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ADCSS-LABELING OF SOME PLANAR GRAPHS

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Abstract. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize few planar graphs for absolute difference of cubic and square sum labeling.

Introduction

All graphs in this paper are finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p, q) graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1, 2, 3]. Some basic concepts are taken from Frank Harary [1]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph in [4]. In [4, 5, 6, 7, 8, 9, 10, 11, 12, 13], it is shown that planar grid, web graph, kayak paddle graph, snake graphs, armed crown, fan graph, friendship graph , windmill graph, cycle graphs, wheel graph, gear graph, helm graph, 2-tuple graphs, middle graphs, total graphs and shadow graphs have an adcss-labeling.

In this paper we proved that some planar graphs admit adcss-labeling.

Definition 1.1. Let $G = (V(G), E(G))$ be a graph. A graph G is said to be an absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection $f : V(G) \to \{1, 2, \ldots, p\}$ such that the induced function $f^*_{adcss} : E(G) \to$ multiples of 2 is given by $f^*_{adcss}(uv) = |f(u)^3 + f(v)^3 - (f(u)^2 + f(v)^2)|$ is injective.

Definition 1.2. A graph in which every edge is labeled with the sum of the cubes of the vertices and the sum of the squares of the vertices is called an absolute difference of cubic and square sum labeling or adcss-labeling.

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Main Results.

Definition 1.3. Let α be a sequence of 'n' symbols of S, that is $\alpha \in S^n$. We will construct a graph by tiling n- blocks side by side, with their positions indicated by α. We will denote the resulting graph by $TB(\alpha)$ and refer to it as triangular belt.

Theorem 1.4. *For $\alpha \in S^n$, $n > 1$, the triangular belt $TB(\alpha)$ admits adcss labeling.*

Proof. Let $G = TB(\alpha)$ and let $v_1, v_2, \ldots, v_{2n+2}$ are the vertices of G.
Here $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$
Define a bijection $f : V \to \{1, 2, 3, \ldots, 2n + 2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, 2n + 2.$$

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned}
f^*_{adcss}[v_{2i-1}v_{2i+1}] &= 16i^3 - 8i^2 + 12i - 2, && 1 \le i \le n\\
f^*_{adcss}[v_{2i}v_{2i+2}] &= 16i^3 + 16i^2 + 16i + 4, && 1 \le i \le n\\
f^*_{adcss}[v_{2i-1}v_{2i}] &= 16i^3 - 20i^2 + 10i - 2, && 1 \le i \le n + 1
\end{aligned}$$

Case (i) $\alpha = (\uparrow\uparrow\uparrow - - - - - - - - \uparrow)$

$$f^*_{adcss}[v_{2i}v_{2i+1}] = 16i^3 + 4i^2 + 2i, \qquad 1 \le i \le n.$$

Case (ii) $\alpha = (\downarrow\uparrow\downarrow - - - - - - - -)$

$$\begin{aligned}
f^*_{adcss}[v_{4i-3}v_{4i}] &= 128i^3 - 176i^2 + 132i - 36, && 1 \le i \le \frac{n+1}{2}, \text{ where n is odd.}\\
f^*_{adcss}[v_{4i-3}v_{4i}] &= 128i^3 - 176i^2 + 132i - 36, && 1 \le i \le \frac{n}{2}, \text{ where n is even.}\\
f^*_{adcss}[v_{4i}v_{4i+1}] &= 128i^3 + 16i^2 + 4i, && 1 \le i \le \frac{n-1}{2}, \text{ where n is odd.}\\
f^*_{adcss}[v_{4i}v_{4i+1}] &= 128i^3 + 16i^2 + 4i, && 1 \le i \le \frac{n}{2}, \text{ where n is even.}
\end{aligned}$$

All edge values of G are distinct, which are multiples of 2. The edge values of G are in the form of an increasing order. Hence $TB(\alpha)$ admits adcss-labeling. □

Definition 1.5. Let $P_n(+)N_m$ be the graph with $p = n + m$ and $q = n + 2m - 1$.
$V(P_n(+)N_m) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m\}$,
where $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $V(N_m) = \{u_1, u_2, \ldots, u_m\}$.
$E(P_n(+)N_m) = E(P_n) \cup \{v_1u_1, \ldots, v_1u_m, v_nu_1, \ldots, v_nu_m\}$.

Theorem 1.6. *The graph $P_n(+)N_m$ admits adcss labeling.*

Proof. Let $G = P_n(+)N_m$ and let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m$ are the vertices of G.

Here $|V(G)| = n + m$ and $|E(G)| = 2m + n - 1$.
Define a bijection $f : V \to \{1, 2, 3, \ldots, n + m\}$ by

$$\begin{aligned} f(v_i) &= i, & i &= 1, 2, \ldots, n. \\ f(u_i) &= n + i, & i &= 1, 2, \ldots, m. \end{aligned}$$

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned} f^*_{adcss}[v_i v_{i+1}] &= 2i^3 + i^2 + i, & i &= 1, 2, \ldots, n - 1. \\ f^*_{adcss}[v_1 u_i] &= (n + i)^2(n + i - 1), & i &= 1, 2, \ldots, m. \\ f^*_{adcss}[v_n u_i] &= (n + i)^2(n + i - 1) + n^2(n - 1), & i &= 1, 2, \ldots, m. \end{aligned}$$

All edge values of G are distinct, which are multiples of 2. The edge values of G are in the form of an increasing order. Hence $P_n(+)N_m$ admits adcss-labeling. □

Definition 1.7. For integers $m, n \geq 0$, we consider the graph $J(m, n)$ with vertex set $V(J(m, n)) = \{u, v, x, y\} \cup \{x_1, x_2, \ldots, x_m\} \cup \{y_1, y_2, \ldots, y_n\}$ and edge set $E(J(m, n)) = \{ux, uv, uy, vx, vy\} \cup \{x_i x, i = 1, 2, \ldots, m\} \cup \{y_i y, i = 1, 2, \ldots, n\}$. We will refer to $J(m, n)$ as Jelly fish graph.

Theorem 1.8. *Jelly fish graph $J(m, n)$ admits adcss-labeling.*

Proof. Let $G = J(m, n)$ and let $u, v, x, y, x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n$ are the vertices of G. Here $|V(G)| = n + m + 4$ and $|E(G)| = m + n + 5$.
Define a bijection $f : V \to \{1, 2, 3, \ldots, n + m + 4\}$ by

$$\begin{aligned} f(x_i) &= i + 4, & i &= 1, 2, \ldots, m. \\ f(yi) &= m + 4 + i, & i &= 1, 2, \ldots, n. \\ f(x) = 1, \quad f(y) &= 2, \quad f(u) = 3, \quad f(v) = 4. \end{aligned}$$

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned} f^*_{adcss}[x_i x] &= (i + 4)^2, & i &= 1, 2, \ldots, m. \\ f^*_{adcss}[y_i y] &= (m + i + 4)^2(m + i + 3) + 4, & i &= 1, 2, \ldots, n. \end{aligned}$$

$f^*_{adcss}(ux) = 18, f^*_{adcss}(uy) = 22, f^*_{adcss}(uv) = 66, f^*_{adcss}(vx) = 48, f^*_{adcss}(vy) = 52$. All edge values of G are distinct, which are multiples of 2. The edge values of G are in the form of an increasing order. Hence $J(m, n)$ admits adcss-labeling. □

Definition 1.9. Two cycles of length n sharing a common edge are called adjacent cycles and is denoted by $A(C_n)$.

Theorem 1.10. *Adjacent cycles, $A(C_n)$ admits adcss labeling.*

Proof. Let $G = A(C_n)$ and let $v_1, v_2, \ldots, v_{2n-2}$ are the vertices of G.

Here $|V(G)| = 2n - 2$ and $|E(G)| = 2n - 1$.
Define a bijection $f : V \to \{1, 2, 3, \ldots, 2n - 2\}$ by

$$f(v_i) = i, \quad i = 1, 2, \ldots, 2n - 2.$$

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned}
f^*_{adcss}[v_i v_{i+1}] &= 2i^3 + i^2 + i, \qquad i = 1, 2, \ldots, 2n - 3.\\
f^*_{adcss}[v_1 v_{2n-2}] &= (2n-2)^2(2n-3).\\
f^*_{adcss}[v_{\frac{n+1}{2}} v_{\frac{3n-1}{2}}] &= \left(\frac{3n-1}{2}\right)^2\left(\frac{3n-3}{2}\right)^2 + \left(\frac{n+1}{2}\right)^2\left(\frac{n-1}{2}\right), \text{ when } n \text{ is odd.}\\
f^*_{adcss}[v_{\frac{n}{2}} v_{\frac{3n-2}{2}}] &= \left(\frac{3n-2}{2}\right)^2\left(\frac{3n-4}{2}\right)^2 + \left(\frac{n}{2}\right)^2\left(\frac{n-1}{2}\right), \text{ when } n \text{ is even.}
\end{aligned}$$

All edge values of G are distinct, which are multiples of 2. The edge values of G are in the form of an increasing order. Hence $A(C_n)$ admits adcss-labeling. □

Theorem 1.11. *The graph* $(P_2 \cup nK_1) + N_2$*, admits adcss labeling.*

Proof. Let $G = (P_2 \cup nK_1) + N_2$ and let $v_1, v_2, \ldots, v_{n+4}$ are the vertices of G.
Here $|V(G)| = n + 4$ and $|E(G)| = 2n + 5$.
Define a bijection $f : V \to \{1, 2, 3, \ldots, n + 4\}$ by

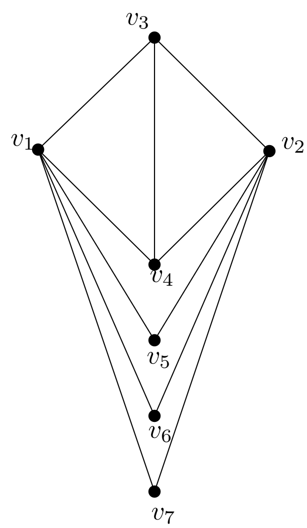
$$f(v_i) = i, \quad i = 1, 2, \ldots, n + 4.$$

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$$\begin{aligned}
f^*_{adcss}[v_1 v_{i+2}] &= i^3 + 5i^2 + 8i + 4, \qquad i = 1, 2, \ldots, n + 2.\\
f^*_{adcss}[v_1 v_{i+2}] &= i^3 + 5i^2 + 8i + 8, \qquad i = 1, 2, \ldots, n + 2.\\
f^*_{adcss}[v_3 v_4] &= 68.
\end{aligned}$$

All edge values of G are distinct, which are multiples of 2. The edge values of G are in the form of an increasing order. Hence $(P_2 \cup nK_1) + N_2$ admits adcss-labeling. □

Example 1.12. Let $G = (P_2 \cup 3K_1) + N_2$



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