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MATHEMATICAL MODELING AND ANALYSIS OF BIOLOGICAL SYSTEMS

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Abstract. This work is a short description of the development of a mathematical model for the dynamics of human elbow movement. The work is presented in a simple way that it can be extended for further models of the dynamics of any human limb. The experiments to be conducted and the processing of the results is done so as to obtain the constant coefficients of the truncated infinite series which is declared as the model representing the dynamics. The differential terms are formulated by the experimental readings and these terms are successfully utilized for the determination of the coefficients. The model is validated by comparing the model output with that of the actual experimental values. The error factors of the model derived through the validation is augmented to the model for improving the performance.

1. Introduction

Models help to study engineering problems indirectly in a simple and easy way without causing any damage, deformation, or disturbance to the existing original system and also is an excellent method to study or analyze the system characteristics where a direct study is practically impossible[19]. Basically, models also help to convert the qualitative and conceptual phenomena of nature or system into representative numerical forms either through well-defined equations, inequalities, or expressions, thereby enabling precise mathematical analysis of the system performance. The obtained analytical results are utilized for the improvement of the existing systems. The place of modeling in technology is in between the qualitative and quantitative concepts as an interfacing tool. There are different types of models available in the scientific field and prominent among them are mathematical models [4, 5]. This paper describes how mathematical models are developed and utilized for the study of biological systems, and uses the knowledge obtained for developing assistive devices for disabled persons by taking an elbow as an illustrative example [16, 20, 23, 8].

1.1. Mathematical models and assistive devices. Development of smart assistive devices for the physically handicapped for their disability compensation is nowadays gaining more and more importance on humanitarian grounds

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[1, 2, 3, 6, 7, 9, 10, 13, 14, 15, 17, 18, 12]. The present research report target towards the development process of such assistive devices to rehabilitate the disabled. The disabled subject is viewed as a biological system and is one in which a direct study is practically not advisable. Assessing the magnitude of disability or in other words measuring the disability and recording it as a numerical quantity is essential for the development of an assistive device for effective compensation. The role of mathematics, especially differential calculus is here, as an effective modeling tool, by viewing the assistive devices as systems, having continuous internal state variables[19, 4, 5, 17]

1.2. Modifying infinite series as a suitable modeling tool. Differential calculus has developed as a science in direct response to the need for modeling systems with continuously changing state variables such as position, velocity, acceleration, etc. in engineering. Accordingly, ordinary linear differential equations with constant coefficients are widely used for studying the dynamics of measurement systems [4, 5]. It is common that linear differential equations are used for defining the input-output relations of dynamic systems. In this way functions having average values may be defined as an infinite series without much error and this idea is used here for formulating the characteristics of assistive devices [16, 20, 23, 8, 11, 22, 21]. Not only that the philosophy is used for defining biological systems like human limb movement characteristics, elbow movement, human locomotion characteristics, etc [16, 20, 23, 8]. also as a continuation. The so-developed differential equations eventually become models, called the average value-based models named by the authors, which are used for the performance evaluation of the developed assistive devices i.e. the smart artificial human limb movement, human knee movement etc.[16, 20, 23, 8]. The algorithm and different steps involved in such modeling are illustrated below. The validation of this type of model in a generic way is already done in [16, 20, 23, 8]. Here the example selected is the elbow dynamics of a hand, hence an exclusive validation for the selected case study is not given exclusively. The entire exercise involves (i) experimental observation of the operational characteristic of the human elbow limb by optical methods,(ii) analyzing the monitored result by advanced software, (iii) finding out the expression for different terms of Taylor's series, (iv) logically formulating appropriate differential equations and (v) solving them for finding different constant coefficients, etc.

2. Development of Average value-based models

The need and importance of models, their utility in the process of smart artificial limb design, etc. are already explained earlier. Mentioning the different types of popular models and the prominence of mathematical models are also done. Among the various mathematical models available the general process of developing the particular suitable model for the current purpose the – "Average value-based model" is explained as under [16, 20, 23, 8].

The idea of an average value-based model is obtained from the n^{th} order linear differential equations. The dynamic characteristic of a system i.e. the input-output function of a system is represented popularly by an n^{th} order linear differential equation with constant coefficients between q_i the input function and q_o the output function. This obviously says that both output as well as input functions are expressed as n^{th} order differential equations.

$$q_{out} = A_m \frac{d^m q_0}{dt^m} + A_{m-1} \frac{d^{m-1} q_0}{dt^{m-1}} + ... + A_1 \frac{dq_0}{dt} + A_0 q_0 \tag{2.1}$$

$$q_{input} = B_n \frac{d^n q_i}{dt^n} + B_{n-1} \frac{d^{n-1} q_i}{dt^{n-1}} + ... + B_1 \frac{dq_i}{dt} + B_0 q_i \tag{2.2}$$

where the constants A's and B's represent the physical parameters of the system. Specifically $A_0 q_0$ is termed as a non-differential base value.

In the case of digital function the variational terms are replaced by their discrete equivalents i.e. digital variational terms.

$$q_{out} = A_m \frac{D^m q_0}{Dt^m} + A_{m-1} \frac{D^{m-1} q_0}{Dt^{m-1}} + ... + A_1 \frac{Dq_0}{Dt} + A_0 q_0 \tag{2.3}$$

where $A_0 q_0$ is the base term, $A_1 \frac{Dq_0}{Dt}$ is the first variational term and $A_2 \frac{D^2 q_0}{Dt^2}$ is the second variational term.

Or q_0 =Average base term +first variational term + second variational term +...

Or in other words

$$q_0 = A\ base\ value + first\ variational\ term + second\ variational\ term + \ldots \tag{2.4}$$

The base value contains a coefficient A_0 multiplied by an average value of the variable, the first variational term contains a coefficient A_1 multiplied by the first variational part of the variable and the second variational term contains another coefficient A_2 multiplied by the second variational part of the variable and this process extends to infinity. While generalizing the model, these coefficients should be defined as constant values representing the physical parameters of the system and the variable is the output of the system (in the case of an engineering system). In the case of the biological system, physical parameters mean both the counterpart of the physical parameters of the engineering system along with the psychological aspects of the biological system. The number of variational terms required to be considered depends upon the accuracy that the researcher demands through the analysis. As mentioned earlier the method of calculation of these terms is explained in detail subsequently.

3. Desirable characteristics of Assistive Devices

In the process of developing a model it is also necessary that the model builder should have a peripheral knowledge about the properties of the system to be modeled, along with that of the modeling tool selected for modeling [19]. The properties of the modeling tool selected are already discussed earlier in section 1.1. Now it is necessary to describe the required characteristics of the smart assistive devices for a disabled biological system. The elbow of a human subject is selected as the biological system and its operational characteristics are given as under.

3.1. Characteristic of elbow movement. The need and importance of assistive devices are already explained in the previous section 1.1. Now the desirable characteristics that the elbow movement is expected to have also to be looked into. The characteristic of an artificial device depends upon the objective or duty of the device to be performed. To be more clear, the devices designed for correcting/assisting the afflicted elbow system necessarily required a characteristic capable of compensating for the affliction. Hence an assistive device intended for the compensation of improper elbow dynamics naturally possesses a different characteristic from that of the healthy one. Moreover, it may differ slightly from person to person. Hence it is obvious that the characteristics are purely customized in nature and not easily explainable generally, even though some common generality is observable. Further, the characteristic of a natural system is closely dependent on the health, the depth of disability, etc. of the subject. This necessitates a correct estimation of the quantified magnitude of the appropriate disability. Suitable testing, experiments, analysis, etc. are the essential procedures to be followed by the rehabilitation engineer to suggest a particular assistive device. These studies about the disability may be done by expert medical practitioners or medical teams, thereby decisions/calculations about the magnitude of the disability may be made. This hurdle is achieved by closely monitoring the operational characteristics of healthy subjects having similar physical and health conditions. This characteristic study of the healthy one is used as a reference for comparison for the afflicted one. For example, here the movement of a healthy elbow is closely monitored by video camera and the video is analyzed by advanced software thereby conducting a detailed positional analysis. The experimental setup is shown in Fig.1. Even though the experiment provides the details of electrical signals, those details are not shown here as it is not relevant. The experiment is repeated in several similar subjects and a database is prepared at an average level. This result is utilized for preparing the characteristics of biological systems. The result of the experiment i.e. the time, position, velocity, acceleration etc. is shown in Table 2. The results are plotted and shown in Fig.2-4. A similar experiment is conducted on the afflicted subject and the afflicted characteristic is also detected. The ideal characteristics of reference one and that of the afflicted one are compared and correct evaluation of the affliction is calculated.

3.2. Experimental set up. The experimental setup for the above experiment is as shown in Fig.1. The instrument used for the measurement of position is a video camera, the method selected is video capturing and the moving object is the elbow. For convenience in measurements, markers/stickers are placed on the moving object – human limb i.e. the elbow. An approximate line diagram is shown in Fig.1.

The elbow is allowed to move and the camera observes the movement at every 0.033 seconds. The video is analyzed with the help of advanced analytical software.

The analyzed reading of position, velocity, and acceleration are tabulated and recorded as shown in Table 2. Having given a small description of the experimental determination of the elbow kinematics, the details of formulating the average value-based model formation are shown below.

3.3. Experimental observation. As mentioned above the experiment is conducted and the results are shown in Table 2 and also in graphical plot Fig.2-4. The relationship of position, velocity, and acceleration with respect to a common base time is shown in Fig.2-4.

It is possible to observe from Fig.2 that a small upward lift is observed between 500 and 600 ms. That is a rise in displacement from 129 degrees to 130 and the displacement is almost steady from 700 to 1600 ms. After 1600 ms a downward gradient is observed till 3500 ms. From 3500 to 3700 ms region, a small upward rise of 18.517 degrees is observed. Similarly upward and downward variations are observed from 3700ms onwards.

While looking at Fig.3,i.e., time versus velocity curve it is interesting to see that up to 500 ms velocity moves between positive and negative values, which makes the position almost a steady increase. Similarly, the acceleration curve swings around positive and negative values while the position shows a dip in values and the velocity curve follows well below negative values. The acceleration swings around positive and negative values.

When the position comes to a low value the velocity shows peak variations with ups and downs and acceleration curves also swing up and down of the zero line. This shows the influence of the inhibitory action of CNS (Central Nervous System). On experimentation of about 300 subjects and that too with different velocities these types of characteristics are observed with minor changes and shifts in regions. Deriving a conclusion by looking at these curves feels to be cumbersome but it is quite easy to illustrate these factors with the help of the equation cited in the following sections. Once again it has to be stressed that these characteristics are the combined effect of the activating signals, feedback signals, and the inhibition signals of the central nervous system (CNS) of the human being. The characteristic is modeled mathematically using the above-mentioned philosophy in section 1. It can be seen that in the total journey of the elbow during the period of 0 to 4000 ms in the upward direction many times upward and downward trends of movements are observed. Between the time interval 0 to 1700 ms, the average rate of variation is smaller compared to variations between 1700 to 3400 ms. Between 3400 ms to 4000 ms another rate of variation is observed. Variations are observed from person to person slightly in the time-based relationship but exhibit some generality in an overall manner throughout the entire time period of the journey.

3.4. Calculations of Parameters. According to the details given above in section 2 the base value and the different variational terms are calculated for the above-explained model in section 2 as follows:

Here the position/ displacement is defined as a function of time and its value at a specific time is defined in the form of a general equation given by

$$\begin{aligned} &\text{Displacement } (D_T) \\ &= (\text{initial position} \times A_0) + (\text{velocity} \times \text{time} \times A_1) \\ &\qquad\qquad + (\text{acceleration} \times \text{time} \times A_2) \end{aligned} \tag{3.1}$$

Now the entire Table 2 is divided into three sectors:

a) Between 0 and 1766 ms

b) 1766 to 3400 ms
c) 3400 ms to 4199 ms

In the first sector (a) the initial value is taken as 129.3711395° (Table 3), and the velocity is considered as the average velocity between 0 to 1766 ms. Again in the third term of equation 4, the acceleration is determined in a similar way. Time is 1766 ms. Total displacement for this sector denoted by D_{T1} is taken as the average of the displacement values up to 1766 ms. From the table, it is clear that

D_{T1} = 129.1660279°, initial position = 129.3711395°, velocity = -1.387695487 deg/ms, acceleration = -15.88878616 deg/ms^2.

Substituting these values and incorporating the details given in section 4, the equation becomes (**??**).

Similarly, equations are formulated for D_{T2} and D_{T3} for the other two sectors. The corresponding time duration is 1634 ms and 800 ms respectively. i.e.

$$\begin{aligned} 129.1660279^\circ &= 129.3711395^\circ\ A_0 + \\ &\quad (-1.387695487) \times 1766\ A_1 (-15.88878616) \times 1766\ A_2 \end{aligned} \tag{3.2}$$

$$\begin{aligned} 75.32182659^\circ &= 126.1541138^\circ\ A_0 + (-67.01076002) \times 1634\ A_1 + \\ &\quad (2.541000417) \times 1634\ A_2 \end{aligned} \tag{3.3}$$

$$\begin{aligned} 24.66854543^\circ &= 17.18432617^\circ\ A_0 + (17.70265923) \times 800\ A_1 + \\ &\quad (33.48061506) \times 800\ A_2 \end{aligned} \tag{3.4}$$

Solving the above equations, the values of A_0, A_1 and A_2 are obtained as 1.011609, 0.0004783384 and 0.00001905717 respectively.
The model-building philosophy involved some approximation in the case of physical and health parameters. Hence to make the model more and more realistic, repeated measurements and calculations are done. The experiment is repeated three times and accordingly three different close values of A_0, A_1, and A_2 are obtained. Hence in the general model an average of A_0, A_1 and A_2 are taken. The corresponding equations and values of each are shown below.
Equations for the second repeated experiment:

$$\begin{aligned} 123.2093632^\circ &= 129.3711395^\circ\ A_0 + (-46960.8399)\ A_1 \\ &\quad + (-83519.24195)\ A_2 \end{aligned} \tag{3.5a}$$

$$\begin{aligned} 114.0705685^\circ &= 129.2966003^\circ\ A_0 + (-62219.004635)\ A_1 \\ &\quad + (-82688.714744)\ A_2 \end{aligned} \tag{3.5b}$$

$$\begin{aligned} 37.7687636^\circ &= 83.97006226^\circ\ A_0 + (-52827.88434)\ A_1 \\ &\quad + (-87278.99557)\ A_2 \end{aligned} \tag{3.5c}$$

Solving the above equations, the values of A_0, A_1 and A_2 are obtained as 1.030767, 0.0005827376 and -0.0002062349 respectively.

Equations for the third repeated experiment:

$$113.067424° = 129.3711395°\ A_0 + (-85289.445705)\ A_1 + (-46962.771523)\ A_2 \tag{3.6a}$$

$$74.7802743° = 129.4569397°\ A_0 + (-108155.26328)\ A_1 + 62439.092748\ A_2 \tag{3.6b}$$

$$37.76873636° = 83.9700626°\ A_0 + (-52827.88434)\ A_1 + (-87278.99557)\ A_2 \tag{3.6c}$$

Solving the above equations, the values of A_0, A_1 and A_2 are obtained as 0.9004185, 0.0002091306 and -0.000306964 respectively.
Now three sets of values for A_0, A_1 and A_2 are obtained as tabulated below in Table 1. For the general model the average of A_0, A_1 and A_2 are taken for the reason stated above.

Coefficients	Expt. No.1	Expt. No.2	Expt. No.3	Average
A_0	1.011609	1.030767	0.9004185	0.980932
A_1	0.0004783384	0.0005827376	0.0002091306	0.0004234
A_2	0.00001905717	-0.0002062349	-0.000306964	-0.00016471

TABLE 1. Tabulated values of the experimentally detected average values for getting the coefficients

4. General Model for the dynamics of elbow movement

The general property of any model is that it must be as general as possible. Hence in this context, the above-mentioned values of A_0, A_1, A_2 will become the constant coefficient, and the variational terms will take the variable part. So the generalized model of the elbow dynamics of our present interest is obtained by substituting the average values of A_0, A_1 and A_2 from Table 1. The values of A_0,A_1 and A_2 are 0.980932, 0.0004234 and -0.00016471 respectively.

4.1. Model of the elbow dynamics. The general model of the elbow dynamics can be written as a function of time $q_0(t)$ and can be represented as an infinite series

$$q_0(t) = C_0(\text{base value}) + C_1(\text{first differential}) + C_2(\text{second differential}) + \ldots\ldots. \tag{4.1}$$

Substituting the values of A_0, A_1, and A_2 it is possible to get the actual model of the selected subject as below.

$$q_0(t) = 0.980932(\text{base value}) + 0.0004234(\text{first differential}) - 0.00016471(\text{second differential}) + \ldots\ldots.. \tag{4.2}$$

As conclusion, the general model of the elbow dynamics is the one mentioned below with proper modification of equation 1.

$$q_{out} = A_0 q_0 + A_1 \frac{dq_0}{dt} + A_2 \frac{d^2 q_0}{dt^2}$$

where A_0, A_1 and A_2 are the constants representing the physical, environmental, mental and other health conditions of the subjects, q0 is the average positional value obtained from the experiment, $\frac{dq_0}{dt}$ is the first variational term representing the velocity factor of the elbow and $\frac{d^2 q_0}{dt^2}$ is the second variational term representing the acceleration component of the elbow movement.

5. Conclusion

The average value-based model for the dynamics of the selected elbow movement is developed satisfactorily. The value of the coefficients of the truncated differential equations is identified and the model is generalized by using these constant coefficients.

The model is validated successfully by comparing the values obtained from the solution of the modeled equations and that of the real values obtained by experiments. The average error is found to be in a tolerable magnitude. The average of the error values are used as a separate term and augmented to the model as a correcting/tuning term in the model and this makes an up gradation. The updated model by taking into account the error term is also given in the paper. The detailed steps right from the beginning of the experiment up to the final model formation is presented clearly. In conclusion, it can be stated that this model is capable of representing the system dynamics satisfactorily.

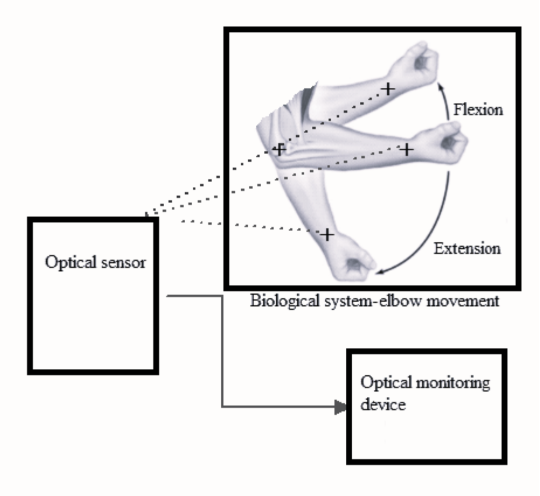

FIGURE 1. Experimental setup for detecting position, velocity, and acceleration with respect to time

Figure 2. Relationship showing the angular displacement w.r.to time

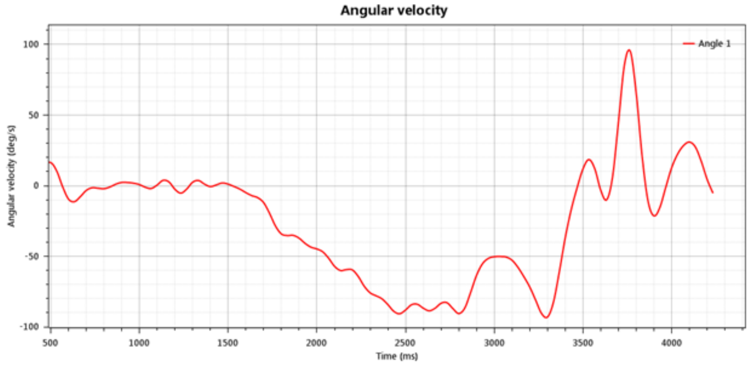

Figure 3. Relationship showing the angular velocity w.r.to time

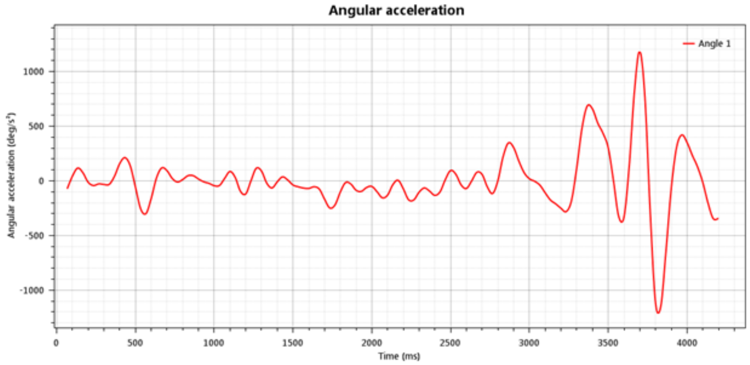

Figure 4. Relationship showing the angular velocity w.r.to time

Sl. No.	Time (ms)	Angle 1	Angular velocity	Angular Acceleration	Sl. No.	Time (ms)	Angle 1	Angular velocity	Angular Acceleration
1.	0	129.3711395			65.	2133	111.5188217	-57.3444862	-34.35156631
2.	33	129.3877563	-4.86553288		66.	2166	109.5104675	-58.6417694	6.724002838
3.	67	129.256546	-5.12726593	-72.65158081	67.	2199	107.5632858	-61.5526047	-72.95439911
4.	100	129.0045624	-4.23753691	43.74778366	68.	2233	105.5414581	-66.369873	-172.3342285
5.	133	128.8192291	-1.63578153	118.5766068	69.	2266	103.2836838	-70.8825684	-173.407486
6.	167	128.8157196	0.935435057	74.7273407	70.	2299	100.8069153	-73.3960876	-102.6009674
7.	200	128.9086304	1.719200015	-12.313097	71.	2333	98.23377991	-75.4072571	-63.81187439
8.	233	128.9588013	1.267059445	-41.9610939	72.	2366	95.6166153	-79.2756577	-93.30770111
9.	267	128.9433441	0.911249816	-27.09558105	73.	2399	92.90042114	-85.0124741	-132.2720184
10.	300	128.9154968	0.740108609	-31.99129868	74.	2433	90.01183319	-90.6357422	-98.37245178
11.	333	128.8576965	0.490877956	-31.6991539	75.	2466	86.97949219	-93.5989456	13.07579994
12.	367	128.7300873	1.098992705	42.11485291	76.	2499	83.97006226	-92.5256729	95.83238983
13.	400	128.6312256	3.258650541	158.4680786	77.	2533	81.11665344	-88.8488007	60.8443718
14.	433	128.7317657	5.65298748	213.8032074	78.	2566	78.35399628	-85.171936	-36.22586823
15.	467	129.1087036	6.530623436	139.0020752	79.	2599	75.524086	-82.4023209	-69.37345123
16.	500	129.6831665	5.420153618	-56.67284393	80.	2633	72.57701111	-80.2059708	-0.710389495
17.	533	130.2036438	2.623481274	-255.7919769	81.	2666	69.62335205	-79.217453	79.6476593
18.	566	130.3828125	-1.01536918	-299.2107239	82.	2699	66.79686737	-80.7541351	59.2514267
19.	600	130.1623077	-3.82597828	-159.8027191	83.	2733	64.07643127	-84.9879837	-55.42432022
20.	633	129.7533569	-4.34514093	26.62814331	84.	2766	61.2799263	-90.2690735	-115.8388901
21.	666	129.4111328	-2.86471868	119.6775208	85.	2799	58.28330231	-92.8822556	2.953670979
22.	700	129.242157	-1.11149526	94.35326385	86.	2832	55.24841309	-89.0667267	223.0118866
23.	733	129.1915741	-0.48862335	23.19622803	87.	2866	52.5032959	-79.185112	348.2594299
24.	766	129.1501007	-1.28386617	-10.2742424	88.	2899	50.20754623	-67.3290329	305.0105896
25.	800	129.0750427	-2.57839012	13.37404156	89.	2932	48.27029419	-56.7302704	180.9839172
26.	833	129.0124054	-2.61815238	48.19144821	90.	2966	46.5215683	-48.7042122	75.67908478
27.	866	129.0179291	-0.74174827	48.17607498	91.	2999	44.84124756	-44.9560661	20.28863144
28.	900	129.0887909	1.527767897	17.76235008	92.	3032	43.17176819	-46.4730225	-3.109798908
29.	933	129.1748199	2.275050163	-7.903263092	93.	3066	41.50232315	-50.9167976	-38.6506424
30.	966	129.2440643	1.151872277	-22.25829124	94.	3099	39.80815125	-55.0116844	-108.5391006
31.	1000	129.2966003	-0.65197611	-43.12044144	95.	3132	37.99170685	-57.9492645	-174.4176331
32.	1033	129.3004608	-1.56896484	-41.54286575	96.	3166	35.96239471	-61.2543945	-209.5302734
33.	1066	129.2268372	-0.77228731	24.61697006	97.	3199	33.70631027	-67.8172455	-249.6689301
34.	1100	129.1723328	1.169293761	87.13269806	98.	3232	31.18587875	-79.3576202	-281.2554321
35.	1133	129.2664185	2.428154945	31.38548088	99.	3266	28.31185722	-92.0763016	-181.3563385
36.	1166	129.4312439	1.634346724	-95.45726013	100.	3299	25.15999603	-96.9907455	116.3755798
37.	1200	129.4454346	-0.18282385	-118.8810577	101.	3332	22.11179733	-87.0382156	478.2468872
38.	1233	129.2661438	-0.76449883	2.986016035	102.	3366	19.65106583	-64.1669998	684.8522339
39.	1266	129.0963593	0.315637589	116.6915131	103.	3399	18.03616333	-38.4862633	659.0390015
40.	1300	129.1142883	1.366874695	90.33397675	104.	3432	17.18432617	-18.1229515	530.9016113
41.	1333	129.2656403	1.049187541	-20.29016304	105.	3466	16.88805389	-1.75991011	436.5403137
42.	1366	129.3637085	-0.16273782	-64.56073761	106.	3499	17.07591057	13.84941959	303.8850403
43.	1400	129.3447876	-0.90120673	-8.161579132	107.	3532	17.67910576	22.61235809	3.056355476
44.	1433	129.326355	-0.70711017	37.80649567	108.	3566	18.31738663	15.61360741	-324.4811707
45.	1466	129.3876801	-0.4107388	8.112376213	109.	3599	18.48373413	-2.59061098	-330.8810425
46.	1500	129.4569397	-0.56466329	-37.06845093	110.	3632	18.11747932	-13.535037	134.9678192
47.	1533	129.4665985	-0.82639658	-53.49308777	111.	3666	17.81855392	1.1568681	822.4420166
48.	1566	129.4228516	-1.34801841	-65.05891418	112.	3699	18.51711655	45.23916626	1186.92041
49.	1599	129.3079071	-2.75650001	-68.82603455	113.	3732	20.80675125	95.34857941	757.5088501
50.	1633	129.099762	-4.83847809	-53.34855652	114.	3766	24.18918419	107.9182358	-285.7497864
51.	1666	128.8434906	-7.46728849	-74.1645813	115.	3799	27.15988731	66.59661102	-1108.342896
52.	1699	128.5397034	-11.7911139	-166.4297943	116.	3832	28.59192085	8.755177498	-1149.727417
53.	1733	128.0496063	-18.4951324	-247.9963837	117.	3866	28.58963966	-23.6569843	-642.2434082
54.	1766	127.2403488	-26.1653309	-219.0556488	118.	3899	27.88743973	-25.3892021	-72.55510712
55.	1799	126.1541138	-32.4840279	-101.7583313	119.	3932	27.1664772	-11.5696039	302.1533203
56.	1833	124.9679108	-36.3190231	-14.77019882	120.	3966	26.86066055	4.109078407	421.4814148
57.	1866	123.8065872	-38.4024353	-27.95969963	121.	3999	27.08551216	15.14668369	353.2423706
58.	1899	122.6298752	-39.9269257	-85.01390839	122.	4032	27.70615005	21.70651627	236.0707397
59.	1933	121.3348694	-41.142128	-95.60488129	123.	4066	28.57368851	26.24498367	129.6027222
60.	1966	119.9141922	-42.2692032	-59.96330261	124.	4099	29.60029221	28.41099548	-12.45223522
61.	1999	118.4384613	-44.3351898	-49.5906601	125.	4132	30.63757515	25.29784012	-201.5220184
62.	2033	116.9321442	-47.9287491	-99.22129059	126.	4166	31.43911743	16.5381279	-344.9881287
63.	2066	115.3217621	-52.2189598	-154.4857025	127.	4199	31.80627823	6.943872929	-342.2950134
64.	2099	113.509346	-55.6075554	-129.6858063					

Table 2. Main Table

position vs velocity set																	
1		2		3		4		5		6		7		8		9	
Range 128.6 to 130.4		**Range 126.2 to 129.4**		**Range 58.2 to 86.98**		**Range 25.16 to 58.28**		**Range 18.12 to 25.16**		**Range 18.12 to 20.81**		**range 20.81 to 24.19**		**Range 24.19 to 27.89**		**Range 27.89 to 31.81**	
128.63123	3.2586505	129.41113	-2.8647187	86.979492	-93.598946	58.283302	-92.882256	25.159996	-96.990746	18.117479	-13.535037	20.806751	95.348579	24.189184	107.91824	27.88744	-25.389202
128.73177	5.6529875	129.24216	-1.1114953	83.970062	-92.525673	55.248413	-89.066727	22.111797	-87.038216	17.818554	1.1568681	24.189184	107.91824	27.159887	66.596611	27.166477	-11.569604
129.1087	6.5306234	129.19157	-0.4886234	81.116653	-88.848801	52.503296	-79.185112	19.651066	-64.167	18.517117	45.239166			28.591921	8.7551775	26.860661	4.1090784
129.68317	5.4201536	129.1501	-1.2838662	78.353996	-85.171936	50.207546	-67.329033	18.036163	-38.486263	20.806751	95.348579			28.58964	-23.656984	27.085512	15.146684
130.20364	2.6234813	129.07504	-2.5783901	75.524086	-82.402321	48.270294	-56.73027	17.184326	-18.122952					27.88744	-25.389202	27.70615	21.706516
130.38281	-1.0153692	129.01241	-2.6181524	72.577011	-80.205971	46.521568	-48.704212	16.888054	-1.7599101							28.573689	26.244984
		129.01793	-0.7417483	69.623352	-79.217453	44.841248	-44.956066	17.075911	13.84942							29.600292	28.410995
		129.08879	1.5277679	66.796867	-80.754135	43.171768	-46.473022	17.679106	22.612358							30.637575	25.29784
		129.17482	2.2750502	64.076431	-84.987984	41.502323	-50.916798	18.317387	15.613607							31.439117	16.538128
		129.24406	1.1518723	61.279926	-90.269073	39.808151	-55.011684	18.483734	-2.590611							31.806278	6.9438729
		129.2966	-0.6519761	58.283302	-92.882256	37.991707	-57.949265	18.117479	-13.535037								
		129.30046	-1.5689648			35.962395	-61.254395										
		129.22684	-0.7722873			33.70631	-67.817245										
		129.17233	1.1692938			31.185879	-79.35762										
		129.26642	2.4281549			28.311857	-92.076302										
		129.43124	1.6343467			25.159996	-96.990746										
		129.44543	-0.1828239														
		129.26614	-0.7644988														
		129.09636	0.3156376														
		129.11429	1.3668747														
		129.26564	1.0491875														
		129.36371	-0.1627378														
		129.34479	-0.9012067														
		129.32636	-0.7071102														
		129.38768	-0.4107388														
		129.45694	-0.5646633														
		129.4666	-0.8263966														
		129.42285	-1.3480184														
		129.30791	-2.7565														
		129.09976	-4.8384781														
		128.84349	-7.4672885														
		128.5397	-11.791114														
		128.04961	-18.495132														
		127.24035	-26.165331														
		126.15411	-32.484028														

Table 3. Showing different sectors experimental readings obtained from Table 2 to find out variational terms

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