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COST OPTIMIZATION IN PRESERVATION TECHNOLOGY FOR PRODUCTS IN STORAGE

M GOVINDHAN* AND M MUTHUKUMAR**

Abstract. Deterioration rate is assumed to be an uncontrolled variable, but the deterioration of the nature of the items can be controlled to a certain level by means of preservation techniques. Hence it is also important to study the impact of preservation methods on the inventory control process. It is generally seen that enterprises purchase more goods than can be held in their owned warehouses (OW) for many reasons, such as discounts on bulk purchasing, seasonality, higher ordering cost, etc., which force a retailer to purchase more quantities than needed or exceed the storage capacity. So in this situation the retailer has to purchase an extra warehouse named as rented warehouse (RW) to stock the extra quantity. In this paper, we have formulated a double warehouse inventory model for the deterioration item. The constant rate of deterioration is assumed to be under control by applying some preservation technology. Numerical illustrations and sensitivity analysis are provided.

♣ Note to author: Use 2000 Mathematics Subject Classification.

1. Introduction

Each organization stores items to maintain a long-term relationship with its customers. It is a very common phenomenon that the amount of stock exceeds the storage capacity of the owned warehouse, which is restricted to a certain level. In this situation, the organization hires a rented warehouse at more expensive holding cost. In literature, the concept of two-warehouse has first been introduced by Hartley (1976). Sarma (1987) is the first who has used the concept of limited storage capacity of owned warehouse in inventory control modelling. After that, many authors have studied the two-warehouse inventory problem. To reflect this phenomenon, Abad (1996) discussed a pricing and lot-sizing problem for a product with a variable rate of deterioration, allowing shortages and partial backlogging. The backlogging rate depends on the time to replenishment the longer customers must wait, the greater the fraction of lost sales. The demand rate was assumed to be constant. Subsequently, the ideas of time-varying demand and stock-dependent demand were considered by some authors, such as Goswami and Chaudhuri (1998). The concept of inflation has first been studied by Buzacott (1975). After that, many authors have extended the work of Buzacott. Wee et al. (2005) and Yang (2006) focused on two-warehouse problem with partially

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backlogged shortages under the effect of inflation. Singh and Rathore (2014) have studied the effect of preservation technology for the deteriorating items under the effect of inflation. Leea and Dye (2012) have developed a inventory model for deteriorating items with controllable deterioration rate using preservation techniques and stock dependent demand rate. Mishra (2013) has developed an inventory model for instantaneous deteriorating items with controllable deterioration rate using preservation techniques and demand and holding cost both are time dependent functions.Shaikh et al.(2019) have been discussed investigated preservation related inventory models for deteriorating item. Chandra Das et.al (2020) have been discussed An application of preservation technology in inventory control system with price dependent demand and partial backlogging.In the present article we have formulated a double warehouse inventory model for the deterioration item. The constant rate of deterioration is assumed to be under control by applying some preservation technology.In this section, we have focusing on the constant rate of deterioration. The mathematical model solution is stated in the following section. Numerical illustration and sensitivity analysis are provided.

2. Mathematical Model Formulations

In this section, we will discuss the backlogged of previous period the inventory level at time t = 0 is S out of which w_2 units are stored in OW and remaining $S - W_2$ units are stored in RW. Inventory level of RW decrease due to demand and deterioration during the time interval $[0, t_1]$ and reach at zero level when time $t = t_1$. After time t_1 demand of items is fulfilled by using inventory of OW during time interval $[t_1, t_2]$, at time period $[t_2, T]$ shortages will occur and are partially backlogged. The fig (1) shows the description of this model.

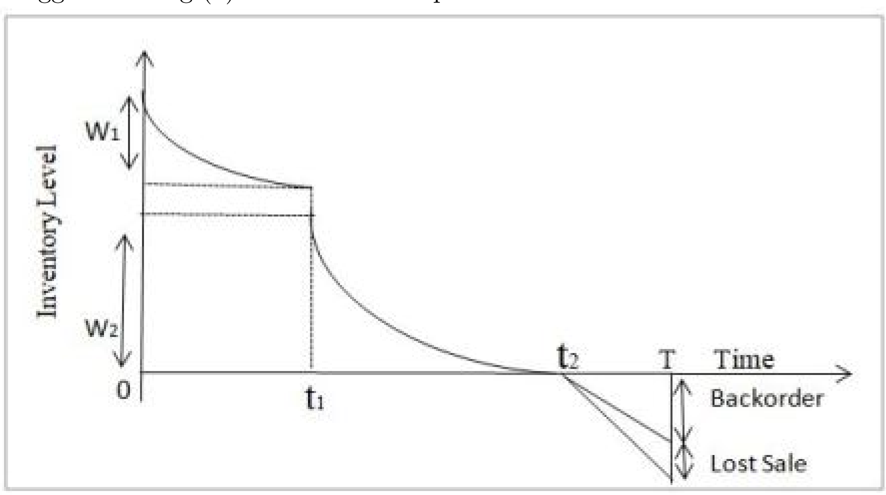

Fig.1 The functioning in two warehouse inventory Level.

Under the boundary conditions

$$I_1(t = 0) = w_1, \quad I_1(t = t_1) = 0,$$

$$I_2(t = 0) = w_2,$$

$$I_2(t = t_1) = w_2 = I_3(t = t_1)$$
$$I_3(t = t_d) = I_4(t = t_d),$$
$$I_4(t = t_2) = 0,$$
$$I_5(t = t_2) = 0$$

Therefore,

$$I_1(t) = \frac{\alpha}{\lambda + \tau\theta}\left(e^{(\lambda+\tau\theta)(t_1-t)} - 1\right) \tag{1}$$
$$I_2(t) = w_2 e^{-t\tau\theta} \tag{2}$$
$$I_3(t) = \frac{\alpha}{\lambda + \tau\theta}\left(e^{(\lambda+\tau\theta)(t_2-t)} - 1\right) \tag{3}$$
$$I_4(t) = -\alpha\,(t - t_2) \tag{4}$$
$$w_1 = \frac{\alpha}{\lambda + \tau\theta}\left(e^{(\lambda+\tau\theta)t_1} - 1\right) \tag{5}$$

Since

$$I_1\,(t = t_1) = I_3\,(t = t_1)$$

Therefore,

$$t_2 = t_1 + \left(\frac{1}{\tau\theta + \lambda}\right) log\left(w_2 e^{t_1\tau\theta} + \frac{\alpha}{\tau\theta + \lambda}\right)$$

On differentiating (1), (2), (3), (4) and (5) w.r.t ‘t’ we get

$$\frac{dI_1\,(t)}{dt} = -\frac{\alpha}{(\tau\theta + \lambda)}\frac{e^{(\tau\theta+\lambda)(t_1-t)}}{(\tau\theta + \lambda)}$$
$$= -\frac{\alpha e^{(\tau\theta+\lambda)(t_1-t)}}{(\tau\theta + \lambda)^2}$$
$$\frac{dI_2\,(t)}{dt} = -\frac{w_2 e^{-t\tau\theta}}{\tau\theta}$$
$$\frac{dI_3\,(t)}{dt} = \frac{-\alpha}{(\lambda + \tau\theta)}\frac{e^{(\lambda+\tau\theta)(t_2-t)}}{(\lambda + \tau\theta)}$$
$$= \frac{-\alpha}{(\lambda + \tau\theta)^2}e^{(\lambda+\tau\theta)(t_2-t)}$$
$$\frac{dI_4\,(t)}{dt} = -\alpha$$

Also, the order quantity

$$Q = w_1 + w_2 + B$$
$$Q = \frac{\alpha}{(\lambda + \tau\theta)}\left(e^{(\lambda+\tau\theta)t_1} - 1\right) + w_2 + \alpha\,(T - t_2)$$

Where

$$\alpha\,(T - t_2) = -I_4\,(t = T) = B,$$

is the maximum backlogged amount

The total relevant cost can has the following cost parameters.

(i). The Ordering Cost =A

(6)

(ii). The Purchase Cost = CQ

$$\tag{7}$$

(iii). The present worth of Shortage Cost (SC) is given by

$$SC = C_S\left(-\int_{t_1}^{T} I_4\left(t\right)e^{-rt}dt\right)$$

$$= C_S\left(-\int_{t_1}^{T} -\alpha\left(t-t_2\right)e^{-rt}dt\right)$$

$$= C_S\left(\alpha\int_{t_1}^{T} te^{-rt}dt - \alpha t_2\int_{t_1}^{T} te^{-rt}dt\right)$$

$$= C_S\left(\alpha\left(\frac{te^{-rt}}{r} - \frac{e^{-rt}}{r^2}\right)_{t_1}^{T} - \alpha t_2\left(\frac{e^{-rt}}{-r}\right)_{t_1}^{T}\right)$$

$$= C_S\left(\alpha\left(\frac{rTe^{-rT} - rt_1e^{-rt_1} + e^{-rt_1} - e^{-rT}}{r^2}\right) + \frac{\alpha t_2}{r}\left(e^{-rT} - e^{-rt_1}\right)\right)$$

$$= C_S\left(\frac{\alpha}{r^2}\left(e^{-rT}(rT-1) - e^{-rt_1}(rt_1-1)\right) + \frac{\alpha t_2}{r}\left(e^{-rT} - e^{-rt_1}\right)\right) \tag{8}$$

The present worth of lost sale cost (LC)

$$LC = C_L\left(\alpha\int_{t_2}^{T}\left(1-\alpha\right)e^{-rt}dt\right)$$

$$= C_L\left(\alpha\int_{t_2}^{T} e^{-rt}dt - \alpha^2\int_{t_2}^{T} e^{-rt}dt\right)$$

$$= C_L\left(\alpha\left(\frac{e^{-rt}}{-r}\right)_{t_2}^{T} - \alpha^2\left(\frac{e^{-rt}}{-r}\right)_{t_2}^{T}\right)$$

$$= C_L\left(\frac{-\alpha}{r}\left(e^{-rT} - e^{-rt_2}\right) + \frac{\alpha^2}{r}\left(e^{-rT} - e^{-rt_2}\right)\right)$$

$$= C_L\left(\frac{\alpha(\alpha-1)}{r}\left(e^{-rT} - e^{-rt_2}\right)\right) \tag{9}$$

The present worth of holding cost (HC)

$$HC = h_1\left(\int_{0}^{t_1} I_1(t)e^{-rt}dt\right) + h_2\left(\int_{0}^{t_1} I_2(t)e^{-rt}dt + \int_{t_1}^{t_2} I_3(t)e^{-rt}dt\right)$$

$$= h_1 \int_0^{t_1} \frac{\alpha}{(\lambda+\tau\theta)} \left(e^{(\lambda+\tau\theta)(t_1-t)} - 1\right) e^{-rt} dt + h_2 \int_0^{t_1} \left(w_2 e^{t\tau\theta} - 1\right) e^{-rt} dt$$

$$+ h_2 \int_{t_1}^{t_2} \frac{\alpha}{(\lambda+\tau\theta)} \left(e^{(\lambda+\tau\theta)(t_2-t)} - 1\right) e^{-rt} dt$$

$$= \frac{-\alpha}{(\lambda+\tau\theta)(\lambda+\tau\theta+r)} h_1 \left(e^{-rt_1} - e^{(\lambda+\tau\theta)t_1}\right)$$
$$+ \frac{\alpha}{r(\lambda+\tau\theta)} h_1 \left(e^{-rt_1} - 1\right) + w_2 h_2 \left(\frac{1-e^{-(r+\tau\theta)t_1}}{(r+\tau\theta)}\right)$$
$$+ \frac{\alpha}{(\lambda+\tau\theta)} h_2 \left(\frac{e^{(\lambda+\tau\theta)t_2 - t_1(\lambda+\tau\theta+r)} - e^{(\lambda+\tau\theta)t_2 - t_2(\lambda+\tau\theta+r)}}{-(\lambda+\tau\theta+r)} + \frac{e^{-rt_2} - e^{-rt_1}}{r}\right) \tag{10}$$

The total relevant cost can be obtained as

$$TRC(t_1, t_2, T) = \left(\frac{A + HC + PC + SC + LC}{T}\right) \tag{11}$$

On substituting the equations (6), (7), (8), (9) and (10) in equation (11), we get the value of present worth of total relevant cost (TRC). In order to minimize equation (11), we differentiate w.r.t the parameter t_1, t_2, T and equate to zero, we get

$$\frac{\partial TRC}{\partial t_1} = 0, \quad \frac{\partial TRC}{\partial t_2} = 0, \quad \frac{\partial TRC}{\partial T} = 0,$$

And the Hessian matrix of TRC is as follows

$$\begin{bmatrix} \frac{\partial^2 TRC}{\partial {t_1}^2} & \frac{\partial^2 TRC}{\partial t_1 \partial t_2} & \frac{\partial^2 TRC}{\partial t_1 \partial T} \\ \frac{\partial^2 TRC}{\partial t_2 \partial t_1} & \frac{\partial^2 TRC}{\partial t_2^2} & \frac{\partial^2 TRC}{\partial t_2 \partial T} \\ \frac{\partial^2 TRC}{\partial T \partial t_1} & \frac{\partial^2 TRC}{\partial T \partial t_2} & \frac{\partial^2 TRC}{\partial T^2} \end{bmatrix}$$

Numerical Illustration For the illustration of proposed model we consider following inventory system in which values of different parameters in proper units as follows

If $A = 400, \alpha = 275, \beta = 25, C = 3, C_S = 1.5, C_L = 1.5, \theta = 0.05, \lambda = 0.2, r = 0.03, h_1 = 0.5, h_2 = 0.1, W_2 = 50, a = 3$ Using Mathematica 7 Mathematical software we get the optimal values of $t_1^* = 0.10275$, t_2^*= 1.0392, T* = 1.2433, Q* = 246.378, TRC* = 673.367

Sensitivity Analysis To test the sensitivity of this model we have performed a sensitivity analysis by varying values of some important parameters like demand parameters α, and β deterioration rate θ etc. the effect of change in parameters is shown in table 1.

Table 1 Two Warehouse Inventory Model

	t_1^*	t_2^*	T*	Q*	$TRC*$
Variations in α					
225	0.1068	1.0032	1.5462	235.301	651.245
235	0.1058	1.0058	1.5181	231.547	664.124
245	0.1048	1.0079	1.4864	228.943	675.219
Variations in β					
10	0.1174	1.3294	1.5297	324.861	516.946
15	0.1087	0.7312	1.5273	367.154	624.124
20	0.065	0.3467	1.5251	395.793	731.11
Variations in θ					
0.02	0.1023	1.0081	1.5834	246.654	679.477
0.04	0.1023	1.0098	1.5834	246.378	681.486
0.06	0.1023	1.0172	1.5834	246.217	683.983
Variations in λ					
0.15	0.1027	1.0094	1.5041	234.645	657.792
0.2	0.1016	1.0082	1.4915	233.598	653.135
0.25	0.0542	1.0071	1.4587	232.871	650.876
Variations in r					
0.01	0.1038	1.0056	1.4942	228.478	648.153
0.015	0.1146	1.0069	1.5024	236.462	683.162
0.02	0.1202	1.0073	1.5947	249.719	715.127
Variations in W					
30	0.1079	0.8341	1.5647	293.331	718.09
40	0.1092	1.2765	1.6712	198.245	553.245
50	0.1118	1.3912	1.7934	115.487	347.477

The keep observations of table 1 reveals following facts. Increasing in demand parameter α result in decreases in t_1^*, T^* while increasing in t_2^* TRC*, Q*. Increasing in demand parameter β result in decreases in t_1^*, t_2^*, T^* while increasing in $TRC*$, $Q*$. Increasing in deterioration rate θ result in decreases in $Q*$ while increasing in t_1^*, t_2^*, T^*, TRC^*. Increasing in demand parameter λ result in decreases in t_1^*, t_2^*, T^*, TRC^*, and Q^* . Increasing in inflation rate r result in decreases in T^*, TRC^* while increasing in t_1^*, t_2^*, Q^*. Increasing in W result in decreases in, $TRC*$, $Q*$ while increasing in t_1^*, t_2^*, T^*.

Conclusion In this study, we have studied the preservation technology investment to secure the items. It is essential to identify that up to what value of money they should invest for the preservation technology so that it keeps the total cost at its minimum value. For this purpose, we have obtained an optimal value of preservation technology cost. To make these models applicable in real life situations, we have considered the two-warehouse inventory system in which storage capacity of owned warehouse is limited where as there is an infinite storage space at rented warehouse. Finally, the sensitivity analysis is performed to sensitize the model and the optimal values of total cost, total cycle length, order quantity are calculated. Our study can be used in the case of fashionable products, cosmetics products etc.

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M GOVINDHAN: Department of Statistics, PSG College of Arts and Science, Coimbatore,India

Email address: govindhan@psgcas.ac.in

M MUTHUKUMAR: Department of Statistics, PSG College of Arts and Science, Coimbatore,India

Email address: muthukumar@psgcas.ac.in