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# THE FOURTH CUBE-ROOT MULTIPLICATIVE ATOM BOND CONNECTIVITY INDEX

S. R. RAMACHANDRA* AND M. J. JYOTHI

Abstract. In this paper, we introduce the fourth cube-root multiplicative atom bond connectivity indices of a graph. A topological index is a numeric quantity derived from the structural graph of a molecule. We compute the fourth cube-root multiplicative atom bond connectivity index of line graphs of subdivision graphs of two-dimensional lattice of $TUC_4C_8[p, q]$.

## 1. Introduction

One of the well-known and widely used topological indices is the product connectivity index or Randi index, introduced by Randi in [4]. Motivated by the definition of the product connectivity index and its wide applications, Kulli [5] introduced the first multiplicative atom bond connectivity index of a graph $G$, which is defined as

$$ABCII_1(G) = \prod_{uv \in E(G)} \frac{d_G(u) + d_G(v) - 2}{d_G(u) d_G(v)}.$$

Recently, many other multiplicative indices have been studied (see [6, 7, 8, 9, 10]). The second multiplicative atom bond connectivity index of a graph $G$ is defined as

$$ABCII_2(G) = \prod_{uv \in E(G)} \frac{n_u n_v}{n_u + n_v - 2},$$

where $n_u$ denotes the number of vertices of $G$ lying closer to the vertex $u$ than to the vertex $v$ for the edge $uv \in E(G)$.
The fourth multiplicative atom bond connectivity index of a graph $G$ is defined as

$$ABCII_4(G) = \prod_{uv \in E(G)} \frac{S_G(u) S_G(v)}{S_G(u) + S_G(v) - 2},$$

where $S_G(u)$ is the sum of the degrees of all vertices adjacent to a vertex $u$.

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*Corresponding author.

Recently kulli introduced the fourth multiplicative atom bond connectivity index of a graph $G$ is defined as

$$ABCII^{(4)}(G) = \prod_{uv\in E(G)} \sqrt{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}},$$

where $S_G(u)$ is the sum of the degrees of neighbors of vertex $u$. Motivated by the definition of the The second multiplicative atom bond connectivity index , the fourth multiplicative atom bond connectivity index and by previous research on topological indices, we now introduce the Cube-root Multiplicative Atom Bond Connectivity Indices of a graph as follows.

## 2. Fourth Cube-root Multiplicative Atom Bond Connectivity Index

The fourth cube-root multiplicative atom bond connectivity index of a graph $G$ is defined as

$$ABCII_3^{(4)}(G) = \prod_{uv\in E(G)} \sqrt[3]{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}},$$

where $S_G(u)$ is the sum of the degrees of neighbors of vertex $u$.

**Lemma 2.1** ([1])**.** *Let $G$ be a $(p,q)$ graph. Then $L(G)$ has $q$ vertices and*

$$\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$$

*edges.*

**Lemma 2.2** ([1])**.** *Let $G$ be a $(p,q)$ graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.*

**Theorem 2.3.** *Let $G$ be the line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then*

$$ABCII_3^{(4)}(G) = \left(\frac{3}{8}\right)^{\frac{4}{3}} \times \left(\frac{7}{20}\right)^{\frac{8}{3}} \times \left(\frac{8}{25}\right)^{\frac{4(p+q-4)}{3}} \times \left(\frac{8}{25}\right)^{\frac{4(p+q-2)}{3}} \times \left(\frac{15}{72}\right)^{\frac{8(p+q-2)}{3}} \times \left(\frac{4}{9}\right)^{\frac{2(9pq+10)-19(p+q)}{3}},$$

*if $p>1, q>1$,*

$$ABCII_3^{(4)}(G) = \left(\frac{3}{8}\right)^{\frac{6}{3}} \times \left(\frac{7}{20}\right)^{\frac{4}{3}} \times \left(\frac{8}{25}\right)^{\frac{2\times(p-2)}{3}} \times \left(\frac{11}{40}\right)^{\frac{4\times(p-1)}{3}} \times \left(\frac{14}{64}\right)^{\frac{2\times(p-1)}{3}} \times \left(\frac{15}{72}\right)^{\frac{4\times(p-1)}{3}} \times \left(\frac{4}{9}\right)^{\frac{(p-1)}{3}}.,$$

*if $p>1, q=1$.*

*Proof.* The 2D-lattice of $TUC_4C_8[p,q]$ is a graph $G$ with $4pq$ vertices and $6pq-p-q$ edges where p and q denote the number of squares in a row and the number of rows of squares respectively. By Lemma 2, the subdivision graph of the 2D-lattice of $TUC_4C_8[p,q]$ is a graph with $10pq-p-q$ vertices and $2(6pq-p-q)$ edges. Thus, by Lemma 1, $G$ has $2(6pq-p-q)$ vertices and $18pq-5p-5q$ edges.

| $S_G(u), S_G(v)$ $(uv \in E(G))$ | Number of edges |
|---|---|
| $(4,4)$ | $4$ |
| $(4,5)$ | $8$ |
| $(5,5)$ | $2(p+q-4)$ |
| $(5,8)$ | $4(p+q-2)$ |
| $(8,9)$ | $8(p+q-2)$ |
| $(9,9)$ | $2(9pq+10)-19(p+q)$ |

TABLE 1. Edge partition of $G$ with $p>1$ and $q>1$.

| $S_G(u), S_G(v)$ $(uv \in E(G))$ | Number of edges |
|---|---|
| $(4,4)$ | $6$ |
| $(4,5)$ | $4$ |
| $(5,5)$ | $2(p-2)$ |
| $(5,8)$ | $4(p-1)$ |
| $(8,8)$ | $2(p-1)$ |
| $(8,9)$ | $4(p-1)$ |
| $(9,9)$ | $p-1$ |

TABLE 2. Edge partition of $G$ with $p>1$ and $q=1$.

**Case 1.** Suppose $p>1$ and $q>1$. By algebraic method, we obtain $|V_4|=8$, $|V_5|=4(p+q-2)$, $|V_8|=4(p+q-2)$, and $|V_9|=2(6pq-5p-5q+4)$ in $G$. Thus, the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 1.

$$ABCII_3^{(4)}(G)=\prod_{uv\in E(G)}\left(\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}\right)^{\frac{1}{3}}$$

$$ABCII_3^{(4)}(G)=\left(\frac{4+4-2}{4\times 4}\right)^{\frac{4}{3}}\times\left(\frac{4+5-2}{4\times 5}\right)^{\frac{8}{3}}\times\left(\frac{5+5-2}{5\times 5}\right)^{\frac{2\times(p+q-4)}{3}}\times\times$$
$$\left(\frac{5+8-2}{5\times 8}\right)^{\frac{4\times(p+q-2)}{3}}\times\left(\frac{8+9-2}{8\times 9}\right)^{\frac{8\times(p+q-2)}{3}}\times\left(\frac{9+9-2}{9\times 9}\right)^{\frac{2(9pq+10)-19(p+q)}{3}}.$$
$$=\left(\frac{3}{8}\right)^{\frac{4}{3}}\times\left(\frac{7}{20}\right)^{\frac{8}{3}}\times\left(\frac{8}{25}\right)^{\frac{4(p+q-4)}{3}}\times\left(\frac{8}{25}\right)^{\frac{4(p+q-2)}{3}}$$

$$\times \left(\frac{15}{72}\right)^{\frac{8(p+q-2)}{3}} \times \left(\frac{4}{9}\right)^{\frac{2(9pq+10)-19(p+q)}{3}}.$$

**Case 2.** Suppose $p > 1$ and $q = 1$. The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 2.

$$ABCII_3^{(4)}(G) = \left(\frac{4+4-2}{4\times 4}\right)^{\frac{6}{3}} \times \left(\frac{4+5-2}{4\times 5}\right)^{\frac{4}{3}} \times \left(\frac{5+5-2}{5\times 5}\right)^{\frac{2\times(p-2)}{3}} \times$$

$$\left(\frac{5+8-2}{5\times 8}\right)^{\frac{4\times(p-1)}{3}} \times \left(\frac{8+8-2}{8\times 8}\right)^{\frac{2\times(p-1)}{3}} \times \left(\frac{8+9-2}{8\times 9}\right)^{\frac{4\times(p-1)}{3}}$$

$$\times \left(\frac{9+9-2}{9\times 9}\right)^{\frac{(p-1)}{3}}.$$

$$= \left(\frac{3}{8}\right)^{\frac{6}{3}} \times \left(\frac{7}{20}\right)^{\frac{4}{3}} \times \left(\frac{8}{25}\right)^{\frac{2\times(p-2)}{3}} \times \left(\frac{11}{40}\right)^{\frac{4\times(p-1)}{3}} \times \left(\frac{14}{64}\right)^{\frac{2\times(p-1)}{3}}$$

$$\times \left(\frac{15}{72}\right)^{\frac{4\times(p-1)}{3}} \times \left(\frac{4}{9}\right)^{\frac{(p-1)}{3}}.$$

□

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S. R. Ramachandra: Department of Mathematics, Government College, M.C. Road, Mandya-571 401, India.
*E-mail address*: srrc16@gmail.com

M. J. Jyothi: Department of Mathematics, Government first Grade College, Chamarajanagar-571 313, India.
*E-mail address*: jyothimj49@gmail.com