Global and Stochastic Analysis
Vol8 No. 2 (July-December, 2021)

MUK PUBLICATIONS
Open Access Publisher

CONSTRUCTION OF GEOMETRY PROOF ON THE PRE-SERVICE ELEMENTARY SCHOOL TEACHER

MUSRIKAH, CHOLIS SA'DIJAH*, SWASONO RAHARDJO, SUBANJI

Abstract: Proof of geometry is a difficult material for students. Various Skills are required to prove any theorems. The aim of study is indentify the ability of students in constructing the proof of geometry. This ekplorative qualitative research uses the subjects of Preservice elementary teacher at UIN Sayyid Ali Rahmatullah Tulungagung. A total of 35 students participated in this study and 3 research subjects were selected consisting of one subject for each type of response. The results showed that there were three types of responses found, namely empirical, rational, and formal types. The most type is the rational type and the least type is the formal type. The students with non-proof construction in the form of empirical argument to prove the premises are very tied to the image, and use the empirical facts to construction argument. Students with non-proof constructions of rational arguments prove with a formal evidentiary framework, yet they have not been able to reach the right conclusions. Students with fomal proof construction can perform a good formal proof. This shows that they have not been able to think formally perfectly. so that it takes a serious effort from the teacher to present the geometry proof material well.

Keywords: Proof, geometry, pre service elementary teacher

Introduction

Students as pre service elementary teachers need to have proofing ability and good reasoning. Because the reasoning and proofing is an important part in the process of mathematical thinking (Stylianou & Knuts, 2010). Reasoning is an important process in mathematics and has been studied by many researchers (DeJarnette & González, 2013; Ellis, 2007; Lee & Hackenberg, 2014; Liu & Manouchehri, 2008). Ellis (2007) examined students' reasoning regarding to the process of generalization and justification in linear relations. DeJarnette & González (2013) conducted research on how the process of building reasoning in the algebra class on the students. While Liu (2014) conducted research on the reasoning and proofing process on the students. The number of the researchers who are interested in doing research on reasoning shows that reasoning is interesting to investigate. Reasoning also includes extensive material on mathematics.

Reasoning and proofing as a mathematical process that needs to be taught in school and requires adjustment when taught in school. Proof includes material that is difficult for learners to understand (Cirillo & Hummer, 2021). Stylianou & Knuts (2010) stated that reasoning and proofing in school should use logical deductions, but in constructing it can use informal elements. So in practice, the teaching of reasoning and proofing can be begun informally toward formal evidence. This is understandable because reasoning and proofing is not an easy thing to learn.The reasoning and proofing tend to be formal and rigid.So it is often difficult to be controlled by students and students experience failure in reasoning and proofing.Whereas reasoning and proofing is an important process in mathematics. While Quaresma (2019) states that the proof produced by students can be in the form of: unreadable proof, unsystematic proof, semi-systematic proof, and semi-synthetic proof.

The importance of reasoning in possession can be demonstrated by some research results (DeJarnette.et.all, 2013; Moore, 2014; Mueller & Yankelewitz, 2014). The results of DeJarnette.et.all (2013) research showed that through the development of environment-based reasoning can help students in doing reflection. The results of Moore's (2014) research showed that reasoning can improve students' understanding. The results of Mueller& Yankelewitz (2014) research showed that reasoning can bind ideas and develop arguments. The results of this study show that reasoning is a thing that should be owned by students.The teachers can facilitate the development of student or college students' reasoning. So their reasoning become better toward an optimal understanding. So the reasoning and good understanding need to be owned by students so that they do not experience failure in reasoning. Failure to do the reasoning will have an adverse effect. This is argued by the results of Liu & Manouchehri (2014) research which stated that the scheme of reasoning that is not materialized will result an adverse effects. Gunhan (2014) stated that the teachers's knowledge of students' reasoning can help teachers develop learning.

The process of reasoning can be known through the process of proofing is done. Geometric proof construction is an activity that involves mathematical ability, language use, and spatial understanding (Nathan, Schenck, Vinsonhaler, Michaelis, Swart, Walkington, 2021). Stylianou & Knuts (2010) stated that the preparation of mathematical evidence shows how each step, from the initial premise to the conclusion, is justified by a definition, fact, or principles established. NCTM (2000) stated that mathematical evidence is the steps toward formal conclusions. According to Stylianou & Knuts (2010), mathematical evidence showed how the steps of the first statement come to the conclusion, according to the definions, facts, or principles established. So the mathematical evidence tends to be formal. The tracking of the proofing process can be seen from the construction of the argument being made. Arguments can be classified in several types. Roger & Steele (2016) stated that there are three types of arguments: (1) non-proof arguments in the form of empirical arguments: (2) non-proof arguments in the form of rational arguments; (3) construction of argument in the form of evidence.

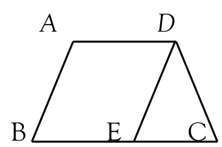
The research on argument construction has been done by several (Conner, Singletary Smith, Wagner, & Francisco, 2014; Mueller & Yankelewitz, 2014; Rogers & Steele, 2016; Maya & Sumarno, 2014). Conner.et.all (2014) did research on the kind of students' reasoning in collective argumentation. As for Mueller & Yankelewitz (2014) did research on invalid argument analysis. While Rogers & Steele (2016) did research on reasoning and proofing of the primary school teachers in constructing the evidence. Maya & Sumarno (2014) examined the differences in students' ability to prove the learning using the Moore approach conventionally.

Based on existing research, it appears that argument construction has been studied by many researchers, but the research on argument construction in students tends to be done on students of mathematics.The research on the construction of mathematical evidence on pre service elementary teachers is relatively limited. In fact, much of the material in elementary schools involves substantiating mainly geometric material including in elementary schools. If the pre service teacher is not interested and does not have sufficient knowledge on the geometric material, it will result in less optimal geometric learning process that will be implemented in elementary school. It will cause problems when students learn geometric material at the next level. So it needs to be traced, as far as the ability of pre service teachers on the ability to construct arguments on geometric material through research. It can be known by doing research on argument construction by prospective elementary school teachers. The purpose of this research is to identify students' ability to construct proof. This will provide important information on how efforts need to be done to assist students in constructing geometric evidence.

Method

This research is a descriptive qualitative research. The students who participated in this study were 35 students of Madrasah Ibtidaiyah Teacher Education at UIN ONE Tulungagung. They were given two questions about the proof of geometry. Furthermore, 3 selected subjects were taken from each group of students with empirical proof, rational proof, and formal proof. The research subjects were three students. The subjects consisted of: (1) one student with non-proof argument construction in the form of empirical proof (SE): (2) one student with non-proof argument construction in the form of rational argument (SR); (3) one student with argument construction in the form of formal evidence (SF). The research was conducted in the odd semester of 2020. The research instrument was two geometry test questions for all students who participated in the research and in-depth interviews on selected subjects. The test questions include material on paralel line and triagles congruent.The Problems of proofing as follows:

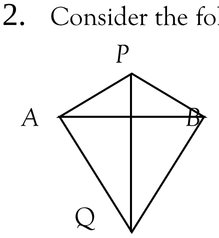
1. Consider the following picture!



Given: Trapezoid $ABCD; AB \cong CD;$ AB // DE; $\angle B \cong \angle C$.

Prove: Δ *DEC is isosceles triangle!*

2. Consider the following picture!



Given: $PA \cong PB;$ $QA \cong QB.$

Prove: $\angle A \cong \angle B$!

Result

The results showed that the student's ability in constructing evidence is quite diverse. There are 3 types of capabilities in constructing evidence that is empirical, rational, and formal. Of the 35 students who participated in this study, it was found that 2 students were able to construct formal proof, 23 students constructed rational proofs, and 10 students constructed empirical proof. One person was taken from each group as the research subject. The research subjects tracked the process of finding evidence from a given problem.

2.1 The proof constructed by SE

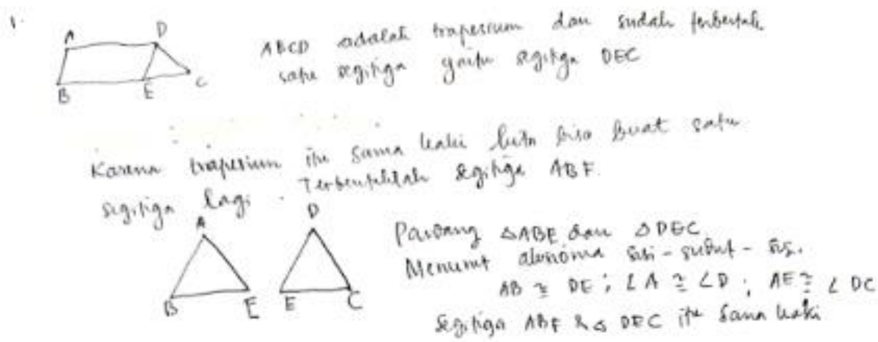


Figure 1: *The proof constructed by SE on question number 1*

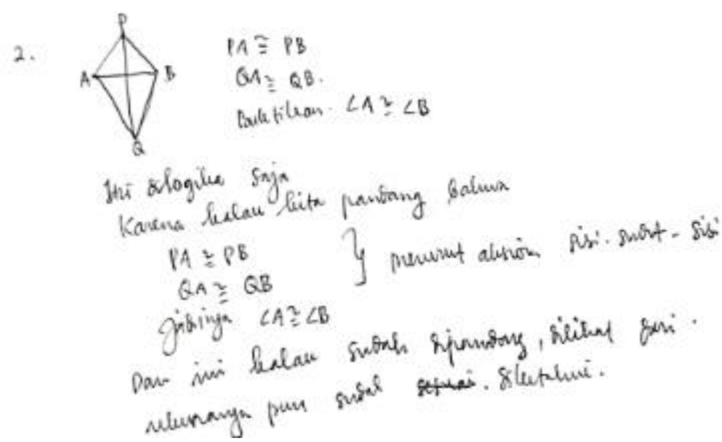


Figure 2: *The proof constructed by SE on question number 2*

SE tries to solve the problem by constructing two triangles from the existing picture. Therefore, he made a line from point A to E to form triangle ABE and Triangle DEC. Because the problem is asked to show that triangle DEC is isosceles, so another triangle must be made that is congruent with triangle DEC. This can be seen from the results of the interview with SE when asked about how he came up with the idea in the following interview excerpt.

> "I have to show that the triangle DEC is isosceles, so I make another triangle in the trapezoid. Because there needs to be another triangle for that. So I made a line AE to form a second triangle, namely triangle ABE. Because in the material studied, if there is one triangle, there needs to be another triangle in the congruence discussion."

The proofs made by SE have errors. He did not analyze all the possibilities first but immediately focused on looking for other triangles, because his understanding of the congruence material was not good. This is evident from the results of interviews which show that the choice of ideas he has is very limited. In addition, he also forced the idea of using the side-angles axiosm to show the congruence of triangles ABE and DEC. Even though it cannot be done because there is no reason to state that AE is congruent with DC. He did not understand the concept of proof and only wrote answers as closely as possible to the proving process being taught. It can be seen from the following interview results.

> "Actually I don't know how to answer, because in my view to show that the triangle is isosceles, we don't need to bother writing down complicated steps. But it is enough to measure directly. But I didn't do that, because the examples given during the lecture showed that the steps used to prove is to compose sentences with reasons. So I tried to write down my answers in a way that was exemplified in my lectures."

In question number 2, SE also experienced the same thing, he just wrote an answer like an example he had encountered. It appears that the resulting solution is false, because he uses the side-angles axiom to prove. In fact he should have used the side-by-side theorem. He only wrote down the random steps. Because according to him the question is strange, in the question he is asked to prove that angle A is congruent to angle B. According to him, it is clear that the two angles are congruent when viewed from the picture. The following interview results state this

> "Angle A and angle B are clearly congruent, from the picture it can be seen that the two angles are congruent. When I was asked to prove it, I was confused. What to prove. Therefore, I just write what comes to my mind about the preparation of evidence. "

The results of tests and interviews with SE showed that the subject did not understand the meaning of proof. SE understanding of geometry is still on the visual form of objects. He had not been able to understand the proof of geometry formally so he did not succeed in making a correct proof. The resulting evidence only includes the type of empirical proof. Students with non-proof constructions in the form of empirical arguments showed that they solve the problem given by drawing first. Images become their main reference.When they corresponded two polygons, they are fixated on the form of polygons and ignore the corresponding pair of correspondence. So the correspondence is also wrong. Though as if they were using formal proofing measures, but the arguments were arbitrary.

2.2 The proof constructed by SR

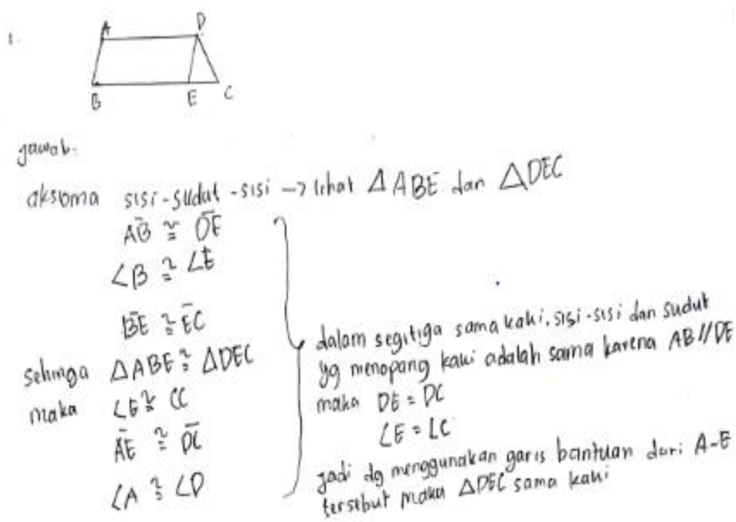


Figure 3: *The proof constructed by SR on question number 1*

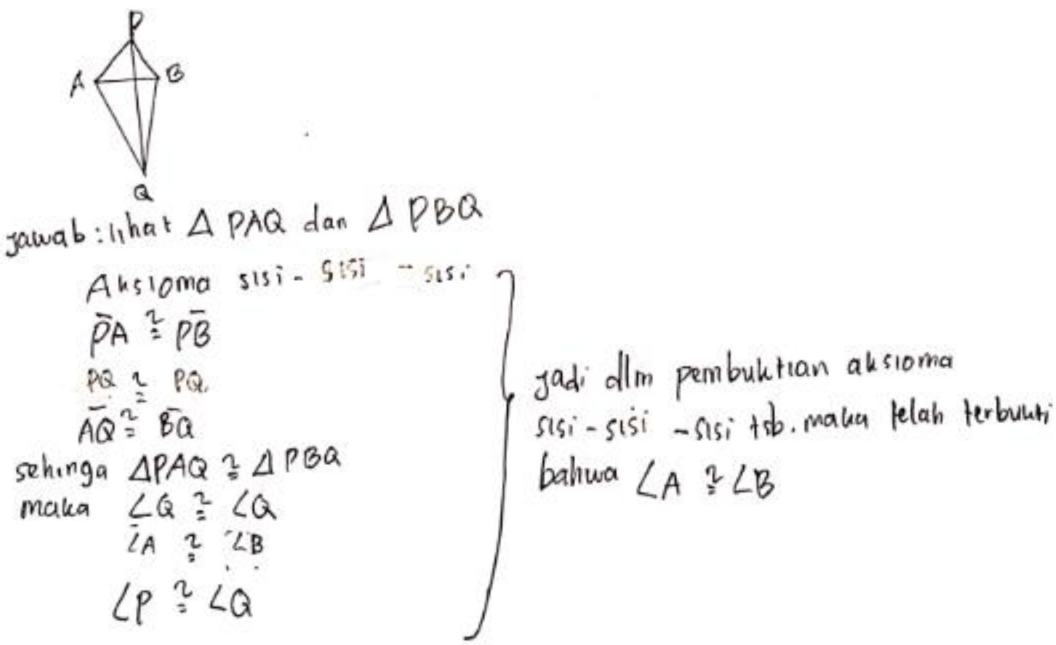


Figure 4: *The proof constructed by SR on question number 2*

SR solves the first problem by making a line AE so that two triangles are formed, namely triangle ABE and triangle DEC. He wanted to show that the two triangles were congruent using the sides-angles axiom. At this stage he has carried out the process correctly. The corresponding sides and angles found are also true. However, the explanation at the end of the answer does not mention the right argument so that it has the potential to cause confusion for the reader. The following is SR's statement.

> "I use the side angle axiom to show that triangle ABE is congruent to triangle DEC. From there I found that angle E is congruent to angle C.

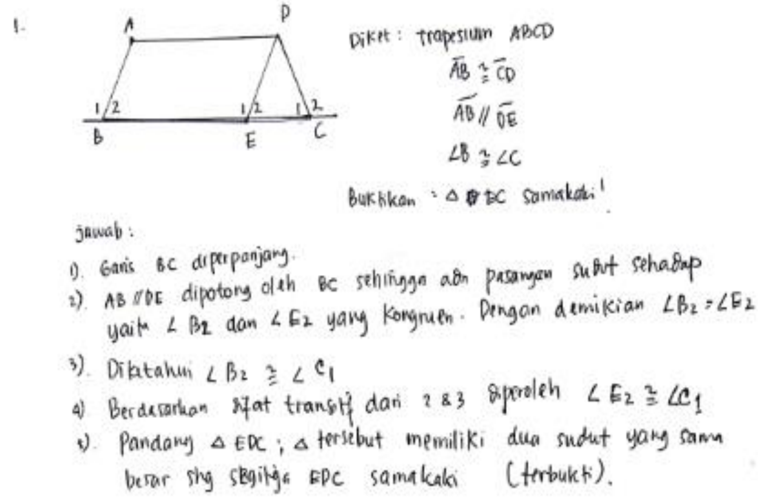
> When the two angles are congruent it is proven that triangle DEC is isosceles. But I find it difficult to describe my idea clearly. "

In question number 2 SR also did the same thing with the answer to the first question. He had pointed his answer to the right idea. He shows that triangle PAQ is congruent with triangle PBQ using the sides-side theorem so that it is proven that angle A is congruent with angle B. However, the writing of the argument is not clear, so it takes a lot of effort to understand the meaning. It is also seen in the result of the congruence of the PAQ triangle with the PBQ triangle, where it is not clear which is the angle P, whether the angle in question is the angle APQ or angle BPQ. Another mistake is when he writes that angle P is congruent with angle Q. How SR came up with his idea is revealed in the following interview results.

> "I solved the second problem using the side-by-side theorem, because AP is congruent to PB, PQ is congruent to PQ, and AQ is congruent to BQ. From there it is proven that angle A is congruent with angle B. But I made a mistake in writing that angle P is congruent with angle Q, because it has no reasoning."

Based on the completion and results of interviews with SR, it shows that this subject is able to carry out proof in the right direction. However, he did not present sentences of evidence that were less systematic so that they could not be understood by others. In writing the name of the angle, the subject is often less explicit, namely only writing the corner point. However, there are two angles at this point. In addition, there is an error in determining the corresponding angle. But when interviewed he realized his mistake. Students with non-proof constructions are rational arguments. This type of student has the correct perception of the steps of proof. However, the knowledge he has has not been able to achieve the correct verification process. This student realized that something was wrong with his answer, and he was able to correct the answer.

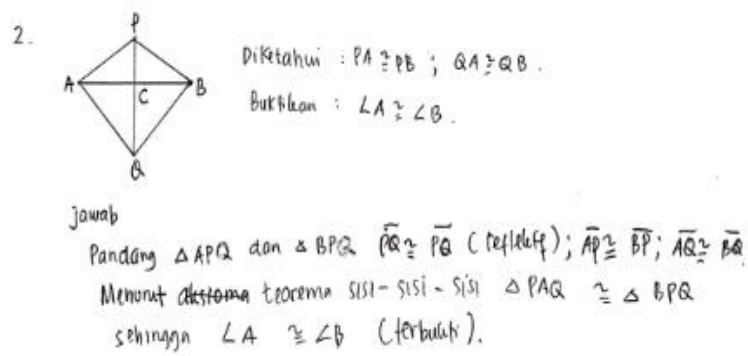
2.3 The proof constructed by SF



1.
Diket : Trapesium ABCD
$\overline{AB} \cong \overline{CD}$
$\overline{AB} \parallel \overline{DE}$
$\angle B \cong \angle C$
Buktikan : △ DEC samakaki!

jawab :
1). Garis BC diperpanjang.
2). AB // DE dipotong oleh BC sehingga ada pasangan sudut sehadap yaitu $\angle B_2$ dan $\angle E_2$ yang kongruen. Dengan demikian $\angle B_2 = \angle E_2$
3). Diketahui $\angle B_2 \cong \angle C_1$
4) Berdasarkan sifat transitif dari 2 & 3 diperoleh $\angle E_2 \cong \angle C_1$
5). Pandang △ EDC ; △ tersebut memiliki dua sudut yang sama besar shg segitiga EDC samakaki (terbukti).

Figure 5: *The proof constructed by SF on question number 1*



2.
Diketahui : $PA \cong PB$; $QA \cong QB$.
Buktikan : $\angle A \cong \angle B$.

Jawab
Pandang △APQ dan △BPQ $\overline{PQ} \cong \overline{PQ}$ (refleksif); $\overline{AP} \cong \overline{BP}$; $\overline{AQ} \cong \overline{BQ}$
Menurut ~~aksioma~~ teorema sisi-sisi-sisi △PAQ ≅ △BPQ
sehingga $\angle A \cong \angle B$ (terbukti).

Figure 5: *The proof constructed by SF on question number 2*

SF solves problem number 1 by seeing that there are two parallel lines AB and DE which are cut by the transverse BC so that a pair of opposite angles is formed, namely angles B2 and E2. Using the transitive property of the congruence relation he proved that the legs of the angle DEC are congruent. So that the triangle DEC is isosceles. SR wrote precisely with a clear explanation of the proof steps. The results of the interview about the problem solving process by SF below also show that he understands this material well.

> "In the known section it is stated that the line AB is parallel to the line DE. I saw the picture, it turns out that line AB intersects the two parallel lines. If two parallel lines are cut by a transversal, then the angles opposite the pliers are congruent. So angle B2 is congruent to angle E2. It is known that angle B2 is congruent to angle C1. Using transitive properties, it can be concluded that angle E2 is congruent with angle C1. If angle E2 is congruent with angle C1 according to the definition of a isosceles triangle, then triangle DEC is an isosceles triangle"

In question number 2, this subject writes down the completion steps briefly, but correctly. He showed that triangle APQ is congruent with triangle BPQ using the sides-by-side theorem. As a result it can be shown that angle A is congruent with angle B. Here is SR's explanation of how he came up with the idea to answer.

> "Initially I wanted to prove by showing that the two upper triangles are congruent so that the upper angle A and the upper angle B are congruent. In the same way I can do it on the bottom two triangles. But I revisited the problem and came up with an easier idea, which is to show that triangle PAQ is congruent to triangle PBQ using the sides-and-side theorem so that it can be proven that angle A is congruent with angle B."

Based on the results of the tests and interviews, it appears that SF is able to compose evidence sentences correctly. He can also and can choose the most effective way to solve the problem. This shows that SF is able to understand the material well, is able to present his ideas correctly and easily understood, has many ideas to answer a question, and is able to choose the most effective way. Students with construction of evidence in the form of formal argument found only two subject. The student's ability in constructing the evidence is good. This type of student has the right idea in writing proof arguments. He can think abstractly. It doesn't split the image into new images, but only adds lines in the image if needed. The idea is correct, the reason is correct, but the reasons used are not written in detail because he assumes that other people already understand what he means.

Discussion

In general, students have difficulty in constructing evidence. Because the proofing of geometry is new for them. At secondary and high school levels they are not taught formal proofing. So they need time and knowledge to adapt to the new thing. This is in accordance with the results of Oflaz, Bulut, & Akcakin (2016) research stated that pre service teachers have difficulty in doing the proofing. It can be seen from the representations made by students. This is understandable because formal proof is a complex material in education (Quaresma, 2019). So that teachers need to have good abilities in teaching proof, namely the ability to present proof with a detailed, logical, reasonable process and draw appropriate conclusions (Cirillo & Hummer, 2021).

Students represent arguments using the type of representation in the form of paragraphs. This is due to the way the example given by the lecturer in preparing the argument in the form of representation in paragraph form. So the students follow the example presented by the lecturer. This is in according to the opinion of Aydogdu & Baki (2011) and Machisi (2020), student pre service teachers will follow the way of the lecturers and the way also that he will use to teach. Although there are actually 4 types of representations that can be used to write evidence as Cirillo & Herbst's (2012) opinions there are 4 types of geometric proof representation: (1) two columns evidence; (2) evidence in the form of paragraphs; (c) tree-shaped evidence, and (4) flow-shaped evidence, but the lecturer uses only one type of representation, is a paragraph representation. Because lecturers want students to focus on the proofing process and not stuck on the choice of representation. Lecturers see that formal proofing is more important than the type of evidence representation. Because the lecturer hopes students can construct the evidence not vice versa.

Students with non-proof constructions in the form of empirical arguments show that they solve the problem given by drawing first. Images become their main reference. This makes sense because geometry in everyday life is common.The Students are less aware that geometry is actually abstract. This is in line with the opinions of previous researchers who suggest that geometry is related to abstract concepts, but empirically geometry can be found in this realm (Seah, 2015; Magajna, 2013). It appears that these students do not understand the concept of proof well so they are confused about what to write. If the concept is not well understood, then their ability to solve problems is also weak (Utami, Sa'dijah, Subanji, & Irawati, 2019). False perceptions make them trapped in empirical evidence. They only memorize the steps they have learned without understanding the meaning. Subanji (2013) states that such conditions are included in the category of learning that is less meaningful.

The visualization can be basically help students in learning geometry. But in the proofing of geometry, visualization is just the first step that can help students in abstracting. A more important step is the formal formulation of the argument to the conclusion. Because if students rely solely on visualization they will fail in proofing. They do not use formal proofing measures, and the arguments are randomly prepared. So visualization is needed, but visualization is only a bridge to the formal. This is in line with the opinion of Seah (2015) who stated that visualization is very important in helping children understand the shape and nature of geometry. Magajna (2013) stated that bad problems in spatial, poor and disorganized knowledge preclude the process of drawing deductive arguments. Proof of geometry becomes a difficult subject for students because it involves various components in constructing proof sentences. So it is necessary to develop a learning design that takes into account the complexity of the problem in proving (Miyazaki, Fujita, Jones, 2016).

Students with non-proof constructions in the form of rational arguments show that their ability to do proving is not yet complete. The results of their completion and interviews show that they already have the correct direction of proofing, but in the process of completion they are difficult to continue. The students have with non-proof construction of rational arguments already have a pretty good idea because their point of view leads to formal proofing. But they have not been able to develop the idea to construct the complete and correct evidence. There are parts that are wrong and they know that it is wrong, but they don't fix it because they still don't understand the material completely. This is consistent with the Tansili's (2016) opinion proved in geometry at school rather than simply validating the truth in the claim.

Students with construction of evidence in the form of formal argument found only two subject. The student's ability in constructing the evidence is good. The ideas found developed so that they could produce multiple correct methods. They are able to choose the most effective method to solve the problem. This shows their comprehensive understanding. Although many experts claim that the proof of geometry is not an easy thing. Stylianou & Knuts. Eds. (2010) stated that compiling a coherent mathematical argument and formulating valid evidence is a complex task for students because it requires a logical sequence. However, there are students who are able to do the proof correctly and systematically. Stylianou & Knuts (2010) stated that composing coherent mathematical arguments and formulating valid evidence is a complex task for students because it requires a logical sequence.

The data obtained show that the proof of geometry is a difficult material. Because only a small number of students are able to do the proof correctly. This is in accordance with the results of Musrikah's research (2019) which states that several things that cause geomatritic proofs to be difficult for students to understand are the many rules that must be understood and the preparation of proof sentences for which a dissertation must be made with proper reasons. The systematic preparation of proof sentences is the biggest problem for students in compiling proof sentences (Iftanti, Zahroh, Musrikah, 2020). So the lecturers also need to provide support to the students as stated by Roger & Steele (2016), five things that can support students' success in the task of reasoning and proofing: (1) time; (2) emphasizes explanations and meanings; (3) scaffolding; (4) student participation, and (5) tasks focused on the reasoning and proofing process.

The teachers or lecturers play an important role for the success of students in conducting the proofing of geometry. So the teachers need to consider the form of the evidentiary task as proposed by Roger & Steele (2016), three practices that it can be used in designing the task of proof are: (a) implementing tasks that promote reasoning and problem solving; (b) using and linking mathematical representations;) support the productive struggle in mathematics. The results also show that students' ability in constructing evidence is still weak. This is caused by many factors, among others: the process of lecturing proofing of geometry, formal proofing has not been encountered in the previous ladder. So this is required good design, planned, and appropriate planning in the teaching of reasoning and proofing. Bieda (2010) stated that teaching reasoning and proofing can build the students' understanding. Kospentaris, Vosniadou, & Thanou (2016) stated that analytical method can be a good choice in teaching geometry.

3. Conclusion

Based on the research results obtained, there are 3 types of arguments produced in the task of constructing the evidence are: non-proof in the form of an empirical argument, non-proof in the form of rational argument, and evidence of formal argument.Its characteristics as follows: (a) Students with non-proof construction in the form of argument empris to prove by visualizing first through the image and view empirically the picture, then states the conclusion based on empirical data; (b) Students with non-proof constructions in the form of rational arguments seek to prove formally, but their knowledge on notation, axioms, and theorems are very limited make it an obstacle for them to do proofing; (c) Students with construction of evidence in the form of formal arguments indicate that students are able to construct formal evidence, but in writing the evidence there is little difficulty. Based on the results of research appears that the construction of evidence is difficult to be mastered by students. So the next researcher can do research on appropriate geometry proofing strategy, development of geometry proofing teaching materials, or other research on proofing.

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MUSRIKAH: FACULTY OF MATHEMATICS AND SCIENCE UNİVERSİTAS NEGERİ MALANG, JL. SEMARANG NO.5, SUMBERSARI, KEC. LOWOKWARU, KOTA MALANG, JAWA TIMUR 65145, INDONESİA, e-mail: *musrikah.1703119@students.um.ac.id*. AND MATHEMATICS EDUCATION PROGRAM UNIVERSITAS SAYYID ALI RAHMATULLAH TULUNGAGUNG, JL. MAYOR SUDJADI TIMUR NO 26 TULUNGAGUNG, JAWA TIMUR 66221, INDONESIA.

CHOLIS SA'DIJAH: FACULTY OF MATHEMATICS AND SCIENCE UNİVERSİTAS NEGERİ MALANG, JL. SEMARANG NO.5, SUMBERSARI, KEC. LOWOKWARU, KOTA MALANG, JAWA TIMUR 65145, INDONESİA, E-mail: *cholis.sadijah.fmipa@um.ac.id*.

SWASONO RAHARDJO: FACULTY OF MATHEMATICS AND SCIENCE UNİVERSİTAS NEGERİ MALANG, JL. SEMARANG NO.5, SUMBERSARI, KEC. LOWOKWARU, KOTA MALANG, JAWA TIMUR 65145, INDONESİA, E-mail: *swasono.rahardjo.fmipa@um.ac.id*.

SUBANJI: FACULTY OF MATHEMATICS AND SCIENCE UNİVERSİTAS NEGERİ MALANG, JL. SEMARANG NO.5, SUMBERSARI, KEC. LOWOKWARU, KOTA MALANG, JAWA TIMUR 65145, INDONESİA. E-mail: subanji.fmipa@um.ac.id.