctly. But there has been a difficulty to untangle the mystery as how decisions are made in more complex settings. As a result, there are few general rules which are not available on how to construct automated decision-making devices able to replace human beings in their creative capacity. Thus, we are mainly concerned with problems in which every possible alternative is assessed against a set of at least two quality criteria.

The comparison of algorithms in evolutionary computation is a thing of difficulty. Generally, algorithms have been utilized numerously to solve the multiple benchmarks. At that point, the results are examined with the help of statistical hypothesis tests [9, 10]. Statistical tests can be identified in the variances among the performance of algorithms. Main problem occurs which algorithm is the most suitable? It is essential to create comparisons which are pairwise among the algorithms as how to use statistical tests on this stage. A number of tests are required to increases it impressively through the various algorithms which are being analyzed. Firstly, the problem arises when the annoying thing of relating both the pairs of algorithms; secondly, the probability of committing an error upsurge, vagueness is also an important aspect which is required to be addressed in MCDM. This is a situation which is explained as an individual, not required to have fitting, qualitative and quantitative info to define, suggest or envisage a system statistically, deterministically and the performance, behavior and its features.

In recent years a lot of sophisticated meta-heuristics have been introduced to solve the most complex problems by transforming them into problems of optimization [11-14]. Improvisations to differential equations of these suggested metaheuristics and their application of these meta-heuristics [15-18] to widely distributed and disease models are difficult to see [19-22]. The study conducted by Farhan et al. for the treatment of the epidemic model by using evolutionary Pad´e-approximation, extend this work to solve under line measles dynamical model [23-26].

A number of MCDM methods are recognized in the literature. Both are unlike from each other in the context of required quantity and quality of supplementary data, the procedures, the user-friendliness of the approaches and their associated software, the sensitivity tools used, and the mathematical properties. So far as the TOPSIS was advanced by Hwang and Yoon [27] and it is a method to assess the presentation and performance of alternatives with the generated resemblance with best possible solution. The finest alternative is the kind of closest with the Positive Ideal Solution (PIS) [28] and it is distant from the Negative Ideal Solution (NIS) [29]. PIS makes the most of the benefit criterion reduces the cost criteria, and NIS increases the cost criteria and decreases the benefit criteria. Besides, the reader who are interested shall refer to [30] for a broad survey about the TOPSIS.

Research has been done on the Single Objective Optimization Algorithms (SOOA) which is the foundation of the additional multifaceted optimization algorithms such as, niching algorithms, constrained optimization algorithms, multi-objective optimizations algorithms and so on. These entire novel and advanced algorithms have been experimented on one and single objective standard problems. Additionally, these benchmark single objective problems may transform into computationally well-appointed and numerous other types of problems [31].

Moreover, our goal is to construct the mechanism to test the optimization function to handle a decision matrix with regarding to evaluated best, worst, median, mean and standard deviations and present a tool to selecting the best algorithm when applied TOPSIS. The remaining part of the article has been organized as: Section 2 describes the classical TOPSIS. In Section 3, we present a novel methodology created on TOPSIS to deal with a decision matrix with ratings in terms of best, worst, median, mean and standard deviations. In Section 4, simulation results for a CEC 2017 concerning to 30 optimization problems involving different categories. In the Section 5 conclusion and directions are done for future tasks.

Classical Technique for Order Preference by Similarity to Ideal Solution

TOPSIS had been established by Hwang and Yoon [27]. It is comparatively easy, rapid, and has an organized process. TOPSIS is considered a top method in solving the rank reversal issue. TOPSIS decision should be nearest to the PIS and farthest from the NIS. Such PIS and NIS are being computed by seeing the other alternatives. The PIS composes all apt values which attain these criteria, and the NIS is consisted on all the poor standards possible to get the criteria. It offers considerate help to comprehend the variances and similarities between the alternatives. These characteristics are or must be numeric, constantly growing or lessening, and must have commensurable units. TOPSIS uses these attribute and information, which provide major ranking of alternatives, do not require attribute preferences to be self-governing after applying this technique.

In this section, we review the TOPSIS method and benchmark test functions. For convenience, we first let $E = \{1,2,3,\cdots,e\}$, $F = \{1,2,3,\cdots,f\}$ and $G = \{1,2,3,\cdots,g\}$; $i \in E, j \in F, k \in G$. Let $R = \{R_1, R_2, \cdots, R_e\}$ $(e \geq 2)$ be a discrete set of e feasible alternatives,

$S = \{s_1, s_2, \cdots, s_f\}$ be a finite set of attributes,

$T = \{t_1, t_2, \cdots, t_g\}$ be a group of decision makers, and

$\emptyset$ is the weight vector of DMs, where $\emptyset_k \geq 0; \sum_{k=1}^{f} \emptyset_k = 1$

Every alternative for n attributes is evaluated and constitute a decision matrix for a MADM problem denoted by

$$U = (u_{ij})_{e\times f} = \begin{array}{c c} & \begin{array}{cccc} s_1 & s_2 & \cdots & s_f \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ \vdots \\ R_e \end{array} & \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1f} \\ u_{21} & u_{22} & \cdots & u_{2f} \\ \vdots & \vdots & \ddots & \vdots \\ u_{e1} & u_{e2} & \cdots & u_{ef} \end{pmatrix} \end{array} \tag{1}$$

The TOPSIS method involves of the following phases [32-34]:

Normalize the Decision Matrix: In general, cost attributes and benefits attributes are existed in the MCDM problems. To convert attributes in dimensionless units, the following equations are introduced to normalize every attribute value u_{ij} in decision matrix $U = (u_{ij})_{e\times f}$ into a corresponding element v_{ij} in normalized decision matrix given by Eq. (2).

$$V = (v_{ij})_{e\times f} = \begin{array}{c c} & \begin{array}{cccc} s_1 & s_2 & \cdots & s_n \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ \vdots \\ R_e \end{array} & \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1f} \\ v_{21} & v_{22} & \cdots & v_{2f} \\ \vdots & \vdots & \ddots & \vdots \\ v_{e1} & v_{e2} & \cdots & v_{ef} \end{pmatrix} \end{array} \tag{2}$$

where $v_{ij} = \frac{u_{ij}}{\sqrt{\sum_{i=1}^{e}(u_{ij})^2}}$, for benefit attribute $u_{ij}, i \in E, j \in F.$ (3)

$and\ v_{ij} = 1 - \frac{u_{ij}}{\sqrt{\sum_{i=1}^{e}(u_{ij})^2}},$ for cost attribute $u_{ij}, i \in E, j \in F.$ (4)

Calculate the Weighted Normalized Decision Matrix: $Z = (z_1, z_2, \cdots, z_f)^T$ is the weighted vector of the attributes, where $z_k \geq 0; \sum_{j=1}^{f} z_j = 1$, for the construction of weighted normalized decision matrix as

$$W = (z_j v_{ij})_{e \times f} = (w_{ij})_{e \times f} = \begin{array}{c} \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & \cdots & s_f \end{array} \\ \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1f} \\ y_{21} & y_{22} & \cdots & y_{2f} \\ \vdots & \vdots & \ddots & \vdots \\ y_{e1} & y_{e2} & \cdots & y_{ef} \end{pmatrix} \end{array} \quad (5)$$

Determine the PIS and NIS: The PIS R^+ and NIS R^- are determined, respectively, as follows:

$R^+ = \{w_1^+, w_2^+, \cdots, w_f^+\}$ (6)

and $R^- = \{w_1^-, w_2^-, \cdots, w_f^-\}$ (7)

where $w_1^+ = \max_{1 \leq i \leq e}\{w_{ij}\}\ (j \in F)$ and

$w_1^- = \min_{1 \leq i \leq e}\{w_{ij}\}\ (j \in F)$

Measure the Distance from PIS and NIS: D_i^+ , is given as for separation of each alternative form the PIS

$D_i^+ = \sqrt{\sum_{j=1}^{f}(w_{ij} - w_j^+)^2}, i \in E$ (8)

Similarly, D_i^- , is given as t0he separation form the NIS,

$D_i^- = \sqrt{\sum_{j=1}^{f}(w_{ij} - w_j^-)^2}, i \in E$ (9)

Compute the Closeness Coefficient to the Ideal Solutions: The closeness coefficient of the i^{th} alternative R^i with respect to the ideal solutions is defined as

$C_i = \frac{D_i^-}{D_i^+ + D_i^-}, i \in E$ (10)

Since $D_i^+ \geq 0\ (i \in E)$ and $D_i^- \geq 0\ (i \in E)$then, clearly, $C_i \in [0,1] (i \in E)$.

Rank the Preference Order: Alternatives can be graded by priority and according to the downward order of C_i; in other words, larger C_i means better alternative.

RANK BASED TOPSIS

Nonparametric hypothesis tests are less efficient than parametric tests. Nonparametric tests replace the actual data values with either sign (positive or negative). Thus, the exact data values are wasted. To overcome this issue, an approach based on TOPSIS has been developed for algorithm ranking named as Rank based TOPSIS (R_B-TOPSIS). Since the TOPSIS is not able to handle directly this kind of data, so convert the data as, algorithms into alternatives, test functions into decision makers and measures into criteria. The ranks are taken in ascending order. The following formula has been used to convert the ranks form maximum to minimum.

$Rank = 1 - \frac{r_i}{r_{i\,max}}, i = 1, 2, 3, \ldots, 12$ (11)

where r_i is the rank of i^{th} algorithm and $r_{i\,max}$ is the maximum rank of i^{th} algorithm? To construct the decision matrix after taking the average value of 30 test problems for each algorithm; best, worst, median, mean and standard deviations has been considered.

This algorithm represents the comprehensive detail of rank based TOPSIS.

Algorithm of R_B-TOPSIS

Step 1: Set alternatives as algorithms and criteria as test functions.
Step 2: Collect and rank all the minimum values of 30 test functions for 12 algorithms.
Step 3: To convert the ranks from maximum to minimum by using equation 11.
Step 4: Construct the decision matrix for best, worst, median, mean and standard deviations.
Step 5: Construct the normalize decision matrix by using equations 2-4.
Step 6: Construct the weighted normalized decision matrix by using equation 5.
Step 7: Determine the PIS and NIS by using equations 6-7.
Step 8: Calculated separation measure by using equations 8-9.
Step 9: Calculated relative closeness to PIS by using equation 10.
Step 10: Rank to obtained optimal solution.

The pictorial representation of rank based TOPSIS is illustrated in figure 1.

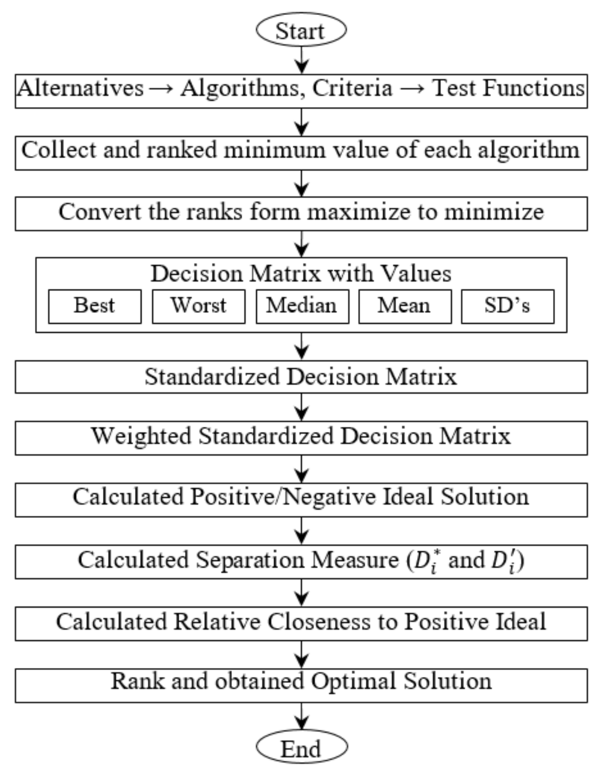

FIGURE 1
Flow chart of R_B-TOPSIS

The figure 2 has been established in hierarchical framework for selecting the best algorithm which is evaluated from twelve algorithms and thirty benchmark functions.

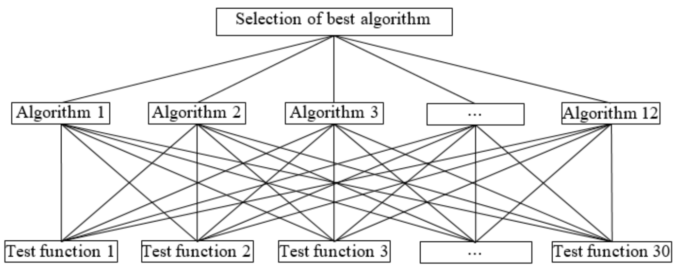

FIGURE 2
The hierarchical model of best algorithm selection

NUMERICAL RESULTS AND DISCUSSION

Several types of novel optimization algorithms are proposed to solve real parameter optimization problems. In IEEE Congress on Evolutionary Computation 2017 (CEC' 2017) competition, twelve algorithms were selected and thirty benchmark problems with several novel types are developed. There are 30 benchmark test functions, three uni-modal, seven multimodal, ten hybrid, ten composition functions were included in CEC'17 test suite, detailed summary is available in [31] with the following experimental setup:
$number\ of\ optimization\ problems\ =\ 30$,
$Dimensions\ D\ =\ 10,\ 30,\ 50,\ 100$,
$execution\ of\ each\ problem\ =\ 51$,
$maximum\ function\ evelutions\ =\ 10000 * D$,
$range\ of\ the\ problems\ =\ [-100, 100]$,
$Initial\ guess: Randomly$,
$Termination\ criteria\ < 10^{-8}$,
$Global\ Optimum: within\ the\ given\ bounds$.
Finally, 12 algorithms were selected for CEC'17 competition. The comprehensive details of all algorithms were provided as follows: $Algo1 = jSO$ [35], $Algo2 = MM\_OED$ [36], $Algo3 = IDEbestNsize$ [37], $Algo4 = RB - IPOP - CMA - ES$ [38], $Algo5 = LSHADE\_SPACMA$ [39], $Algo6 = DES$ [40], $Algo7 = DYYPO$ [41], $Algo8 = TLBO - FL$[42], $Algo9 = PPSO$ [43], $Algo10 = MOS - SOCO2011/13$ [44], $Algo11 = LSHADE - cnEpSin$ [45], $Algo12 = EBOwithCMAR$ [46]. As a standard procedure in computation, the twelve different algorithms have been applied to the thirty benchmarks test problems is to govern the superior algorithm in terms of effectiveness among the twelve algorithms analyzed. The comprehensive description of rank based TOPSIS is applied on ten dimensional problems only, in remaining 30, 50 and 100 dimensional problems directly result are used. Under the above settings the ranked based TOPSIS is as follows:

Rank the Preference Order: In first step the optimization results and algorithms are set according to the TOPSIS methods. Set algorithms as alternatives, test functions as decision makers and measures as criteria.
Collect and Rank the Minimum Values: In this step collect the best, worse, median, mean and standard deviation of thirty test functions for twelve algorithms. Firstly, collect the best values from twelve algorithms [35-46] see appendix A.1. After collecting the best values applying equation 11 on the dataset and rank the algorithms which are presenting in Table 1. In similar fashion secondly collect the worst values from twelve algorithms [35-46] see appendix A.2, and rank the algorithms by using the equation 11 which are presenting in Table 2. Thirdly collect the median values from twelve algorithms [35-46] see appendix A.3, and rank the algorithms by using the equation 11 which are presenting in Table 3. Fourthly collect the mean values from twelve algorithms [35-46] see appendix A.4, and rank the algorithms by using the equation 11 which are presenting in Table 4. Lastly collect the standard deviation from twelve algorithms [35-46] see appendix A.5, and rank the algorithms by using the equation 11 which are presenting in Table 5.

TABLE 1
Ranks of best values of thirty test functions for twelve algorithms

Algorithms	*Algo1*	*Algo2*	*Algo3*	*Algo4*	*Algo5*	*Algo6*	*Algo7*	*Algo8*	*Algo9*	*Algo10*	*Algo11*	*Algo12*
F_1	4.5	4.5	4.5	4.5	4.5	4.5	12	11	10	9	4.5	4.5
F_2	6	6	6	6	6	6	6	6	6	12	6	6
F_3	5.5	5.5	5.5	5.5	5.5	5.5	12	11	5.5	5.5	5.5	5.5
F_4	4.5	4.5	4.5	4.5	4.5	4.5	10	12	11	9	4.5	4.5
F_5	3.5	3.5	8	3.5	7	3.5	11	9	12	10	3.5	3.5
F_6	5.5	5.5	5.5	11	5.5	5.5	12	5.5	5.5	5.5	5.5	5.5
F_7	10	5	2	1	6.5	9	3	12	4	11	8	6.5
F_8	3.5	3.5	8	3.5	3.5	3.5	11	10	12	9	7	3.5
F_9	6	6	6	6	6	6	12	6	6	6	6	6
F_{10}	3	5	6	8	4	1	9	12	11	10	7	2
F_{11}	4.5	4.5	4.5	4.5	4.5	4.5	11	9	12	10	4.5	4.5
F_{12}	3	3	3	6.5	3	8	10	12	11	9	6.5	3
F_{13}	3.5	7	8	3.5	3.5	3.5	11	12	9	10	3.5	3.5
F_{14}	4	4	4	8	4	4	9	12	11	10	4	4
F_{15}	5	2	4	6	7	8	10	12	11	9	1	3

F_{16}	4	3	2	11	7	9	10	12	6	8	1	5
F_{17}	6	1	7	11	5	9	8	12	10	2	3	4
F_{18}	1	3	2	7	6	8	9	12	11	10	4	5
F_{19}	3.5	3.5	3.5	7	3.5	10	9	12	11	8	3.5	3.5
F_{20}	4	4	4	11	4	9	8	10	12	4	4	4
F_{21}	5.5	5.5	5.5	5.5	5.5	11	5.5	5.5	5.5	5.5	5.5	12
F_{22}	10	10	1	7	10	10	2	4	5	3	10	6
F_{23}	7	2.5	7	2.5	7	7	1	7	12	11	7	7
F_{24}	7.5	7.5	7.5	2	7.5	7.5	7.5	7.5	1	7.5	7.5	7.5
F_{25}	4.5	2.5	6	1	8.5	2.5	11.5	11.5	8.5	8.5	4.5	8.5
F_{26}	10.5	6.5	10.5	4	10.5	6.5	6.5	2	2	2	10.5	6.5
F_{27}	6	4.5	3	11	1	10	8	8	12	4.5	2	8
F_{28}	9	9	9	3	9	9	4	9	1.5	5	9	1.5
F_{29}	7	9	10	3	5	5	8	12	11	1	2	5
F_{30}	4	4	8	6	1	4	12	11	10	9	2	7

TABLE 2
Ranks of worst values of thirty test functions for twelve algorithms

Algorithms	***Algo1***	***Algo2***	***Algo3***	***Algo4***	***Algo5***	***Algo6***	***Algo7***	***Algo8***	***Algo9***	***Algo10***	***Algo11***	***Algo12***
F_1	4	4	4	8	4	4	12	11	9	10	4	4
F_2	5.5	5.5	5.5	5.5	5.5	5.5	5.5	12	5.5	11	5.5	5.5
F_3	5.5	5.5	5.5	5.5	5.5	5.5	11	12	5.5	5.5	5.5	5.5
F_4	4.5	4.5	4.5	4.5	4.5	4.5	12	11	10	9	4.5	4.5
F_5	2	3	7	8	4	6	12	10	11	9	5	1
F_6	4	4	4	9	4	12	10	8	11	4	4	4
F_7	6	5	8	3	2	7	11	12	9	10	4	1
F_8	3	4.5	7	8	2	4.5	12	11	9	10	6	1
F_9	5	5	5	5	5	5	12	11	5	10	5	5
F_{10}	6	3	7	11	2	1	10	12	9	8	4	5
F_{11}	2.5	2.5	5	8	6.5	6.5	11	10	12	9	2.5	2.5
F_{12}	2	5	1	7	6	8	11	12	9	10	4	3
F_{13}	3	6	1	8	2	7	12	10	9	11	5	4
F_{14}	2.5	6	1	9	4.5	7	10	11	8	12	2.5	4.5
F_{15}	3.5	3.5	1	7	6	8	12	10	9	11	3.5	3.5
F_{16}	6	1	3	12	5	8	10	7	11	9	4	2
F_{17}	3	1	4	12	6	9	11	10	8	7	5	2
F_{18}	2	4	1	7	6	8	11	12	9	10	5	3
F_{19}	2	1	3	8	5	7	12	11	10	9	6	4
F_{20}	6.5	5	3	12	4	8	10	9	11	1.5	6.5	1.5
F_{21}	5	3	7	9	6	8	1	11	10	12	4	2
F_{22}	3.5	7	3.5	9	3.5	3.5	12	3.5	10.5	10.5	8	3.5
F_{23}	4	5	7	8	2	6	11	10	12	9	3	1
F_{24}	4	1	6	9	7	2	10	8	12	11	5	3
F_{25}	4.5	6.5	9	4.5	10	6.5	11.5	11.5	1.5	8	3	1.5
F_{26}	4.5	4.5	4.5	12	4.5	4.5	11	9	4.5	10	4.5	4.5
F_{27}	1	2	3	7	5.5	9	11	8	12	10	4	5.5
F_{28}	6.5	10.5	8	6.5	3	10.5	2	12	1	9	5	4
F_{29}	3	5	6	12	2	7	10	9	8	11	1	4
F_{30}	2	10.5	3	7	1	10.5	6	12	5	9	8	4

TABLE 3
Ranks of median values of thirty test functions for twelve algorithms

Algorithms	***Algo1***	***Algo2***	***Algo3***	***Algo4***	***Algo5***	***Algo6***	***Algo7***	***Algo8***	***Algo9***	***Algo10***	***Algo11***	***Algo12***
F_1	4.5	4.5	4.5	4.5	4.5	4.5	11	12	9	10	4.5	4.5
F_2	5.5	5.5	5.5	5.5	5.5	5.5	5.5	12	5.5	11	5.5	5.5
F_3	5.5	5.5	5.5	5.5	5.5	5.5	11	12	5.5	5.5	5.5	5.5
F_4	4.5	4.5	4.5	4.5	4.5	4.5	10	12	11	9	4.5	4.5
F_5	6	3.5	8	2	5	3.5	11	10	12	9	7	1
F_6	4	4	4	8	4	10	12	9	11	4	4	4
F_7	5	4	8	2	3	6	11	12	9	10	7	1
F_8	5	3.5	8	2	3.5	6	12	11	10	9	7	1
F_9	5.5	5.5	5.5	5.5	5.5	5.5	11	12	5.5	5.5	5.5	5.5
F_{10}	4	3	7	10	2	1	9	12	11	8	6	5
F_{11}	4	4	4	4	4	8	11	10	12	9	4	4
F_{12}	1.5	5	1.5	6	7	8	11	12	9	10	4	3
F_{13}	5	8	2	5	7	3	12	11	9	10	5	1
F_{14}	3.5	3.5	3.5	8	3.5	9	7	12	11	10	3.5	3.5
F_{15}	4	2	1	7	5	8	9	12	11	10	6	3
F_{16}	5	1	3	12	6	7	9	10	11	8	4	2

F_{17}	5	1	6	12	4	9	8	11	10	7	3	2
F_{18}	3	2	1	7	6	8	12	11	10	9	5	4
F_{19}	1.5	1.5	3	7	6	8	9	12	11	10	5	4
F_{20}	6.5	2.5	2.5	12	5	10	8	9	11	2.5	6.5	2.5
F_{21}	4.5	4.5	4.5	4.5	9	11	4.5	10	4.5	12	4.5	4.5
F_{22}	6.5	6.5	6.5	6.5	6.5	6.5	6.5	1	12	6.5	6.5	6.5
F_{23}	2	7	4	6	8	2	11	9	12	10	5	2
F_{24}	9	2.5	7	5	11	8	2.5	6	2.5	12	10	2.5
F_{25}	5	3.5	2	6	11	1	10	9	7	8	12	3.5
F_{26}	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5
F_{27}	4	3	8	9	1	10	11	5.5	12	7	2	5.5
F_{28}	5.5	5.5	5.5	5.5	5.5	12	5.5	11	5.5	5.5	5.5	5.5
F_{29}	5	6	7	8	2.5	4	9	11	12	10	1	2.5
F_{30}	2	4	5	7	1	8	11	12	9	10	6	3

TABLE 4.
Ranks of mean values of thirty test functions for twelve algorithms

Algorithms	***Algo1***	***Algo2***	***Algo3***	***Algo4***	***Algo5***	***Algo6***	***Algo7***	***Algo8***	***Algo9***	***Algo10***	***Algo11***	***Algo12***
F_1	4.5	4.5	4.5	4.5	4.5	4.5	12	11	9	10	4.5	4.5
F_2	6	6	6	6	6	6	6	6	6	12	6	6
F_3	5.5	5.5	5.5	5.5	5.5	5.5	12	11	5.5	5.5	5.5	5.5
F_4	4.5	4.5	4.5	4.5	4.5	4.5	11	12	10	9	4.5	4.5
F_5	7	2	8	5	3	4	11	10	12	9	6	1
F_6	4.5	4.5	4.5	9	4.5	11	10	4.5	12	4.5	4.5	4.5
F_7	5	4	8	1	3	6	11	12	9	10	7	2
F_8	6	3	8	7	2	4	12	11	10	9	5	1
F_9	5.5	5.5	5.5	5.5	5.5	5.5	12	5.5	5.5	11	5.5	5.5
F_{10}	4	3	7	10	2	1	9	12	11	8	6	5
F_{11}	3	3	3	8	6	7	11	10	12	9	3	3
F_{12}	2	5	1	6	7	8	11	12	9	10	4	3
F_{13}	3	8	1	7	5	4	12	10	9	11	6	2
F_{14}	3	6	1	8	2	7	9	11	10	12	5	4
F_{15}	4	2	1	7	6	8	9	12	10	11	5	3
F_{16}	5	1	3	12	6	8	10	7	11	9	4	2
F_{17}	4	1	5	12	6	9	8	11	10	7	3	2
F_{18}	2	4	1	7	5	8	12	11	9	10	6	3
F_{19}	3	1	2	7	6	8	12	11	10	9	5	4
F_{20}	7	3	1	12	5	9	8	10	11	2	6	4
F_{21}	6	3	9	7	10	12	1	4	2	11	8	5
F_{22}	9	9	2	6	9	9	4	9	3	1	12	5
F_{23}	4	2	6	1	7.5	5	9.5	9.5	12	11	7.5	3
F_{24}	9	1	8	4	7	10	2	12	5	6	11	3
F_{25}	3	7	6	1	11	4	9	12	2	8	10	5
F_{26}	9.5	5	9.5	4	9.5	6	9.5	9.5	3	1	9.5	2
F_{27}	2	3.5	7	9	1	10	11	5	12	8	3.5	6
F_{28}	6	5	4	11	7	12	2	9	1	8	10	3
F_{29}	4	5	7	9	2	6	8	11	12	10	1	3
F_{30}	2	11	3	5	1	12	7	8	6	10	9	4

TABLE 5
Ranks of standard deviation of thirty test functions for twelve algorithms

Algorithms	***Algo1***	***Algo2***	***Algo3***	***Algo4***	***Algo5***	***Algo6***	***Algo7***	***Algo8***	***Algo9***	***Algo10***	***Algo11***	***Algo12***
F_1	4	4	4	8	4	4	12	11	9	10	4	4
F_2	5.5	5.5	5.5	5.5	5.5	5.5	5.5	12	5.5	11	5.5	5.5
F_3	5.5	5.5	5.5	5.5	5.5	5.5	11	12	5.5	5.5	5.5	5.5
F_4	4.5	4.5	4.5	4.5	4.5	4.5	12	11	10	9	4.5	4.5
F_5	5	3	7	8	2	6	10	12	11	9	4	1
F_6	4	4	4	9	4	12	10	8	11	4	4	4
F_7	4	5	7	9	2	6	12	11	8	10	3	1
F_8	2	6	5	8	3	7	12	11	9	10	4	1
F_9	5	5	5	5	5	5	12	11	5	10	5	5
F_{10}	5	3	7	11	2	1	10	12	9	8	6	4
F_{11}	2.5	2.5	5	8	6	7	11	10	12	9	2.5	2.5
F_{12}	2	3	1	7	4	8	11	12	9	10	5	6
F_{13}	2	6	1	8	3	7	12	10	9	11	5	4
F_{14}	4	6	1	9	2	7	11	10	8	12	5	3
F_{15}	4	2	1	7	6	8	11	10	9	12	5	3
F_{16}	5	3	1	12	4	8	10	7	11	9	6	2
F_{17}	3	1	5	12	6	10	11	9	8	7	4	2

F_{18}	2	4	1	7	5	8	12	11	9	10	6	3
F_{19}	3	1	2	8	5	7	12	11	10	9	6	4
F_{20}	3	4	1	12	6	11	9	10	8	2	7	5
F_{21}	6	3	12	7	8	2	1	11	4	9	10	5
F_{22}	1	4	10	6	5	2	9	11	8	12	3	7
F_{23}	3	10	6	12	2	5	11	8	9	7	4	1
F_{24}	7	1	6	11	8	4	2	5	12	9	3	10
F_{25}	2	6	8	12	10	3	11	7	1	4	9	5
F_{26}	2.5	6	2.5	12	2.5	5	7	8	10	11	2.5	9
F_{27}	2	1	4	3	9	6	11	10	12	8	5	7
F_{28}	7	8	4	12	5	10	2	11	1	6	9	3
F_{29}	3	6	5	12	2	7	10	9	8	11	1	4
F_{30}	1	10	2	7	4	11	6	12	5	9	8	3

Ranked Average Value: Use the equation 11 to convert the ranks from maximum to minimum values. After converting best, median, mean, worst and standard deviation, Table 6 represented the average ranked values of best, median, mean, worst and standard deviation of 12 algorithms and 5 criteria.

TABLE 6

Average ranked values of best, median, mean, worst and standard deviation of 12 algorithms

Algorithms	Best	Worst	Median	Mean	SD
Algorithm 1	0.52029	0.675845	0.6	0.595833	0.695833
Algorithm 2	0.567952	0.626993	0.643056	0.640789	0.630556
Algorithm 3	0.513998	0.616969	0.584722	0.598611	0.630556
Algorithm 4	0.504959	0.328623	0.447222	0.438743	0.284722
Algorithm 5	0.508696	0.62657	0.544444	0.55	0.611111
Algorithm 6	0.430452	0.446437	0.431944	0.40117	0.465278
Algorithm 7	0.259272	0.156944	0.215278	0.240278	0.204167
Algorithm 8	0.17754	0.148611	0.1375	0.190278	0.158333
Algorithm 9	0.282624	0.287319	0.213889	0.30614	0.316667
Algorithm 10	0.357161	0.233756	0.279167	0.299269	0.268056
Algorithm 11	0.54529	0.619082	0.536111	0.484722	0.579167
Algorithm 12	0.545839	0.72343	0.683333	0.697149	0.655556

Construct the Decision Matrix: To construct the decision matrix for best, median, mean, worst and standard deviation taking the transpose of the values. The detail results of best, median, mean, worst and standard deviation for all 12 algorithms and 5 criteria are available in Table 7.

TABLE 7

Representation of best, median, mean, worst and standard deviation for all algorithms

Criteria	Best	Worst	Median	Mean	SD
Algorithm 1	5.20E-01	6.76E-01	6.00E-01	5.96E-01	6.96E-01
Algorithm 2	5.68E-01	6.27E-01	6.43E-01	6.41E-01	6.31E-01
Algorithm 3	5.14E-01	6.17E-01	5.85E-01	5.99E-01	6.31E-01
Algorithm 4	5.05E-01	3.29E-01	4.47E-01	4.39E-01	2.85E-01
Algorithm 5	5.09E-01	6.27E-01	5.44E-01	5.50E-01	6.11E-01
Algorithm 6	4.30E-01	4.46E-01	4.32E-01	4.01E-01	4.65E-01
Algorithm 7	2.59E-01	1.57E-01	2.15E-01	2.40E-01	2.04E-01
Algorithm 8	1.78E-01	1.49E-01	1.38E-01	1.90E-01	1.58E-01
Algorithm 9	2.83E-01	2.87E-01	2.14E-01	3.06E-01	3.17E-01
Algorithm 10	3.57E-01	2.34E-01	2.79E-01	2.99E-01	2.68E-01
Algorithm 11	5.45E-01	6.19E-01	5.36E-01	4.85E-01	5.79E-01
Algorithm 12	5.46E-01	7.23E-01	6.83E-01	6.97E-01	6.56E-01

Given 12 alternatives, 5 criteria, 30 decision-makers, a typical multi-criteria group decision-making problem can be expressed in matrix format as

$$\begin{array}{rc} & \begin{array}{cccccccccccc} Algo_1 & Algo_2 & Algo_3 & Algo_4 & Algo_5 & Algo_6 & Algo_7 & Algo_8 & Algo_9 & Algo_{10} & Algo_{11} & Algo_{12} \end{array} \\ \begin{array}{r} Best \\ Worst \\ X = Median \\ Mean \\ SD \end{array} & \begin{pmatrix} 5.20E-01 & 5.68E-01 & 5.14E-01 & 5.05E-01 & 5.09E-01 & 4.30E-01 & 2.59E-01 & 1.78E-01 & 2.83E-01 & 3.57E-01 & 5.45E-01 & 5.46E-01 \\ 6.76E-01 & 6.27E-01 & 6.17E-01 & 3.29E-01 & 6.27E-01 & 4.46E-01 & 1.57E-01 & 1.49E-01 & 2.87E-01 & 2.34E-01 & 6.19E-01 & 7.23E-01 \\ 6.00E-01 & 6.43E-01 & 5.85E-01 & 4.47E-01 & 5.44E-01 & 4.32E-01 & 2.15E-01 & 1.38E-01 & 2.14E-01 & 2.79E-01 & 5.36E-01 & 6.83E-01 \\ 5.96E-01 & 6.41E-01 & 5.99E-01 & 4.39E-01 & 5.50E-01 & 4.01E-01 & 2.40E-01 & 1.90E-01 & 3.06E-01 & 2.99E-01 & 4.85E-01 & 6.97E-01 \\ 6.96E-01 & 6.31E-01 & 6.31E-01 & 2.85E-01 & 6.11E-01 & 4.65E-01 & 2.04E-01 & 1.58E-01 & 3.17E-01 & 2.68E-01 & 5.79E-01 & 6.56E-01 \end{pmatrix} \end{array}$$

Standardized Decision Matrix: For standardized decision matrix, converts the various dimensional criteria into dimensionless criteria in TOPSIS. By using Equations 2-4 the normalized decision matrix V is thus calculated as

$$\begin{array}{rc} & \begin{array}{cccccccccccc} Ago_1 & Ago_2 & Ago_3 & Ago_4 & Ago_5 & Ago_6 & Ago_7 & Ago_8 & Ago_9 & Ago_{10} & Ago_{11} & Ago_{12} \end{array} \\ V = \begin{array}{r} Best \\ Worst \\ Median \\ Mean \\ SD \end{array} & \begin{pmatrix} 3.32E-01 & 3.62E-01 & 3.28E-01 & 3.22E-01 & 3.24E-01 & 2.75E-01 & 1.65E-01 & 1.13E-01 & 1.80E-01 & 2.28E-01 & 3.48E-01 & 3.48E-01 \\ 3.89E-01 & 3.61E-01 & 3.55E-01 & 1.89E-01 & 3.60E-01 & 2.57E-01 & 9.03E-02 & 8.55E-02 & 1.65E-01 & 1.34E-01 & 3.56E-01 & 4.16E-01 \\ 3.62E-01 & 3.88E-01 & 3.53E-01 & 2.70E-01 & 3.29E-01 & 2.61E-01 & 1.30E-01 & 8.30E-02 & 1.29E-01 & 1.69E-01 & 3.24E-01 & 4.13E-01 \\ 3.57E-01 & 3.84E-01 & 3.59E-01 & 2.63E-01 & 3.30E-01 & 2.41E-01 & 1.44E-01 & 1.14E-01 & 1.84E-01 & 1.80E-01 & 2.91E-01 & 4.18E-01 \\ 4.05E-01 & 3.67E-01 & 3.67E-01 & 1.66E-01 & 3.56E-01 & 2.71E-01 & 1.19E-01 & 9.21E-02 & 1.84E-01 & 1.56E-01 & 3.37E-01 & 3.81E-01 \end{pmatrix} \end{array}$$

Construct the Weighted Normalized Decision Matrix: Different weight has been considered for each criterion; the weighted normalized decision matrix can be calculated: evaluated weights × values of normalized decision matrix. Mostly in optimization problems, the main concentration is on optimal value and secondly on mean value that's why the weights are assigned as follows: *Best= 0.5, Worst=0.1, Median=0.1, Mean=0.2, SD=0.1*. The weighted normalized decision matrix can be calculated by using Equation 5. Radar images of best, worst, median mean and standard deviations of weighted normalized decision matrix are represented in figure 3.

$$\begin{array}{rc} & \begin{array}{cccccccccccc} Ago_1 & Ago_2 & Ago_3 & Ago_4 & Ago_5 & Ago_6 & Ago_7 & Ago_8 & Ago_9 & Ago_{10} & Ago_{11} & Ago_{12} \end{array} \\ W = \begin{array}{r} Best \\ Worst \\ Median \\ Mean \\ SD \end{array} & \begin{pmatrix} 1.66E-01 & 1.81E-01 & 1.64E-01 & 1.61E-01 & 1.62E-01 & 1.37E-01 & 8.27E-02 & 5.66E-02 & 9.01E-02 & 1.14E-01 & 1.74E-01 & 1.74E-01 \\ 3.89E-02 & 3.61E-02 & 3.55E-02 & 1.89E-02 & 3.60E-02 & 2.57E-02 & 9.03E-03 & 8.55E-03 & 1.65E-02 & 1.34E-02 & 3.56E-02 & 4.16E-02 \\ 3.62E-02 & 3.88E-02 & 3.53E-02 & 2.70E-02 & 3.29E-02 & 2.61E-02 & 1.30E-02 & 8.30E-03 & 1.29E-02 & 1.69E-02 & 3.24E-02 & 4.13E-02 \\ 7.15E-02 & 7.69E-02 & 7.18E-02 & 5.26E-02 & 6.60E-02 & 4.81E-02 & 2.88E-02 & 2.28E-02 & 3.67E-02 & 3.59E-02 & 5.82E-02 & 8.37E-02 \\ 4.05E-02 & 3.67E-02 & 3.67E-02 & 1.66E-02 & 3.56E-02 & 2.71E-02 & 1.19E-02 & 9.21E-03 & 1.84E-02 & 1.56E-02 & 3.37E-02 & 3.81E-02 \end{pmatrix} \end{array}$$

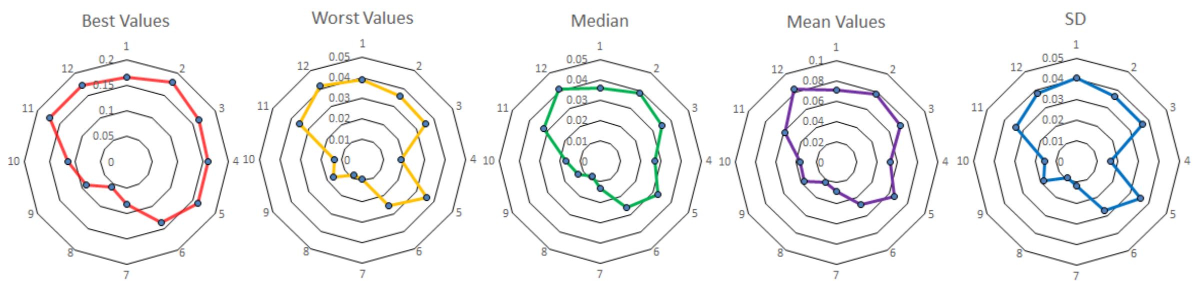

FIGURE 3
Radar images of best, median, mean, worst and standard deviation of weighted normalized decision matrix

Determine the Positive and Negative Ideals: In the weighted normalized decision matrix, the positive ideal reference point R^+ is calculated for best scores, and the negative ideal reference point R^- is calculated for worst scores. Equations 6 – 7 have been used to find the value of R^+ *and* R^-

Positive ideal solution = {0.181131178, 0.041622163, 0.041268749, 0.083653173, 0.040482367}
Negative ideal solution = {0.056620929, 0.008550262, 0.008304078, 0.022832044, 0.009211557}

Calculate the Separation Measure: To measure the separation distances of each alternative from the PIS and NIS Euclidean distance method is applied. Thus, for the distance from the PIS (D_i^+) Equation 8 has been used. Similarly, for the distance from the NIS (D_i^-) Equation 9 has been used, detailed results are in Table 8.

TABLE 8
Detailed results for calculate the separation measure

Criteria	*Algo1*	*Algo2*	*Algo3*	*Algo4*	*Algo5*	*Algo6*	*Algo7*	*Algo8*	*Algo9*	*Algo10*	*Algo11*	*Algo12*
(D_i^+)	0.02028976	0.009842	0.022876	0.05154028	0.028189	0.062038	0.124001	0.149534	0.111363	0.093848	0.029379	0.007432
D_i^-	0.13287584	0.146494	0.129291	0.11191083	0.125065	0.092135	0.029645	0.009212	0.041723	0.061587	0.132103	0.145364

Compute the Relative Closeness to the Ideal Solution: After calculating all alternatives, rank them and select the most feasible alternative. Using equation 10 to calculate closeness coefficient of each alternative, detail results are present in Table 9.

TABLE 9
Detailed results for calculate relative closeness to the ideal solution

Criteria	*Algo1*	*Algo2*	*Algo3*	*Algo4*	*Algo5*	*Algo6*	*Algo7*	*Algo8*	*Algo9*	*Algo10*	*Algo11*	*Algo12*
C_i	0.867531	0.937047367	0.849666	0.684675	0.816065	0.597607	0.192944	0.058027	0.272547	0.396224	0.818066	0.951363

Rank the Preference Order: The best alternative has been chosen on the base of shortest distance of the ideal solution, preference rank order of C_i, detail results are present in Table 10. Similarly, 30, 50 and 100 dimensions have been calculated and results have been represented in Table 11, 12 and 13 respectively.

TABLE 10
Final ranking of the algorithms for 10 dimensions

Criteria	***Algo1***	***Algo2***	***Algo3***	***Algo4***	***Algo5***	***Algo6***	***Algo7***	***Algo8***	***Algo9***	***Algo10***	***Algo11***	***Algo12***
Ranks	3	2	4	7	6	8	11	12	10	9	5	1

TABLE 11
Detailed *results* of algorithms for 30 dimensions

Criteria	**(D_i^+)**	**D_i^-**	**C_i**	**Ranks**
Algorithm 1	0.030379	0.151449	0.832925	3
Algorithm 2	0.035962	0.142963	0.79901	4
Algorithm 3	0.068991	0.10928	0.612999	8
Algorithm 4	0.055928	0.123622	0.688511	7
Algorithm 5	0.010021	0.168471	0.943855	2
Algorithm 6	0.053354	0.125041	0.700923	6
Algorithm 7	0.172714	0.00729	0.040497	12
Algorithm 8	0.174276	0.011015	0.059445	11
Algorithm 9	0.170677	0.015868	0.085064	10
Algorithm 10	0.153632	0.025528	0.142486	9
Algorithm 11	0.043212	0.139227	0.763142	5
Algorithm 12	0.003903	0.17723	0.978451	1

TABLE 12
Detailed results of algorithms for 50 dimensions

Criteria	**(D_i^+)**	**D_i^-**	**C_i**	**Ranks**
Algorithm 1	0.032486	0.167711	0.83773	4
Algorithm 2	0.0552	0.144563	0.723672	6
Algorithm 3	0.099696	0.099765	0.500173	8
Algorithm 4	0.063767	0.136308	0.681284	7
Algorithm 5	0	0.199189	1	1
Algorithm 6	0.030276	0.169157	0.848191	3
Algorithm 7	0.180436	0.019543	0.097727	10
Algorithm 8	0.190078	0.011068	0.055027	12
Algorithm 9	0.195451	0.016726	0.07883	11
Algorithm 10	0.165729	0.034719	0.173205	9
Algorithm 11	0.050416	0.150589	0.74918	5
Algorithm 12	0.026235	0.173464	0.86863	2

TABLE 13
Detailed results of algorithms for 100 dimensions

Criteria	**(D_i^+)**	**D_i^-**	**C_i**	**Ranks**
Algorithm 1	0.038796	0.163921	0.808621	5
Algorithm 2	0.046873	0.154883	0.767675	6
Algorithm 3	0.104256	0.09791	0.484305	8
Algorithm 4	0.048874	0.155451	0.760803	7
Algorithm 5	0	0.201653	1	1
Algorithm 6	0.011259	0.190539	0.944206	2
Algorithm 7	0.182068	0.020157	0.099675	11
Algorithm 8	0.200511	0.005803	0.028126	12
Algorithm 9	0.187275	0.023114	0.109864	10
Algorithm 10	0.147793	0.054053	0.267794	9
Algorithm 11	0.033709	0.169203	0.833874	4
Algorithm 12	0.028161	0.178873	0.863978	3

TABLE 14
Final ranks of algorithms for 10, 30, 50 and 100 dimensions

	10 D	30 D	50 D	100 D	Score	Rank
jSO	3	3	4	5	15	3
MM_OED	2	4	6	6	18	4
IDEbestNsize	4	8	8	8	28	7
RB-IPOP-CMA-ES	7	7	7	7	28	7
LSHADE_SPACMA	6	2	1	1	10	2
DES	8	6	3	2	19	5
DYYPO	11	12	10	11	44	11
TLBO-FL	12	11	12	12	47	12
PPSO	10	10	11	10	41	10
MOS-SOCO2011/13	9	9	9	9	36	9
LSHADE-cnEpSin	5	5	5	4	19	5
EBOwithCMAR	1	1	2	3	7	1

Algorithms DYYPO, TLBO-FL, PPSO, MOS-SOCO2011/13 having the maximum vales in various dimensions while algorithms EBOwithCMAR, LSHADE_SPACMA, jSO, MM_OED having the minimum value, remaining algorithms having the average values.

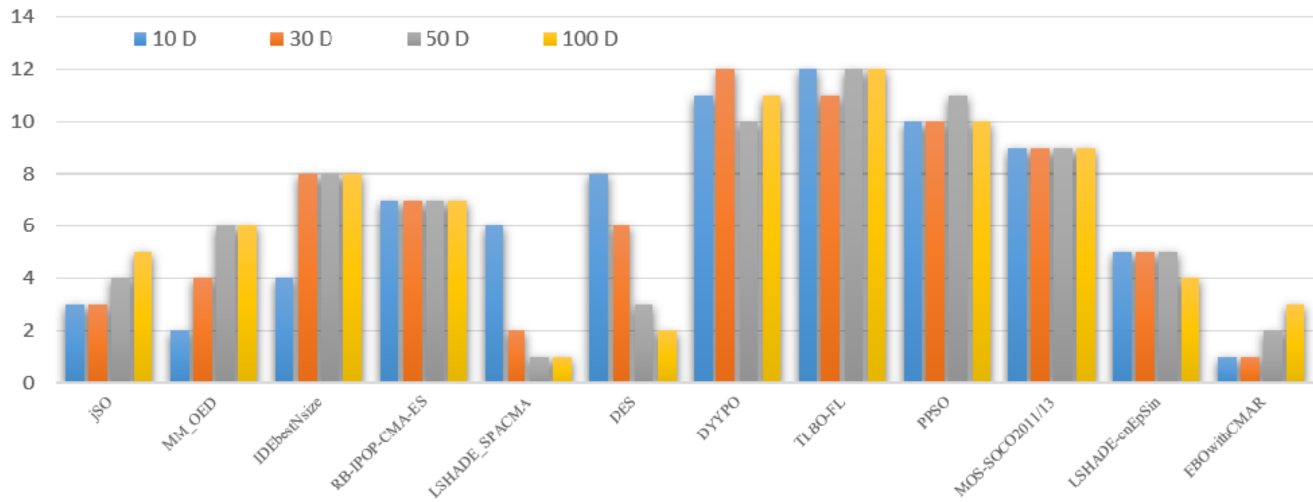

FIGURE 4
Rank comparison of all the algorithms for 10, 30, 50, 100 dimensions

COMPARISON OF RB-TOPSIS AND CEC 2017

In CEC'2017 the evaluation method for each algorithm is based on a score of 100 which is based on two criteria one 50% summation of error values for all dimensions and other 50% rank based for each problem in each dimension. The final score has to be found by adding the above two criteria as follows:

$$Score=Score1+Score2$$

The cumulative scores of the algorithms are presented in table 15 with the comparison of R_B-TOPSIS.

TABLE 15
Final ranks of R_B-TOPSIS and CEC 2017

Algorithms	R_B-TOPSIS Ranks	CEC'2017 Ranks
jSO	3	2
MM_OED	4	6
IDEbestNsize	7	7
RB-IPOP-CMA-ES	7	7
LSHADE_SPACMA	2	4
DES	5	5
DYYPO	11	11
TLBO-FL	12	12
PPSO	10	10
MOS-SOCO2011/13	9	9
LSHADE-cnEpSin	5	3
EBOwithCMAR	1	1

In both the strategies the winner algorithm is EBOwithCMAR, LSHADE_SPACMA got rank 2 in R_B-TOPSIS but got the rank 4 in CEC'2017, jSO has rank 3 in R_B-TOPSIS and rank 2 in CEC'2017, MM_OED has ranks 4 in R_B-TOPSIS and has rank 6 in CEC'2017, which are different from each other because the weights of decision make. All the other algorithms having the same ranks in both R_B-TOPSIS and CEC'2017 which shows that the rank based TOPSIS is also well executed for multi-criteria decision making in optimization algorithms with less computational cost.

CONCLUSION

The current study introduces a Rank Based Technique for Multiple Criteria Decision Making, in already existing 'Technique for Order Preference by Similarity to Ideal Solution' (TOPSIS) for the selection of best optimization algorithms. Since the TOPSIS was not able to handle directly such kind of data, so to construct the decision matrix for TOPSIS, a new R_B-TOPSIS has been introduced. After data conversion; algorithms into alternatives, test functions into decision makers and measures into criteria, R_B-TOPSIS was implemented. IEEE Congress on Evolutionary Computation 2017 (CEC's 2017) competition has been taken to evaluate the performance of RB-TOPSIS. The study compares R_B-TOPSIS with the 12 algorithms selected in IEEE Congress on Evolutionary Computation 2017 (CEC 2017) competition and considers their statistics in terms of best, worst, median, mean and standard deviations. The experimental results demonstrate that R_B-TOPSIS not only overcomes the computational cost but also has better segmentation accuracy.

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Appendix

TABLE A1

Best values of thirty test functions for twelve algorithms

Algorithm	jSO	MM_OED	IDEbest Nsize	RB-IPOP-CMA-ES	LSHADE_SPACMA	DES	DYYPO	TLBO-FL	PPSO	MOS-SOCO 2011/13	LSHADE-cnEpSin	EBOwith CMAR
F_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.60E+00	3.30E+00	1.40E+00	6.18E-02	0.00E+00	0.00E+00
F_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.75E-07	0.00E+00	0.00E+00
F_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.40E-08	1.30E-11	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-02	1.50E-01	1.37E-02	3.18E-06	0.00E+00	0.00E+00
F_5	0.00E+00	0.00E+00	9.95E-01	0.00E+00	2.62E-06	0.00E+00	5.30E+00	2.00E+00	5.97E+00	3.07E+00	0.00E+00	0.00E+00
F_6	0.00E+00	0.00E+00	0.00E+00	1.87E-08	0.00E+00	0.00E+00	1.50E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_7	1.07E+01	1.04E+01	4.70E+00	6.43E-07	1.04E+01	1.07E+01	6.00E+00	2.00E+01	8.52E+00	1.31E+01	1.06E+01	1.04E+01
F_8	0.00E+00	0.00E+00	9.95E-01	0.00E+00	0.00E+00	0.00E+00	3.00E+00	2.20E+00	3.98E+00	9.95E-01	1.70E-03	0.00E+00
F_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.90E-10	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{10}	1.87E-01	2.50E-01	3.12E-01	3.75E-01	1.95E-01	6.25E-02	6.90E+00	3.30E+02	1.22E+02	6.95E+00	3.71E-01	1.25E-01
F_{11}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.30E+00	3.60E-01	1.99E+00	9.95E-01	0.00E+00	0.00E+00
F_{12}	0.00E+00	0.00E+00	0.00E+00	2.08E-01	0.00E+00	1.14E+01	3.60E+02	7.40E+03	4.54E+02	2.68E+02	2.08E-01	0.00E+00
F_{13}	0.00E+00	6.82E-05	1.17E-04	0.00E+00	0.00E+00	0.00E+00	3.70E+01	2.00E+02	1.34E+01	2.13E+01	0.00E+00	0.00E+00
F_{14}	0.00E+00	0.00E+00	0.00E+00	1.18E-07	0.00E+00	0.00E+00	2.20E+00	3.50E+01	5.49E+00	2.51E+00	0.00E+00	0.00E+00
F_{15}	4.85E-05	2.00E-05	4.52E-05	6.51E-04	3.19E-03	7.13E-03	1.80E+00	5.20E+01	4.00E+00	8.03E-01	7.03E-06	3.81E-05
F_{16}	2.48E-02	2.11E-02	2.08E-02	6.55E-01	1.96E-01	2.38E-01	2.40E-01	1.40E+00	1.19E-01	2.37E-01	3.92E-03	2.62E-02
F_{17}	1.98E-02	0.00E+00	5.40E-02	6.69E+00	1.97E-02	1.04E+00	9.90E-02	2.50E+01	3.32E+00	6.33E-04	2.66E-03	1.00E-02
F_{18}	2.64E-06	1.11E-04	3.25E-05	2.64E-02	1.16E-02	4.97E-01	2.70E+01	3.80E+02	4.93E+01	3.24E+01	2.22E-04	3.92E-04
F_{19}	0.00E+00	0.00E+00	0.00E+00	1.94E-02	0.00E+00	1.41E-01	8.80E-02	1.50E+01	2.99E+00	3.79E-02	0.00E+00	0.00E+00
F_{20}	0.00E+00	0.00E+00	0.00E+00	1.62E+00	0.00E+00	3.12E-01	1.50E-03	1.00E+00	2.30E+00	0.00E+00	0.00E+00	0.00E+00
F_{21}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.74E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	2.11E+04
F_{22}	1.00E+02	1.00E+02	0.00E+00	6.03E+01	1.00E+02	1.00E+02	1.10E+01	1.20E+01	1.28E+01	1.13E+01	1.00E+02	2.17E+01
F_{23}	3.00E+02	1.00E+02	3.00E+02	1.00E+02	3.00E+02	3.00E+02	1.50E-04	3.00E+02	3.14E+02	3.06E+02	3.00E+02	3.00E+02
F_{24}	1.00E+02	1.00E+02	1.00E+02	1.65E-06	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E-06	1.00E+02	1.00E+02	1.00E+02
F_{25}	3.98E+02	3.98E+02	3.98E+02	1.00E+02	3.98E+02	3.98E+02	4.00E+02	4.00E+02	3.98E+02	3.98E+02	3.98E+02	3.98E+02
F_{26}	3.00E+02	2.00E+02	3.00E+02	1.14E-06	3.00E+02	2.00E+02	2.00E+02	0.00E+00	0.00E+00	0.00E+00	3.00E+02	2.00E+02
F_{27}	3.89E+02	3.89E+02	3.87E+02	3.94E+02	3.73E+02	3.94E+02	3.90E+02	3.90E+02	4.03E+02	3.89E+02	3.84E+02	3.90E+02
F_{28}	3.00E+02	3.00E+02	3.00E+02	3.04E-07	3.00E+02	3.00E+02	1.30E-03	3.00E+02	0.00E+00	1.84E+02	3.00E+02	0.00E+00
F_{29}	2.29E+02	2.30E+02	2.30E+02	2.26E+02	2.27E+02	2.27E+02	2.30E+02	2.50E+02	2.47E+02	2.00E+02	2.26E+02	2.27E+02
F_{30}	3.95E+02	3.95E+02	3.98E+02	3.95E+02	2.02E+02	3.95E+02	1.20E+03	1.10E+03	9.86E+02	6.11E+02	3.40E+02	3.95E+02

TABLE A2
Worst values of thirty test functions for twelve algorithms

Algorithm	jSO	MM_OED	IDEbest Nsize	RB-IPOP-CMA-ES	LSHADE_SPACMA	DES	DYYPO	TLBO-FL	PPSO	MOS-SOCO2011/13	LSHADE-cnEpSin	EBOwith CMAR
F_{1}	0.00E+00	0.00E+00	0.00E+00	1.49E-08	0.00E+00	0.00E+00	1.30E+04	9.80E+03	6.67E+02	4.43E+03	0.00E+00	0.00E+00
F_{2}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00	0.00E+00	1.26E-02	0.00E+00	0.00E+00
F_{3}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.90E-04	5.60E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{4}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.00E+01	4.70E+00	2.87E+00	2.02E-02	0.00E+00	0.00E+00
F_{5}	2.98E+00	2.99E+00	4.97E+00	9.95E+00	2.99E+00	3.98E+00	3.00E+01	2.10E+01	2.49E+01	1.35E+01	2.99E+00	0.00E+00
F_{6}	0.00E+00	0.00E+00	0.00E+00	3.98E-06	0.00E+00	1.58E+00	2.40E-04	2.90E-06	8.38E-01	0.00E+00	0.00E+00	0.00E+00
F_{7}	1.35E+01	1.31E+01	1.61E+01	1.22E+01	1.17E+01	1.38E+01	3.70E+01	4.00E+01	1.97E+01	2.52E+01	1.29E+01	1.10E+01
F_{8}	2.98E+00	2.99E+00	4.97E+00	1.19E+01	2.98E+00	2.99E+00	2.30E+01	2.10E+01	1.29E+01	1.49E+01	2.99E+00	0.00E+00
F_{9}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.40E-01	4.50E-01	0.00E+00	5.97E-03	0.00E+00	0.00E+00
F_{10}	2.44E+02	1.42E+02	4.59E+02	1.03E+03	1.25E+02	1.19E+02	9.10E+02	1.40E+03	6.95E+02	6.61E+02	1.55E+02	2.17E+02
F_{11}	0.00E+00	0.00E+00	2.24E-07	1.99E+00	9.95E-01	9.95E-01	1.90E+01	7.70E+00	2.59E+01	6.01E+00	0.00E+00	0.00E+00
F_{12}	1.20E+02	2.64E+02	1.70E+01	4.61E+02	3.37E+02	8.97E+02	4.50E+04	3.00E+05	9.38E+03	4.48E+04	2.38E+02	2.37E+02
F_{13}	5.95E+00	9.63E+00	5.84E+00	1.63E+01	5.95E+00	1.32E+01	2.30E+04	1.00E+04	3.96E+03	1.75E+04	8.32E+00	7.95E+00
F_{14}	9.95E-01	1.03E+00	2.29E-04	5.88E+01	9.95E-01	2.70E+01	1.10E+02	1.30E+02	5.09E+01	4.68E+03	9.95E-01	9.95E-01
F_{15}	5.00E-01	5.00E-01	1.02E-01	2.38E+00	1.49E+00	1.79E+01	7.50E+02	2.50E+02	9.20E+01	6.69E+02	5.00E-01	5.00E-01
F_{16}	1.14E+00	8.80E-01	9.64E-01	3.59E+02	1.12E+00	1.21E+02	1.40E+02	1.20E+02	2.37E+02	1.36E+02	1.08E+00	9.35E-01
F_{17}	1.45E+00	3.76E-01	2.35E+00	1.64E+02	2.10E+01	4.48E+01	6.00E+01	5.30E+01	3.73E+01	3.45E+01	2.63E+00	1.01E+00
F_{18}	5.00E-01	2.00E+01	3.01E-01	1.44E+02	2.05E+01	1.67E+02	2.40E+04	2.60E+04	2.36E+03	1.45E+04	2.05E+01	2.00E+01
F_{19}	3.92E-02	1.94E-02	4.25E-02	2.36E+01	1.09E+00	1.55E+01	1.90E+03	1.50E+02	6.54E+01	5.50E+01	1.50E+00	1.22E-01
F_{20}	6.24E-01	6.24E-01	3.12E-01	2.64E+02	6.24E-01	2.50E+01	3.70E+01	2.60E+01	4.24E+01	3.12E-01	6.24E-01	3.12E-01
F_{21}	2.04E+02	2.03E+02	2.07E+02	2.09E+02	2.05E+02	2.07E+02	1.00E+02	2.20E+02	2.11E+02	2.22E+02	2.04E+02	2.02E+02
F_{22}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.10E+02	1.00E+02	1.02E+02	1.02E+02	1.00E+02	1.00E+02
F_{23}	3.06E+02	3.08E+02	3.08E+02	3.09E+02	3.04E+02	3.08E+02	3.30E+02	3.20E+02	3.55E+02	3.19E+02	3.05E+02	3.03E+02
F_{24}	3.31E+02	2.01E+02	3.33E+02	3.45E+02	3.33E+02	3.29E+02	3.50E+02	3.40E+02	3.79E+02	3.54E+02	3.32E+02	3.30E+02
F_{25}	4.43E+02	4.43E+02	4.46E+02	4.43E+02	4.46E+02	4.43E+02	4.50E+02	4.50E+02	4.43E+02	4.45E+02	4.43E+02	4.43E+02
F_{26}	3.00E+02	3.00E+02	3.00E+02	1.18E+03	3.00E+02	3.00E+02	4.40E+02	3.60E+02	3.00E+02	4.12E+02	3.00E+02	3.00E+02
F_{27}	3.90E+02	3.90E+02	3.94E+02	3.99E+02	3.95E+02	4.03E+02	4.10E+02	4.00E+02	4.50E+02	4.04E+02	3.95E+02	3.95E+02
F_{28}	6.12E+02	6.47E+02	6.12E+02	6.12E+02	4.73E+02	6.47E+02	4.20E+02	9.30E+02	3.00E+02	6.12E+02	6.11E+02	5.84E+02
F_{29}	2.42E+02	2.48E+02	2.49E+02	4.82E+02	2.36E+02	2.59E+02	3.30E+02	3.00E+02	2.97E+02	3.32E+02	2.33E+02	2.45E+02
F_{30}	3.95E+02	1.25E+06	4.18E+02	7.47E+04	3.37E+02	1.25E+06	3.70E+04	1.30E+06	4.88E+03	8.37E+05	4.65E+05	4.43E+02

TABLE A3.
Median values of thirty test functions for twelve algorithms

Algorithm	jSO	MM_OED	IDEbest Nsize	RB-IPOP-CMA-ES	LSHADE_SPACMA	DES	DYYPO	TLBO-FL	PPSO	MOS-SOCO2011/13	LSHADE-cnEpSin	EBOwith CMAR
F_{1}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.80E+03	2.00E+03	2.03E+02	3.55E+02	0.00E+00	0.00E+00
F_{2}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.00E-02	0.00E+00	5.99E-03	0.00E+00	0.00E+00
F_{3}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.60E-07	1.10E-04	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{4}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.20E-01	3.00E+00	1.03E+00	8.17E-06	0.00E+00	0.00E+00
F_{5}	1.99E+00	9.95E-01	2.98E+00	9.95E-01	1.00E+00	9.95E-01	1.10E+01	8.80E+00	1.89E+01	6.87E+00	1.99E+00	0.00E+00
F_{6}	0.00E+00	0.00E+00	0.00E+00	7.91E-08	0.00E+00	1.52E-07	3.20E-05	8.40E-08	4.00E-06	0.00E+00	0.00E+00	0.00E+00
F_{7}	1.18E+01	1.15E+01	1.30E+01	1.08E+01	1.09E+01	1.18E+01	2.10E+01	2.80E+01	1.71E+01	1.89E+01	1.20E+01	1.05E+01
F_{8}	1.99E+00	9.95E-01	2.98E+00	9.95E-01	9.95E-01	1.99E+00	1.30E+01	1.20E+01	1.09E+01	7.30E+00	1.99E+00	0.00E+00
F_{9}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.40E-08	8.90E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_{10}	1.03E+01	6.83E+00	1.43E+02	4.07E+02	3.75E+00	4.37E-01	3.60E+02	9.50E+02	5.41E+02	3.54E+02	1.51E+01	1.36E+01
F_{11}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.93E-09	8.70E+00	4.10E+00	1.80E+01	1.99E+00	0.00E+00	0.00E+00
F_{12}	4.16E-01	1.19E+02	4.16E-01	1.20E+02	1.20E+02	4.15E+02	9.30E+03	6.60E+04	4.83E+03	8.51E+03	1.19E+02	1.18E+02
F_{13}	4.84E+00	5.05E+00	7.92E-02	4.84E+00	4.84E+00	2.99E+00	3.00E+03	2.40E+03	8.79E+02	1.03E+03	4.84E+00	3.13E-02
F_{14}	0.00E+00	0.00E+00	0.00E+00	2.00E+01	0.00E+00	2.00E+01	1.30E+01	6.70E+01	4.01E+01	2.61E+01	0.00E+00	0.00E+00
F_{15}	1.79E-01	2.13E-02	1.53E-03	5.00E-01	4.85E-01	1.49E+00	1.00E+01	1.30E+02	5.67E+01	2.84E+01	4.91E-01	3.07E-02
F_{16}	5.19E-01	2.16E-01	4.94E-01	1.19E+02	6.99E-01	1.49E+00	4.10E+00	8.90E+00	1.19E+02	3.91E+00	4.98E-01	4.43E-01
F_{17}	4.03E-01	1.97E-02	7.46E-01	4.18E+01	3.32E-01	2.22E+01	6.80E+00	3.80E+01	2.53E+01	1.27E+00	3.23E-01	4.94E-02
F_{18}	3.79E-01	9.00E-02	2.45E-03	2.05E+01	4.68E-01	2.37E+01	7.40E+03	6.20E+03	7.99E+02	7.19E+02	4.62E-01	4.09E-01
F_{19}	0.00E+00	0.00E+00	5.05E-04	1.21E+00	2.00E-02	2.46E+00	2.60E+00	6.10E+01	1.72E+01	3.63E+00	1.97E-02	1.79E-02
F_{20}	3.12E-01	0.00E+00	0.00E+00	1.38E+02	3.12E-01	2.00E+01	4.30E+00	1.50E+01	2.56E+01	0.00E+00	3.12E-01	0.00E+00
F_{21}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.38E+02	2.03E+02	1.00E+02	1.40E+02	1.00E+02	2.09E+02	1.00E+02	1.00E+02
F_{22}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02	9.30E+01	1.01E+02	1.00E+02	1.00E+02	1.00E+02
F_{23}	3.00E+02	3.03E+02	3.03E+02	3.03E+02	3.03E+02	3.00E+02	3.20E+02	3.10E+02	3.44E+02	3.11E+02	3.03E+02	3.00E+02
F_{24}	3.29E+02	1.00E+02	3.27E+02	2.00E+02	3.30E+02	3.27E+02	1.00E+02	3.10E+02	1.00E+02	3.39E+02	3.30E+02	1.00E+02
F_{25}	3.98E+02	3.98E+02	3.98E+02	3.98E+02	4.43E+02	3.98E+02	4.40E+02	4.30E+02	4.00E+02	4.02E+02	4.43E+02	3.98E+02
F_{26}	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
F_{27}	3.90E+02	3.90E+02	3.94E+02	3.95E+02	3.88E+02	3.95E+02	4.00E+02	3.90E+02	4.25E+02	3.92E+02	3.89E+02	3.90E+02
F_{28}	3.00E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02	5.84E+02	3.00E+02	4.50E+02	3.00E+02	3.00E+02	3.00E+02	3.00E+02
F_{29}	2.33E+02	2.34E+02	2.36E+02	2.54E+02	2.30E+02	2.33E+02	2.60E+02	2.70E+02	2.79E+02	2.68E+02	2.28E+02	2.30E+02
F_{30}	3.95E+02	3.95E+02	4.03E+02	4.43E+02	2.17E+02	4.64E+02	4.40E+03	2.80E+05	2.75E+03	3.99E+03	4.07E+02	3.95E+02

TABLE A4

Mean values of thirty test functions for twelve algorithms

Algorithm	jSO	MM_OED	IDEbest Nsize	RB-IPOP-CMA-ES	LSHADE_SPACMA	DES	DYYPO	TLBO-FL	PPSO	MOS-SOCO2011/13	LSHADE-cnEpSin	EBOwith CMAR
F_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.90E+03	1.30E+03	2.39E+02	6.91E+02	0.00E+00	0.00E+00
F_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.25E-03	0.00E+00	0.00E+00
F_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.20E-05	1.50E-08	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.10E+00	3.30E+00	1.20E+00	6.26E-04	0.00E+00	0.00E+00
F_5	1.76E+00	1.11E+00	3.28E+00	1.58E+00	1.35E+00	1.54E+00	1.10E+01	7.20E+00	1.81E+01	6.95E+00	1.69E+00	0.00E+00
F_6	0.00E+00	0.00E+00	0.00E+00	2.01E-07	0.00E+00	1.17E-01	6.40E-05	0.00E+00	2.26E-01	0.00E+00	0.00E+00	0.00E+00
F_7	1.18E+01	1.15E+01	1.29E+01	1.01E+01	1.10E+01	1.19E+01	2.20E+01	2.70E+01	1.69E+01	1.90E+01	1.20E+01	1.06E+01
F_8	1.95E+00	1.11E+00	2.91E+00	1.97E+00	1.04E+00	1.56E+00	1.30E+01	1.20E+01	9.95E+00	6.97E+00	1.80E+00	0.00E+00
F_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.00E-02	0.00E+00	0.00E+00	2.39E-04	0.00E+00	0.00E+00
F_{10}	3.59E+01	1.79E+01	1.86E+02	4.35E+02	1.05E+01	5.66E+00	3.70E+02	9.90E+02	5.03E+02	3.60E+02	4.30E+01	3.72E+01
F_{11}	0.00E+00	0.00E+00	0.00E+00	1.71E-01	1.95E-02	1.17E-01	9.30E+00	4.30E+00	1.69E+01	2.54E+00	0.00E+00	0.00E+00
F_{12}	2.66E+00	1.02E+02	2.44E+00	1.10E+02	1.14E+02	4.41E+02	1.30E+04	5.90E+04	4.55E+03	1.10E+04	1.01E+02	9.02E+01
F_{13}	2.96E+00	4.19E+00	8.37E-01	4.17E+00	3.37E+00	3.31E+00	5.10E+03	1.80E+03	1.39E+03	3.29E+03	3.66E+00	2.17E+00
F_{14}	5.85E-02	8.80E-02	1.24E-05	1.59E+01	5.31E-02	1.23E+01	2.10E+01	6.50E+01	3.73E+01	1.58E+02	7.80E-02	6.05E-02
F_{15}	2.21E-01	6.71E-02	7.60E-03	4.91E-01	4.11E-01	3.25E+00	4.40E+01	1.20E+02	5.33E+01	8.95E+01	3.24E-01	1.09E-01
F_{16}	5.69E-01	2.53E-01	4.91E-01	9.71E+01	6.42E-01	6.09E+00	4.40E+01	3.80E+00	8.30E+01	3.45E+01	5.37E-01	4.17E-01
F_{17}	5.02E-01	5.64E-02	7.89E-01	5.25E+01	1.54E+00	2.10E+01	1.40E+01	3.80E+01	2.46E+01	2.89E+00	3.07E-01	1.47E-01
F_{18}	3.08E-01	9.69E-01	4.84E-02	1.97E+01	1.97E+00	2.91E+01	8.80E+03	4.30E+03	8.78E+02	1.40E+03	3.86E+00	7.00E-01
F_{19}	1.07E-02	3.80E-03	1.06E-02	1.82E+00	8.13E-02	2.52E+00	9.30E+01	5.30E+01	2.25E+01	9.45E+00	4.47E-02	1.50E-02
F_{20}	3.43E-01	6.73E-02	1.84E-02	1.06E+02	1.84E-01	1.22E+01	8.00E+00	2.10E+01	2.78E+01	2.76E-02	2.57E-01	1.47E-01
F_{21}	1.32E+02	1.04E+02	1.49E+02	1.37E+02	1.54E+02	2.02E+02	1.00E+02	1.10E+02	1.04E+02	1.73E+02	1.46E+02	1.14E+02
F_{22}	1.00E+02	1.00E+02	9.45E+01	9.93E+01	1.00E+02	1.00E+02	9.70E+01	1.00E+02	9.67E+01	8.79E+01	1.00E+02	9.85E+01
F_{23}	3.01E+02	2.98E+02	3.02E+02	2.75E+02	3.02E+02	3.02E+02	3.10E+02	3.10E+02	3.42E+02	3.12E+02	3.02E+02	3.00E+02
F_{24}	2.97E+02	1.04E+02	2.93E+02	1.98E+02	2.92E+02	3.05E+02	1.20E+02	3.30E+02	2.27E+02	2.76E+02	3.16E+02	1.66E+02
F_{25}	4.06E+02	4.14E+02	4.14E+02	4.02E+02	4.26E+02	4.08E+02	4.20E+02	4.40E+02	4.04E+02	4.15E+02	4.26E+02	4.12E+02
F_{26}	3.00E+02	2.94E+02	3.00E+02	2.73E+02	3.00E+02	2.96E+02	3.00E+02	3.00E+02	2.67E+02	2.55E+02	3.00E+02	2.65E+02
F_{27}	3.89E+02	3.90E+02	3.93E+02	3.95E+02	3.88E+02	3.96E+02	4.00E+02	3.90E+02	4.27E+02	3.93E+02	3.90E+02	3.92E+02
F_{28}	3.39E+02	3.37E+02	3.23E+02	4.02E+02	3.63E+02	5.26E+02	3.00E+02	3.70E+02	2.94E+02	3.69E+02	3.85E+02	3.07E+02
F_{29}	2.34E+02	2.36E+02	2.37E+02	2.66E+02	2.30E+02	2.36E+02	2.60E+02	2.70E+02	2.78E+02	2.69E+02	2.28E+02	2.31E+02
F_{30}	3.95E+02	5.69E+04	4.04E+02	2.05E+03	2.30E+02	1.54E+05	6.70E+03	9.10E+03	2.99E+03	2.17E+04	1.76E+04	4.07E+02

TABLE A5

Standard deviation of thirty test functions for twelve algorithms

Algorithm	jSO	MM_OED	IDEbest Nsize	RB-IPOP-CMA-ES	LSHADE_SPACMA	DES	DYYPO	TLBO-FL	PPSO	MOS-SOCO2011/13	LSHADE-cnEpSin	EBOwith CMAR
F_1	0.00E+00	0.00E+00	0.00E+00	2.09E-09	0.00E+00	0.00E+00	3.30E+03	2.50E+03	1.99E+02	9.19E+02	0.00E+00	0.00E+00
F_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.40E-01	0.00E+00	3.97E-03	0.00E+00	0.00E+00
F_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.30E-05	7.80E-04	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F_4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.30E+00	1.20E+00	9.27E-01	3.21E-03	0.00E+00	0.00E+00
F_5	7.60E-01	7.35E-01	1.10E+00	1.96E+00	7.13E-01	9.40E-01	4.20E+00	5.60E+00	5.05E+00	2.24E+00	7.53E-01	0.00E+00
F_6	0.00E+00	0.00E+00	0.00E+00	6.21E-07	0.00E+00	3.48E-01	6.00E-05	4.40E-07	3.05E-01	0.00E+00	0.00E+00	0.00E+00
F_7	6.07E-01	6.71E-01	1.49E+00	2.69E+00	3.51E-01	7.04E-01	6.00E+00	4.00E+00	2.19E+00	3.04E+00	4.80E-01	1.75E-01
F_8	7.44E-01	9.68E-01	9.30E-01	2.32E+00	7.44E-01	1.04E+00	4.50E+00	4.40E+00	2.35E+00	2.74E+00	7.71E-01	0.00E+00
F_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.80E-02	6.40E-02	0.00E+00	9.89E-04	0.00E+00	0.00E+00
F_{10}	5.55E+01	3.64E+01	1.17E+02	1.90E+02	2.40E+01	1.71E+01	1.70E+02	2.10E+02	1.53E+02	1.50E+02	5.57E+01	5.39E+01
F_{11}	0.00E+00	0.00E+00	3.13E-08	4.10E-01	1.39E-01	3.24E-01	4.80E+00	1.50E+00	5.26E+00	1.40E+00	0.00E+00	0.00E+00
F_{12}	1.68E+01	5.96E+01	4.58E+00	9.37E+01	6.57E+01	1.94E+02	1.20E+04	5.50E+04	2.49E+03	9.76E+03	7.30E+01	7.44E+01
F_{13}	2.35E+00	2.66E+00	1.63E+00	3.61E+00	2.42E+00	3.00E+00	5.60E+03	2.20E+03	1.32E+03	4.97E+03	2.66E+00	2.53E+00
F_{14}	2.36E-01	2.74E-01	4.53E-05	1.25E+01	1.95E-01	1.02E+01	2.20E+01	1.80E+01	1.16E+01	6.55E+02	2.70E-01	2.36E-01
F_{15}	2.00E-01	1.19E-01	1.90E-02	4.76E-01	2.66E-01	4.10E+00	1.10E+02	4.30E+01	2.25E+01	1.36E+02	2.16E-01	1.74E-01
F_{16}	2.64E-01	2.01E-01	1.88E-01	1.03E+02	2.30E-01	2.34E+01	5.80E+01	2.20E+01	7.18E+01	5.01E+01	2.93E-01	1.98E-01
F_{17}	3.48E-01	1.13E-01	5.18E-01	3.36E+01	4.44E+00	1.03E+01	1.40E+01	7.80E+00	7.35E+00	5.71E+00	3.81E-01	2.03E-01
F_{18}	1.95E-01	3.89E+00	7.77E-02	2.35E+01	5.44E+00	2.46E+01	6.40E+03	5.60E+03	7.07E+02	2.25E+03	7.63E+00	2.77E+00
F_{19}	1.25E-02	7.49E-03	1.25E-02	3.30E+00	2.08E-01	2.34E+00	2.90E+02	3.20E+01	1.54E+01	1.31E+01	2.09E-01	1.88E-02
F_{20}	1.29E-01	1.57E-01	7.42E-02	6.95E+01	1.67E-01	9.86E+00	9.10E+00	9.40E+00	8.87E+00	8.67E-02	2.31E-01	1.57E-01
F_{21}	4.84E+01	1.99E+01	5.27E+01	4.92E+01	4.93E+01	4.62E+00	9.00E-01	5.20E+01	2.13E+01	5.15E+01	5.17E+01	3.52E+01
F_{22}	0.00E+00	6.88E-02	2.22E+01	5.57E+00	7.82E-02	1.42E-08	2.00E+01	2.30E+01	1.66E+01	2.61E+01	6.80E-02	1.10E+01
F_{23}	1.59E+00	2.84E+01	2.02E+00	7.06E+01	1.50E+00	1.98E+00	4.50E+01	3.80E+00	1.03E+01	3.20E+00	1.64E+00	7.07E-01
F_{24}	7.93E+01	1.97E+01	7.82E+01	1.02E+02	8.29E+01	6.12E+01	5.00E+01	6.90E+01	1.34E+02	9.19E+01	5.45E+01	9.97E+01
F_{25}	1.75E+01	2.19E+01	2.21E+01	6.54E+01	2.24E+01	1.89E+01	2.30E+01	2.20E+01	1.44E+01	2.03E+01	2.24E+01	2.12E+01
F_{26}	0.00E+00	2.38E+01	0.00E+00	1.51E+02	0.00E+00	1.96E+01	3.10E+01	4.60E+01	7.58E+01	9.88E+01	0.00E+00	4.74E+01
F_{27}	2.26E-01	1.22E-01	1.62E+00	1.09E+00	3.13E+00	2.33E+00	3.80E+00	3.30E+00	1.33E+01	2.53E+00	1.96E+00	2.40E+00
F_{28}	9.65E+01	1.02E+02	7.90E+01	1.64E+02	8.29E+01	1.23E+02	5.10E+01	1.60E+02	4.16E+01	9.22E+01	1.19E+02	7.18E+01
F_{29}	2.96E+00	4.19E+00	3.91E+00	4.53E+01	2.24E+00	7.67E+00	1.90E+01	1.40E+01	1.34E+01	2.16E+01	1.72E+00	3.77E+00
F_{30}	4.50E-02	2.34E+05	4.45E+00	1.04E+04	3.25E+01	3.69E+05	6.50E+03	4.90E+05	8.90E+02	1.17E+05	8.61E+04	1.78E+01