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COMPUTATIONAL APPROACH FOR TRANSIENT BEHAVIOUR OF FINITE SOURCE RETRIAL QUEUEING MODEL WITH MULTIPLE VACATIONS, BREAKDOWN AND REPAIR

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Abstract. In this work, we are analysing transient behaviour of finite source retrial with multiple vacation, repair- breakdown model in which the primary arrival rate λ which obeys Poisson distribution,service time µ obeys exponential distribution. We have assumed that the finite calling population size M. vacation time obeys exponential distribution with parameter . All transitions are assembled by using an infinitesimal generator matrix(IGM). Eigen Values and Eigen vectors applied to obtain Time dependent and Steady state solutions. Numerical studies obtained for time dependent mean no. of customers in orbit, transient prob. of server free/busy/vacation/ repair-breakdown for various values of M, λ, μ, p, $\sigma, \alpha, \beta, \gamma$ and t.

1. Introduction

our aim is to analyze transient retrial behaviour finite source retrial queueing model, multiple vacations, breakdown and repair using new computational approach. If the arriving customer who find server busy then go for invisible queue (orbit) retry for service after some random time again and again from orbit till to get service which is known as **Retrial queues**. Application of retrial queues in various areas such as call centres, LAN, Telephone systems, Retrial shopping queue etc. For example, in a call centre, if a customer makes a phone call when all the agents are busy, the customer will try to make phone call again after some random time.

Miller (1983) explained Matrix-geometric stochastic models solutions: An algorithmic approach, Retrial queues and its studies have been found in Falin (1990), Artalejo (2010), Considerable attention on finite source queues studies from various researchers such as Patrick Wüchnera, János Sztrikb and Hermann (2010) discussed Finite-source Retrial and Applications. Dragieva and Tuan (2020) studied a finite-source M/G/1 retrial queue with outgoing calls.

Transient analysis and its studies from Sudhesh, Azhagappan and Dharmaraja (2017) analysed Transient study of M/M/1 working vacation, heterogeneous service, customers impatience, Sherif Ammar(2017) studied transient Solution of

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Key words and phrases. Finite source queues - Multiple vacation - Retrial queue - Classical Retrial policy - Breakdown and Repair- Infinitesimal generator matrix - Eigen Values and Eigen vectors - Multiple Vacation.

M/M/1 Vacation Waiting Server, Impatient Customers, Indhumathi, Muthu Ganapathi Subramanian, Gopal Sekar(2021) analysed Computational approach for transient behaviour of finite source retrial queueing model with exhaustive type single vacation, loss and feedback.
Breakdown and repair studies from Muthu Ganapathi Subramanian, Ayyappan and Gopal Sekar (2011) M/M/c Retrial Queueing System , Breakdown, Repair , Srinivas Chakravarthy and Shruti Rakhee (2020) worked in queueing model, server breakdowns, repairs, vacations, backup server.

2. Mathematical Model and its solution

2.1. Model Description

Consider finite source retrial queuing system, multiple vacation, repair and breakdown in which the primary arrival rate λ obeys Poisson distribution, service time obeys exponential distribution with parameter µ. Further, we have assumed that the calling population is finite of size M.If server idle then primary arrival will served immediately and after completion of service, it leaves the system. An effective algorithm is used to obtain transient probabilities and time dependent system performance measures.

2.2 Retrial queues

If server is not idle then the arriving customer goes to orbit becomes a group of repeated customers. This repeated customers may viewed as sort of queue. it obeys Poisson process of repeated customers with intensity σ. If incoming recurring customers find server free, it served and leaves the system after service, while this source which produced repeated customer disappears.

2.3 Multiple Vacation

The concept of multiple vacations is incorporated in this work in such a way that after completion of a service if server finds no one in orbit then he goes for vacation. After completion of vacation period if server finds atleast one customer in orbit then he will be waiting for customer in the system, otherwise he goes for once again another vacation. it obeys exponential distribution with parameter α.

2.4 Breakdown and Repair

Breakdown of a server in any queueing model is an inevitable. We assume that the breakdown occurs only when server busy (Active breakdown). it obeys exponential distribution with parameter α. When ever breakdown occurs during service of a customer, the server goes to repair state and the customer who is in service will be send to orbit without completing his service. it obeys exponential distribution with parameter β.After completion of repair time, the server returns to the system and becomes idle and waiting for arrival of a customer.

2.5 Retrial Policy

The retrial policy states that the probability of an customers from an orbit to try for service during the time interval $(t,\ t + \Delta t)$ given that there were n customers in orbit at time t is n Δt + O (Δt). This regulation of access for server from retrial group is known as classical retrial policy.

3. REPRESENTATION OF RANDOM PROCESS

Let C(t) be random variable it represents no. of customers in orbit at t, S(t) be random variable it represents status of the server at t.
It is described as
$\{<N(t), C(t)> / N(t) = 1,2,3\ldots, M; C(t) = 0\} \bigcup \{<N(t), C(t)> / N(t) = 0,1,2,3\ldots, M-1; C(t) = 1\} \bigcup \{<N(t), C(t)> / N(t) = 0,1,2,3\ldots, M; C(t) = 2\} \bigcup \{<N(t), C(t)> / N(t) = 0,1,2,3\ldots, M; C(t) = 3\}$, where
C (t) = 0 if server free/idle at t,
C(t) = 1 if server busy at t,
C(t) = 2 if server in vacation at t &
C(t) = 3 if server in repair status at t.
We define, $P_{n0}(t)$: Prob. that server idle when there are n customers in orbit at t.
$P_{n1}(t)$: Prob. that server busy when there are n customers in orbit at t.
$P_{n2}(t)$: Prob. that server in vacation when there are n customers in orbit at t
$P_{n3}(t)$: Prob. that server in repair state when there are n customers in orbit at t

Chapman's balanced equations are given below

The server in idle:

$$\left.\begin{array}{l} \mathrm{P_{10}}'(t) = -[(M-1)\lambda + \sigma]\mathrm{P_{10}}(t) + \mu \mathrm{P_{11}}(t) + \alpha \mathrm{P_{12}}(t) + \gamma \mathrm{P_{13}}(t) \\ \mathrm{P_{20}}'(t) = -[(M-2)\lambda + 2\sigma]\mathrm{P_{20}}(t) + \mu \mathrm{P_{21}}(t) + \alpha \mathrm{P_{22}}(t) + \gamma \mathrm{P_{23}}(t) \\ \ldots. \\ \mathrm{P_{M0}}'(t) = -[M\sigma]\mathrm{P_{M0}}(t) + \alpha \mathrm{P_{M2}}(t) + \gamma \mathrm{P_{M3}}(t) \end{array}\right\} \quad (3.1)$$

The server in busy:

$$\left.\begin{array}{l} \mathrm{P_{01}}'(t) = -[(M-1)\lambda + \mu + \beta]\mathrm{P_{01}}(t) + \sigma \mathrm{P_{10}}(t) \\ \mathrm{P_{11}}'(t) = -[(M-2)\lambda + \mu + \beta]\mathrm{P_{11}}(t) + (M-1)\lambda \mathrm{P_{10}} \\ +(M-1)\lambda P_{01}(t) + 2\sigma \mathrm{P_{20}}(t) \\ \mathrm{P_{21}}'(t) = -[(M-3)\lambda + \mu + \beta]\mathrm{P_{21}}(t) + (M-2)\lambda \mathrm{P_{20}}(t) \\ +(M-2)\lambda P_{11}(t) \\ +3\sigma P_{30}(t) \\ \ldots \end{array}\right\} \quad (3.2)$$

The server in vacation:

$$\left.\begin{array}{l} \mathrm{P_{02}}'(t) = -[M\lambda]\mathrm{P_{02}}(t) + \mu \mathrm{P_{01}}(t) \\ \mathrm{P_{12}}'(t) = -[(M-1)\lambda + \alpha]\mathrm{P_{12}}(t) + M\lambda \mathrm{P_{02}}(t) \\ \mathrm{P_{22}}'(t) = -[(M-2)\lambda + \alpha]\mathrm{P_{22}}(t) + (M-1)\lambda \mathrm{P_{12}}(t) \\ \ldots \\ \mathrm{P_{M2}}'(t) = -\alpha \mathrm{P_{M2}}(t) + (M-(n-1))\lambda \mathrm{P_{M-12}}(t) \end{array}\right\} \quad (3.3)$$

The server in breakdown & repair:

$$\left.\begin{array}{l} \mathrm{P_{13}}'(t) = -[(M-1)\lambda + \gamma]\mathrm{P_{13}}(t) + \beta \mathrm{P_{01}}(t) \\ \mathrm{P_{23}}'(t) = -[(M-2)\lambda + \gamma]\mathrm{P_{23}}(t) + (M-1)\lambda \mathrm{P_{13}}(t) + \beta \mathrm{P_{11}}(t) \\ \ldots. \\ \mathrm{P_{M3}}'(t) = -\gamma \mathrm{P_{M3}}(t) + \lambda \mathrm{P_{M-13}}(t) + \beta \mathrm{P_{M-11}}(t) \end{array}\right\} \quad (3.4)$$

In general,

$$\left.\begin{array}{l}
\mathrm{P_{n0}}'(\mathrm{t}) = -[(\mathrm{M-n})\lambda + \mathrm{n}\sigma]\mathrm{P_{n0}(t)} + \mu\mathrm{P_{n1}(t)} + \alpha\mathrm{P_{n2}(t)} + \gamma\mathrm{P_{n3}(t)};\\
for\ \mathrm{n} = 1,2,...,\mathrm{M}-1\\
\mathrm{P_{n1}}'(\mathrm{t}) = -[(\mathrm{M}-(\mathrm{n}+1))\lambda + \mu + \beta]\mathrm{P_{n1}(t)} + (\mathrm{M-n})\lambda\mathrm{P_{n0}(t)}\\
+(M-n)\lambda P_{\mathrm{n-11}}(\mathrm{t}) + (\mathrm{n}+1)\sigma\mathrm{P_{n+10}(t)}; \mathrm{for\ n} = 1,2,...,\mathrm{M}-1\\
\mathrm{P_{n2}}'(\mathrm{t}) = -[(\mathrm{M-n})\lambda + \alpha]\mathrm{P_{n2}(t)} + (\mathrm{M}-(\mathrm{n}-1))\lambda\mathrm{P_{n-12}(t)};\\
for\ \mathrm{n} = 1,2,...,\mathrm{M}-1\\
\mathrm{P_{n3}}'(\mathrm{t}) = -[(\mathrm{M-n})\lambda + \gamma]\mathrm{P_{n3}(t)} + (\mathrm{M}-(\mathrm{n}-1))\lambda\mathrm{P_{n-13}(t)}\\
+\beta P_{\mathrm{n-11}}(\mathrm{t}); \mathrm{for\ n} = 1,2,...,\mathrm{M}-1
\end{array}\right\} \quad (3.5)$$

Infinitesimal generator matrix for this model is given below

$$R = \begin{pmatrix}
L_{00} & L_{01} & L_{02} & L_{03} & L_{04} & ...... & L_{0M}\\
L_{10} & L_{11} & L_{12} & L_{13} & L_{14} & ...... & L_{1M}\\
L_{20} & L_{21} & L_{22} & L_{23} & L_{24} & ...... & L_{2M}\\
L_{30} & L_{31} & L_{32} & L_{33} & L_{34} & ...... & L_{3M}\\
. & . & . & . & . & ...... & ......\\
L_{M-10} & L_{M-11} & L_{M-12} & L_{M-13} & L_{M-14} & .... & L_{M-1M}\\
L_{M0} & L_{M1} & L_{M2} & L_{M3} & L_{M4} & .... & L_{MM}
\end{pmatrix}$$

The matrices $L_{00}, L_{01}, L_{10}, L_{11}, L_{21}, L_{22}, ..., L_{M-1M-1}, L_{MM}$ are described in L. The infinitesimal transition rates of process $\boldsymbol{X}$ as follows

$$L_{00} = \begin{bmatrix} -((M-1)\lambda + \mu + \beta) & 0 \\ \mu & -M\lambda \end{bmatrix}$$

$$L_{01} = \begin{bmatrix} (M-1)\lambda & 0 \\ 0 & M\lambda \\ \beta & 0 \end{bmatrix}$$

$$L_{ii} = \begin{bmatrix} -(T\lambda + i\sigma) & \mu & \alpha & \gamma \\ -T\lambda & -(T-1)\lambda + \mu + \beta) & 0 & 0 \\ 0 & 0 & -(T\lambda + \alpha) & 0 \\ 0 & 0 & 0 & -(T\lambda + \gamma) \end{bmatrix},$$

where T = M-i;for i=1,2,...,M-1

$$L_{i\ i+1} = \begin{bmatrix} (M-(i+1))\lambda & 0 & 0 \\ 0 & (M-i)\lambda & 0 \\ \beta & 0 & (M-i)\lambda \end{bmatrix} for\ i = 1,2,...,M-1$$

$$L_{i\ i-1} = \begin{bmatrix} i\sigma & 0 \\ 0 & 0 \end{bmatrix} for\ i = 1,2,...,M-1$$

$$L_{MM} = \begin{bmatrix} -M\sigma & \alpha & \gamma \\ 0 & -\alpha & 0 \\ 0 & 0 & -\gamma \end{bmatrix}$$

Remaining all other entries are zero.

The equations (3.1) – (3.5) can be combined and expressed as

$X'(t) = LX(t),\ \ where\ L = \ R^T,$
$[X(t)]^T = [P_{01}(t)P_{02}(t)P_{10}(t)P_{11}(t)P_{12}(t)P_{13}(t)...P_{M0}(t)P_{M2}(t)P_{M3}(t)]$
Solving the equations, we get, $X(t) \ = \ e^{Lt}\ X_0$
When t = 0, $X_0 = \ X(0) \ = \ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 \end{bmatrix}^T.$

4. DESCRIPTION OF COMPUTATIONAL METHOD

An effective computational procedure is used to find Time dependent prob. of no. of customers in orbit at t.Time dependent Prob. is denoted and followed by
$X(t) = [P_{01}(t)P_{02}(t)\, P_{10}(t)P_{11}(t)P_{12}(t)P_{13}(t)...P_{M0}(t)P_{M2}(t)P_{M3}(t)]^T$
Step 1: Find Eigen values and Eigen vectors of finite order matrix tL.

Step 2: Let $d_0, d_1, d_2, ...d_M$ be (M+1) Eigen values and $\overline{c_0}, \overline{c_1}, \overline{c_2}..., \overline{c_M}$ be (M+1) Eigen vectors
Step 3: Represent this Eigen vectors as column vectors of a matrix
$C = (\overline{c_0}, \overline{c_1}, \overline{c_2}..., \overline{c_M})$

Step 4: Let $D = \begin{pmatrix} d_0 & 0 & 0 & . & . & 0 \\ 0 & d_1 & 0 & . & . & 0 \\ 0 & 0 & d_2 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & d_M \end{pmatrix}$

Step 5: Find the Exponential of the matrix L using D and C (i.e.,) $e^{tL} = CDC^{-1}$
Step 6: Extract first column of this Exponential matrix tL and store in X (t).
Step 7: This probability vector **X(t)** provides time dependent probabilities of no. of customers in orbit at t.

5. SYSTEM PERFORMANCE MEASURES

System measures are used to bring out Transient behaviour of finite retrial queueing model, breakdown,repair under Computational study for various values of M, λ, μ, p, $\sigma, \alpha, \beta, \gamma$, t are given below:

(1) Prob. that server idle at
t = $P_{idle}(t) = \sum_{n=1}^{M} P_{n0}(t)$
(2) Prob. that server busy at
t = $P_{busy}(t) = \sum_{n=0}^{M-1} P_{n1}(t)$
(3) Prob. that server in vacation at
t = $P_{vacation}(t) = \sum_{n=0}^{M} P_{n2}(t)$
(4) Prob. that server in breakdown,repair at
t = $P_{b\&r}(t) = \sum_{n=1}^{M} P_{n3}(t)$
(5) Mean no. of customers in orbit at
t = $L_q\ (t) = \sum_{n=1}^{M} n\,(P_{n0}(t) + P_{n2}(t) + P_{n3}(t)) + \sum_{n=1}^{M-1} nP_{n1}(t).$

6. Numerical Computations

Transient Probabilities and System Performance Measures, Transient prob. of this model have been done and expressed in Table forms for Various Values M, λ, μ, p, $\sigma, \alpha, \beta, \gamma$ and t. All numerical values given below are done by using SCILAB.

Table 1: Transient prob. distribution for no. of customers in orbit when server idle in the system for *M =3, λ =2, μ =5, α = 2, σ= 1, β=1,* $\gamma = 2$ and various values of*t*.

t	P10(t)	P20(t)	P30(t)
0.1	0.0578	0.0054	0.0001
0.2	0.1283	0.0260	0.0016
0.3	0.1692	0.0552	0.0058
0.4	0.1859	0.0854	0.0131
0.5	0.1887	0.1125	0.0228
0.6	0.1848	0.1352	0.0339
0.7	0.1783	0.1537	0.0453
0.8	0.1713	0.1686	0.0561
0.9	0.1647	0.1806	0.0657
1	0.1590	0.1904	0.0738

Table 2: Transient prob. distribution for no. of customers in orbit when server idle in the system for *M =5, λ =3, μ =6, α = 3, σ=2, β =2,*$\gamma = 3$ and various values of*t*.

t	P10(t)	P20(t)	P30(t)	P40(t)	P50(t)
0.1	0.0869	0.0324	0.0067	0.0007	0.0000
0.2	0.0923	0.0751	0.0340	0.0082	0.0008
0.3	0.0662	0.0868	0.0623	0.0240	0.0039
0.4	0.0446	0.0818	0.0805	0.0427	0.0097
0.5	0.0305	0.0722	0.0897	0.0599	0.0172
0.6	0.0218	0.0628	0.0933	0.0739	0.0251
0.7	0.0162	0.0549	0.0941	0.0846	0.0324
0.8	0.0127	0.0487	0.0936	0.0928	0.0385
0.9	0.0103	0.0439	0.0927	0.0989	0.0433
1	0.0086	0.0403	0.0917	0.1036	0.0470

Table 3: Transient prob. distribution for no. of customers in orbit when server idle in the system for *M =6, λ =4, μ =7, α = 4, σ= 3, β =3,*$\gamma = 4$ and various values of*t*.

t	P10(t)	P20(t)	P30(t)	P40(t)	P50(t)	P60(t)
0.1	0.0739	0.0498	0.0202	0.0049	0.0007	0.0000
0.2	0.0408	0.0614	0.0547	0.0298	0.0091	0.0012
0.3	0.0180	0.0440	0.0619	0.0534	0.0264	0.0058
0.4	0.0084	0.0288	0.0556	0.0653	0.0442	0.0134
0.5	0.0044	0.0193	0.0470	0.0691	0.0582	0.0220
0.6	0.0025	0.0135	0.0397	0.0693	0.0680	0.0296
0.7	0.0016	0.0101	0.0343	0.0681	0.0747	0.0357
0.8	0.0011	0.0080	0.0305	0.0667	0.0792	0.0403
0.9	0.0008	0.0067	0.0278	0.0656	0.0823	0.0435
1	0.0007	0.0058	0.0260	0.0647	0.0843	0.0457

From Table 1 to 3 show Transient prob. for no. of customers in orbit when server idle in system for various values ofM, λ, μ, p, $\sigma, \alpha, \beta, \gamma$ and t, We infer that as value of t increases then the Transient state goes to steady state (i.e.)$P_{idle}(t) \to P_{idle}$.

Table 4: Transient probability distribution of no. of customers in the orbit when the server is busy in the system for *M=3*, λ *=2*, μ *=5*, α *= 2*, σ*= 1*, β*=1*,$\gamma = 2$ and various values of$t$.

t	P01(t)	P11(t)	P21(t)
0.1	0.3696	0.1707	0.0187
0.2	0.1424	0.1676	0.0388
0.3	0.0621	0.1483	0.0520
0.4	0.0343	0.1383	0.0629
0.5	0.0245	0.1356	0.0743
0.6	0.0208	0.1358	0.0865
0.7	0.0191	0.1364	0.0989
0.8	0.0180	0.1368	0.1108
0.9	0.0172	0.1366	0.1216
1	0.0165	0.1361	0.1311

Table 5: Transient prob. distribution for no. of customers in orbit when server busy in system for *M=5*, λ *=3*, μ *=6*, α *= 3*, σ*=2*, β*=2*,$\gamma = 3$ and various values of$t$.

t	P01(t)	P11(t)	P21(t)	P31(t)	P41(t)
0.1	0.1404	0.2266	0.1158	0.0262	0.0022
0.2	0.0273	0.1389	0.1473	0.0695	0.0127
0.3	0.0102	0.0937	0.1467	0.1039	0.0292
0.4	0.0058	0.0673	0.1382	0.1302	0.0497
0.5	0.0038	0.0497	0.1260	0.1483	0.0719
0.6	0.0026	0.0378	0.1134	0.1592	0.0933
0.7	0.0019	0.0296	0.1021	0.1650	0.1123
0.8	0.0014	0.0241	0.0928	0.1677	0.1281
0.9	0.0011	0.0203	0.0855	0.1687	0.1407
1	0.0009	0.0176	0.0800	0.1689	0.1504

Table 6: Transient prob. distribution for no. of customers in orbit when server busy in system for *M=6*, λ *=4*, μ *=7*, α *= 4*, σ*= 3*, β*=3*,$\gamma = 4$ and various values of$t$.

t	P01(t)	P11(t)	P21(t)	P31(t)	P41(t)	P51(t)
0.1	0.0559	0.1772	0.1652	0.0765	0.0179	0.0017
0.2	0.0076	0.0735	0.1391	0.1331	0.0667	0.0139
0.3	0.0026	0.0354	0.0995	0.1445	0.1129	0.0378
0.4	0.0011	0.0184	0.0686	0.1346	0.1446	0.0683
0.5	0.0006	0.0103	0.0481	0.1185	0.1616	0.0987
0.6	0.0003	0.0064	0.0353	0.1036	0.1688	0.1248
0.7	0.0002	0.0043	0.0273	0.0920	0.1710	0.1451
0.8	0.0001	0.0031	0.0223	0.0836	0.1711	0.1601
0.9	0.0001	0.0025	0.0192	0.0777	0.1705	0.1706
1	0.0001	0.0020	0.0171	0.0737	0.1698	0.1779

From Table 4 to 6 show Transient prob. for no. of customers in orbit when server is busy for various values of $\mathrm{M}, \lambda, \mu, \mathrm{p}, \sigma, \alpha, \beta, \gamma$ and t, We infer that as value of t increases then Transient state goes to steady state (i.e.)$P_{busy}(t) \to P_{busy}$.

Table 7: Transient prob. distribution for no. of customers in orbit when server in vacation in the system for *M=3*, λ *=2*, μ *=5*, α *= 2*, σ*=1*, β*=1*,$\gamma = 2$ and various values of t.

t	P02(t)	P12(t)	P22(t)	P32(t)
0.1	0.2264	0.0724	0.0105	0.0006
0.2	0.2091	0.1413	0.0445	0.0053
0.3	0.1492	0.1571	0.0804	0.0159
0.4	0.0986	0.1404	0.1033	0.0300
0.5	0.0647	0.1125	0.1107	0.0442
0.6	0.0438	0.0854	0.1064	0.0560
0.7	0.0315	0.0633	0.0954	0.0642
0.8	0.0242	0.0470	0.0818	0.0686
0.9	0.0199	0.0356	0.0682	0.0697
1	0.0173	0.0278	0.0560	0.0683

Table 8: Transient prob. distribution for no. of customers in orbit when server in vacation in the system for M=5, λ =3, μ =6, α = 3, σ=2, β=2,$\gamma = 3$ and various values of$t$.

t	P02(t)	P12(t)	P22(t)	P32(t)	P42(t)	P52(t)
0.1	0.1061	0.0858	0.0386	0.0099	0.0014	0.0001
0.2	0.0404	0.0676	0.0677	0.0398	0.0127	0.0017
0.3	0.0136	0.0317	0.0518	0.0520	0.0289	0.0068
0.4	0.0053	0.0130	0.0291	0.0439	0.0378	0.0140
0.5	0.0026	0.0055	0.0143	0.0296	0.0370	0.0202
0.6	0.0015	0.0027	0.0068	0.0177	0.0306	0.0238
0.7	0.0010	0.0015	0.0034	0.0100	0.0227	0.0245
0.8	0.0007	0.0010	0.0018	0.0055	0.0157	0.0230
0.9	0.0006	0.0007	0.0011	0.0030	0.0104	0.0204
1	0.0004	0.0005	0.0007	0.0018	0.0068	0.0173

Table 9: Transient prob. distribution for no. of customers in orbit when server in vacation in the system for M=6, λ =4, μ =7, α=4, σ= 3, β=3,$\gamma = 4$ and various values of$t$.

t	P02(t)	P12(t)	P22(t)	P32(t)	P42(t)	P52(t)	P62(t)
0.1	0.0488	0.0635	0.0491	0.0234	0.0068	0.0011	0.0001
0.2	0.0084	0.0208	0.0362	0.0409	0.0287	0.0114	0.0019
0.3	0.0017	0.0046	0.0122	0.0240	0.0307	0.0226	0.0072
0.4	0.0005	0.0011	0.0033	0.0095	0.0194	0.0237	0.0128
0.5	0.0002	0.0004	0.0009	0.0032	0.0095	0.0181	0.0155
0.6	0.0001	0.0002	0.0003	0.0010	0.0041	0.0115	0.0151
0.7	0.0001	0.0001	0.0001	0.0004	0.0016	0.0065	0.0130
0.8	0.0000	0.0001	0.0001	0.0001	0.0006	0.0035	0.0103
0.9	0.0000	0.0000	0.0000	0.0001	0.0003	0.0018	0.0077
1	0.0000	0.0000	0.0000	0.0000	0.0001	0.0009	0.0056

From Table 7 to 9 show Transient prob. for no. of customers in orbit when server in vacation in the system for various values of M, λ, μ, p, $\sigma, \alpha, \beta, \gamma$, t, We infer that as value of t increases then Transient state goes to steady state (i.e.)$P_{vacation}(t) \to P_{vacation}$.

Table 10: Transient prob. distribution for no. of customers in orbit when server in breakdown, repair in the system for *M=3*, λ *=2*, μ *=5*, $\alpha = 2$, σ*= 1*, β*=1*,$\gamma = 2$ and various values of *t*.

t	P13(t)	P23(t)	P33(t)
0.1	0.0453	0.0202	0.0022
0.2	0.0418	0.0427	0.0105
0.3	0.0298	0.0533	0.0216
0.4	0.0197	0.0554	0.0329
0.5	0.0129	0.0536	0.0431
0.6	0.0088	0.0506	0.0520
0.7	0.0063	0.0475	0.0599
0.8	0.0048	0.0449	0.0669
0.9	0.0040	0.0428	0.0733
1	0.0035	0.0412	0.0791

Table 11: Transient prob. distribution for no. of customers in orbit when server in breakdown, repair in the system for *M=5*, λ *=3*, μ *=6*, $\alpha = 3$, σ*=2*, β*=2*,$\gamma = 3$ and various values of *t*.

t	P13(t)	P23(t)	P33(t)	P43(t)	P53(t)
0.1	0.0354	0.0512	0.0269	0.0062	0.0005
0.2	0.0135	0.0501	0.0613	0.0327	0.0066
0.3	0.0045	0.0330	0.0682	0.0615	0.0210
0.4	0.0018	0.0207	0.0616	0.0811	0.0413
0.5	0.0009	0.0136	0.0520	0.0911	0.0637
0.6	0.0005	0.0094	0.0434	0.0946	0.0858
0.7	0.0003	0.0069	0.0364	0.0941	0.1060
0.8	0.0002	0.0053	0.0311	0.0918	0.1235
0.9	0.0002	0.0043	0.0271	0.0887	0.1382
1	0.0001	0.0036	0.0241	0.0855	0.1501

Table 12: Transient prob. distribution for no. of customers in orbit when server in breakdown, repair in the system for *M=6*, λ *=4*, μ *=7*, $\alpha = 4$, σ*= 3*, β*=3*,$\gamma = 4$ and various values of *t*.

t	P13(t)	P23(t)	P33(t)	P43(t)	P53(t)	P63(t)
0.1	0.0209	0.0548	0.0540	0.0263	0.0064	0.0006
0.2	0.0036	0.0269	0.0637	0.0737	0.0429	0.0101
0.3	0.0007	0.0108	0.0423	0.0827	0.0835	0.0347
0.4	0.0002	0.0048	0.0256	0.0715	0.1073	0.0688
0.5	0.0001	0.0024	0.0159	0.0569	0.1150	0.1041
0.6	0.0001	0.0014	0.0105	0.0449	0.1135	0.1356
0.7	0.0000	0.0009	0.0074	0.0363	0.1080	0.1611
0.8	0.0000	0.0006	0.0056	0.0305	0.1020	0.1805
0.9	0.0000	0.0004	0.0046	0.0266	0.0966	0.1946
1	0.0000	0.0004	0.0039	0.0240	0.0923	0.2047

From Table 10 to 12 show Transient prob. for no. of customers in orbit when server in vacation in the system for various values of $\mathrm{M}, \lambda, \mu, \mathrm{p}, \sigma, \alpha, \beta, \gamma, \mathrm{t}$.
We infer that the value of t increases then the Transient state goes to steady state (i.e.)$P_{b\&r}(t) \rightarrow P_{b\&r}$.

Table 13: Time dependent system measures for $M=3$, $\lambda=2$, $\mu=5$, $\alpha=2$, $\sigma=1$, $\beta=1$, $\gamma=2$ and various values of t

t	$\mathbf{P}_{idle}$(t)	$\mathbf{P}_{busy}$(t)	$\mathbf{P}_{vacation}$ (t)	$\mathbf{P}_{b\&r}$ (t)	$\mathbf{L}_q$(t)
0.1	0.0633	0.5591	0.3098	0.0678	1.5425
0.2	0.1560	0.3488	0.4002	0.0950	1.2551
0.3	0.2302	0.2624	0.4026	0.1047	1.1297
0.4	0.2843	0.2355	0.3722	0.1080	1.1074
0.5	0.3240	0.2344	0.3321	0.1096	1.1381
0.6	0.3539	0.2431	0.2916	0.1113	1.1896
0.7	0.3773	0.2545	0.2545	0.1137	1.2444
0.8	0.3960	0.2656	0.2217	0.1167	1.2942
0.9	0.4110	0.2754	0.1934	0.1201	1.3359
1	0.4232	0.2837	0.1694	0.1237	1.3690

Table 14: Time dependent system measures for $M=5$, $\lambda=3$, $\mu=6$, $\alpha=3$, $\sigma=2$, $\beta=2, \gamma=3$and various values of t

t	$\mathbf{P}_{idle}(\mathbf{t})$	$\mathbf{P}_{busy}(\mathbf{t})$	$\mathbf{P}_{vacation}$ $(\mathbf{t})$	$\mathbf{P}_{b\&r}$ $(\mathbf{t})$	$\mathbf{L}_q(\mathbf{t})$
0.1	0.1268	0.5112	0.2419	0.1202	1.6305
0.2	0.2104	0.3956	0.2299	0.1641	1.5061
0.3	0.2432	0.3837	0.1848	0.1883	1.8007
0.4	0.2593	0.3913	0.1430	0.2064	2.1047
0.5	0.2696	0.3998	0.1093	0.2213	2.3478
0.6	0.2769	0.4063	0.0831	0.2337	2.5307
0.7	0.2823	0.4109	0.0630	0.2437	2.6659
0.8	0.2862	0.4141	0.0478	0.2519	2.7652
0.9	0.2891	0.4163	0.0362	0.2584	2.8380
1	0.2912	0.4178	0.0275	0.2635	2.8912

Table 15: Time dependent system measures for *M=6*, λ *=4*, μ *=7*, α = *4*, σ= *3*, β*=3*,γ = 4 various values of t

t	$\mathbf{P}_{idle}(\mathbf{t})$	$\mathbf{P}_{busy}(\mathbf{t})$	$\mathbf{P}_{vacation}$ $(\mathbf{t})$	$\mathbf{P}_{b\&r}$ $(\mathbf{t})$	$\mathbf{L}_q(\mathbf{t})$
0.1	0.1496	0.4945	0.1928	0.1631	1.6005
0.2	0.1970	0.4339	0.1483	0.2208	2.1055
0.3	0.2094	0.4326	0.1032	0.2548	2.7037
0.4	0.2158	0.4356	0.0705	0.2782	3.1322
0.5	0.2199	0.4378	0.0478	0.2945	3.4233
0.6	0.2227	0.4391	0.0323	0.3058	3.6191
0.7	0.2246	0.4399	0.0218	0.3137	3.7503
0.8	0.2258	0.4403	0.0147	0.3191	3.8382
0.9	0.2267	0.4406	0.0100	0.3228	3.8970
1	0.2272	0.4407	0.0067	0.3253	3.9364

From Table 13 to 15 show Time dependent System performance measures for various values ofM, λ, μ, p, $\sigma, \alpha, \beta, \gamma$ and t.
We infer:

(a) $P_{busy}(t)$ increases as rate of arrival increases for all t.
(b) $P_{vacation}(t)$ decreases as rate of arrival increases for all t.
(c) $P_{b\&r}(t)$ increases as rate of arrival increases for all t.
(d) The value of t increases and for various values of M, λ, μ, p, $\sigma, \alpha, \beta, \gamma$ and t then the Transient state goes to steady state (i.e.)$P_{idle}(t) \rightarrow P_{idle}, P_{busy}(t) \rightarrow P_{busy},\ P_{vacation}(t) \rightarrow P_{vacation},$ $P_{b\&r}(t) \rightarrow P_{b\&r}, L_q(t) \rightarrow L_q$

7. SPECIAL CASE

As β,α, and $\gamma \rightarrow 0$, this model coincides with standard single server finite source queuing model.

8. CONCLUSION

A new computational approach used to evaluate transient solution of finite source retrial queueing model with multiple vacation, breakdown and repair by applying Eigen values and Eigen vectors method and infinitesimal generator matrix. Numerical studies were done in an elaborate manner to determine the transient behaviour of probability distributions and time dependent System performance measures.

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