# MATHEMATICAL MODELING OF DAMAGE OF A CYLINDRICALLY ISOTROPIC THICK PIPE UNDER A COMPLEX STRESS STATE

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**ABSTRACT:** The paper investigates the process of scattered destruction of a thick pipe with uniform pressure on the inner boundary of the pipe. The process of fracture of a cylindrically isotropic pipe under the action of internal pressure, assuming that the pipe material behind the fracture front completely loses its bearing capacity and the acting external pressure is transferred to a new boundary surface, which is a moving fracture front. The process of damaging is described by the kinetic equation. The problem was solved taking into account the residual strength of the pipe material behind the fracture front. The problem was solved taking into account the residual strength of the pipe material behind the fracture front.
**Keywords:** stress intensity, damageability, destruction front, strongly affects, fracture, fracture front, damage volume.

## 1.İntroduction

The specificity of calculating the strength of bodies in a non-uniform stress state is the difference in the time of destruction of its individual parts. Expansion of destroyed parts changes the interface between destroyed and non-destroyed parts. This movable boundary surface is called the fracture front and was first introduced by L.M. Kachanov [1]. As already noted, a similar situation occurs for an inhomogeneous stress state with a structure. Theories of strength are unsuitable for studying the destruction of such bodies, because the element of the material for which the strength criterion is satisfied is considered to be completely destroyed and completely lost its resistance to loading. It is impossible to describe the further behavior of such an element under continued loading within the framework of strength theories. Great opportunities here are opened by damage theories or theories of scattered destruction.

One of the ways to analyze the destruction of a body in an inhomogeneous stress state is the way based on the concept of a fracture front. In this case, in addition to the governing equations and the fracture criterion, additional assumptions are required that do not follow from the deformation and destruction of the model.

Since the stress levels at different points are different in a non-uniform stress state, the degree of damage to these points also differs accordingly. The equations that relate stresses to strains - the defining equations - at each point will be valid until the corresponding failure criterion is met for it. From this moment on, this particle of material is not able to fulfill its functional duty, bear a certain load and collapse. As a result, a redistribution of stresses occurs in the body, which subsequently leads to the destruction of a neighboring particle of the material. Over time, the destroyed part of the body increases until the entire structure loses its bearing capacity.

From this moment on, this particle of material is not able to fulfill its functional duty, bear a certain load and collapse. As a result, a redistribution of stresses occurs in the body, which subsequently leads to the destruction of a neighboring particle of the material. Over time, the destroyed part of the body increases until the entire structure loses its bearing capacity. In the future, this area of the body increases. The movement of the destruction front, which characterizes the increase in the destroyed area, occurs until the moment when the entire construction of the theory, the bearing capacity, completely fails. The period of time from $t_0$ to $t_p$ is called the stage of destruction propagation. Determining the point in time requires $t_p$ additional assumptions. So, for example, it is possible that the velocity of the destruction front should turn to infinity. However, such a condition is not always acceptable, because for some structures the speed of the destruction front during the entire stage of the propagation of destruction remains finite.

The equation of motion of the destruction front is determined by the kinetic equation.

As a basic model of a damaged body, the article takes the model [2], which considers destruction as a critical stage in material deformation. The convenience of using this theory is that the same operator characterizing the process of damage accumulation is included as a kinetic equation.

## 2. Materials and Methods

We take the criterion of the destruction in the form, also following the works [3]:

$$\left(1+M^*\right)\sigma_i = \sigma_0 \tag{1}$$

where $\sigma_i$ - stress intensity, which is for a thick one, that is in a complex stress state, has the form [4]:

$$\sigma_i = \sqrt{3}p\frac{a^2b^2}{b^2-a^2}\cdot\frac{1}{r^2} \tag{2}$$

Here $a$ and $b$ - respectively, the inner and outer radii of the pipe, $r$ - current pipe radius, $p$

– pressure on the inner surface of the pipe, which is created by the placeholder.

Initially, the pipe consists of a single unauthorized material. The maximum value of the stress intensity is reached on the inner surface of the pipe, where the most intense process of damage accumulation takes place. This process leads to the emergence of a destruction zone there at the moment of time $t_0$, which is determined on the basis of the criterion of destruction:

$$\left(1+M^*\right)\sigma_{i\,max} = \sigma_{no}\,. \tag{3}$$

designating $a/b = \beta_0$ and $\frac{\sigma_{no}}{\sqrt{3}p} = g$ and considering (5) in (6), we will get:

$$\int_0^{t_0} M(\tau)d\tau = g\left(1-\beta_0^2\right)-1 \tag{4}$$

Let us give an explicit form for the initial destruction time for two types of nuclei $M(t)$

$$M(t) = m; \qquad t_0 = \frac{1}{m}\left(g(1-\beta_0^2)-1\right) \tag{5}$$

$$M(t) = mt^{-\alpha}; 0<\alpha<1; \qquad t_0 = \left\{\frac{1-\alpha}{m}\left[g\left(1-\beta_0^2\right)-1\right]\right\}^{\frac{1}{1-\alpha}} \tag{6}$$

Further, the boundary of the destruction zone - the destruction front will move towards the outer surface of the pipe. The destruction zone itself is an annular zone. The material of this fracture zone retains its bearing capacity, but to a much lesser extent than the original material ahead of the fracture front. We will assume that at the fracture front, the pipe material sharply changes its instantaneous rheological characteristics; in this problem, the value of the shear modulus is $G$. Let's take for $G_1$ - shear modulus of the pipe material ahead of the fracture front, and $G_0$ - is a destruction front. Let's introduce the notation:

$$\chi = \frac{G_0}{G_1} \tag{7}$$

It's obvious that $\chi < 1$.

The stress state at an arbitrary moment of time is defined as for a two-layer pipe with different elastic characteristics.

For the intensity of stresses in the pipe area ahead of the fracture front, which will supply the index 1, according to the well-known formulas [4], we obtain, taking the material incompressible, we get:

$$\sigma_i^{(1)} = \sqrt{3}q\frac{k^2b^2}{b^2-k^2}\cdot\frac{1}{r^2} \tag{8}$$

Where $q$ - fracture front pressure, $\kappa$ – destruction front radius.

For the radial displacements of the points of the region behind and in front of the fracture front, provided the material is incompressible, again according to [3], we have:

$$u^{(1)} = \frac{1}{2G_1}\left(1+M^*\right)\frac{k^2b^2q}{b^2-k^2}\cdot\frac{1}{r^2}; \quad u^{(0)} = \frac{a^2k^2(p-q)}{2G_0(k^2-a^2)}\cdot\frac{1}{r} \tag{9}$$

From the condition of continuity of displacements at the fracture front

$$u^{(1)}\Big|_{r=k} = u^{(0)}\Big|_{r=k} \tag{10}$$

We get:

$$\frac{b^2k(t)q(t)}{b^2-k^2(t)} + \int_0^t M(t-\tau)\frac{b^2k^2(\tau)q(\tau)}{b^2-k^2(\tau)}\cdot\frac{1}{k(t)}d\tau = \frac{1}{\chi}\frac{a^2k(t)}{k^2(\tau)-a^2}\left(p-q(t)\right); \tag{11}$$

Introducing the dimensionless quantity $\beta(t)=k(t)/b$, with respect to this dimensionless radial coordinate of the fracture front, we obtain the following nonlinear integral equation:

$$\frac{\beta^2(t)\tilde{q}(t)}{1-\beta^2(t)} + \int_0^t M(t-\tau)\frac{\beta^2(\tau)\tilde{q}(\tau)}{1-\beta^2(\tau)}d\tau = \frac{1}{\chi}\frac{\beta_0^2\beta^2(t)(\tilde{p}-\tilde{q}(t))}{\beta^2(t)-\beta_0^2}. \tag{12}$$

The stress intensity in formula (9) in the dimensionless radial coordinate has the form [6]:

$$\tilde{\sigma}_i = \sqrt{3}\tilde{q}(t)\frac{\beta^2(\tau)}{1-\beta^2(t)}\frac{1}{\beta^2(t)} \tag{13}$$

The kinetic equation for the development of the destruction front was adopted as follows:

$$\frac{d\beta(t)}{dt} = \varphi(\varepsilon_i) \tag{14}$$

where $\varphi(\varepsilon_i)$ - strain rate functions, $\beta(t)$ – radius of the destruction front.

Stresses at a point in time $t$ causes elastic deformation $\tilde{\sigma}_i = \sigma_i / E$. Therefore, the total deformation at the moment of time $t$ consists of this deformation and deformation arising from the stresses acting up to the moment of time $t$ [5],

$$\varphi(\varepsilon_i) = \tilde{\sigma}_i + \int_0^t M(t-\tau)\tilde{\sigma}_i(\tau)d\tau \tag{15}$$

Then the expression obtained from (15), substituting into equation (14), we obtain,

$$\frac{d\beta(t)}{dt} = \tilde{\sigma}_i + \int_0^t M(t-\tau)\tilde{\sigma}_i(\tau)d\tau \,. \tag{16}$$

Taking into account formula (13) in equation (16), we obtain,

$$\frac{\beta^2(t)}{\sqrt{3}}\frac{d\beta(t)}{dt} = \frac{\tilde{q}(t)}{1-\beta^2(t)} + \int_0^t M(t-\tau)\frac{\tilde{q}(\tau)}{1-\beta^2(\tau)}\beta^2(\tau)d\tau \tag{17}$$

Thus, we have a system of two nonlinear integral equations (12), (17) relative to the radial coordinate $\beta(t)$ fracture front and pressure $\tilde{q}(t)$ on it. Note that if $t_0$ - time of initial destruction, that is, destruction of the inner surface of the pipe $\beta = \beta_0$, determined according to formula (6), so in the system (12), (17) at $\tau \le t_0$ should be assumed $\beta(\tau) = \beta_0;\ \tilde{q}(\tau) = \tilde{p}$. Integral equation (17) makes sense only for $t > t_0$.

Equations (12), (17) can be reduced to solving one nonlinear integral equation. For this, using the identity of the structure of the integral terms of equations (12) and (17), excluding them, we obtain the following explicit representation of the dependence of the pressure at the fracture front from its radial coordinate:

$$\tilde{q}(t) = \tilde{p} - \frac{\chi(\beta^2(t) - \beta_0^2)}{\sqrt{3}\beta_0^2}\frac{d\beta(t)}{dt} \tag{18}$$

Taking into account this representation by the integro-differential equation (17), we obtain the following nonlinear integral equation:

$$\frac{\beta^2(t)}{\sqrt{3}}\frac{d\beta(t)}{dt} = \frac{1}{1-\beta^2(t)}\left[\tilde{p} - \frac{\chi(\beta^2(t)-\beta_0^2)}{\sqrt{3}\beta_0^2}\frac{d\beta(t)}{dt}\right] + \int_0^t M(t-\tau)\frac{\beta^2(\tau)}{1-\beta^2(\tau)}\left\{\tilde{p} - \frac{\chi(\beta^2(\tau)-\beta_0^2)}{\sqrt{3}\beta_0^2}\frac{d\beta(\tau)}{d\tau}\right\}d\tau$$

(19)

### 3. Solution Method

The solution to equation (19) determines the nature of the expansion of the annular fracture zone $\beta = \beta(t)$. Further, using formula (20), the pressure at the destruction front is determined. It should be noted that the solution to the integral equation is valid as long as the pressure $q(t)$ calculated by formula (18) is positive. Its equality to zero or its negativity means violation of the material overshoot with the formation of an arc crack along the destruction front [7].

In order to clarify the qualitative picture of the destruction process, we will take as the kernel of the damage operator: $M(t - \tau) = m = const$, then, introducing dimensionless time $\theta = mt$ and $\eta = m\tau$, equation (19) takes the form:

$$\frac{\beta^2(\theta)}{\sqrt{3}}\frac{d\beta(\theta)}{d\theta}=\frac{1}{1-\beta^2(\theta)}\left[\tilde{p}-\frac{\chi(\beta^2(\theta)-\beta_0^2)}{\sqrt{2}\beta_0^2}\frac{d\beta(\theta)}{d\theta}\right]+\int_0^{\theta}\frac{\beta^2(\eta)}{1-\beta^2(\eta)}\left\{\tilde{p}-\frac{\chi(\beta^2(\eta)-\beta_0^2)}{\sqrt{2}\beta_0^2}\frac{d\beta(\eta)}{d\eta}\right\}d\eta \tag{20}$$

Differentiating by dimensionless time $\theta$, we obtain the following differential equation:

$$A(\beta)\frac{d^2\beta}{d\theta^2}+B(\beta)\left(\frac{d\beta}{d\theta}\right)^2+C(\beta)\frac{d\beta}{d\theta}=\frac{\beta^2}{1-\beta^2}\tilde{P}$$

$$A(\beta)=\frac{\beta^2\beta_0^2+\chi(\beta^2-\beta_0^2)}{\sqrt{2}\beta_0^2};\ B(\beta)=\frac{2\beta\left(\beta_0^2(1-\beta^2)^2+\chi\beta^2(1-\beta^2)+\chi(\beta^2-\beta_0^2)\right)}{\sqrt{2}\beta_0^2(1-\beta^2)^2}$$

$$C(\beta)=\frac{\chi\beta^2(\beta^2-\beta_0^2)(1-\beta^2)-2\sqrt{2}\tilde{P}\beta\beta_0^2}{\sqrt{2}\beta_0^2(1-\beta^2)^2} \tag{21}$$

The initial conditions for equation (21) are condition (6):

$$\beta\big|_{\theta=\theta_0}=\beta_0;\quad \theta_0=g\left(1-\beta_0^2\right)-1;\quad \frac{d\beta}{d\theta}\Big|_{\theta=\theta_0}=0 \tag{22}$$

The resulting Cauchy problem (21), (22) was solved numerically for the values of the input parameters: $\beta_0=0{,}5;\ \ g=3{,}9;4{,}7;5{,}9$ and $\chi=0;0{,}2;0{,}4;0{,}6$. Figure 1-2 shows the curves of the destruction front movement for three values of the residual strength parameter $\chi$ depending on the parameter $g$.

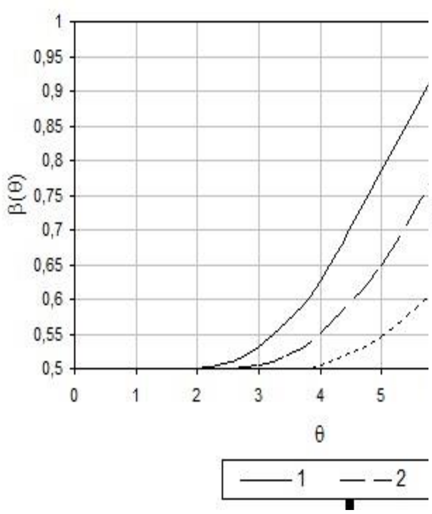


Fig. 1. Curves of the destruction f movement for the damage k $M(t-\tau)=m=const.$ For $\chi=0{,}01$: $g=3{,}9$, 2. $g=4{,}7$, 3. $g=5{,}9$.

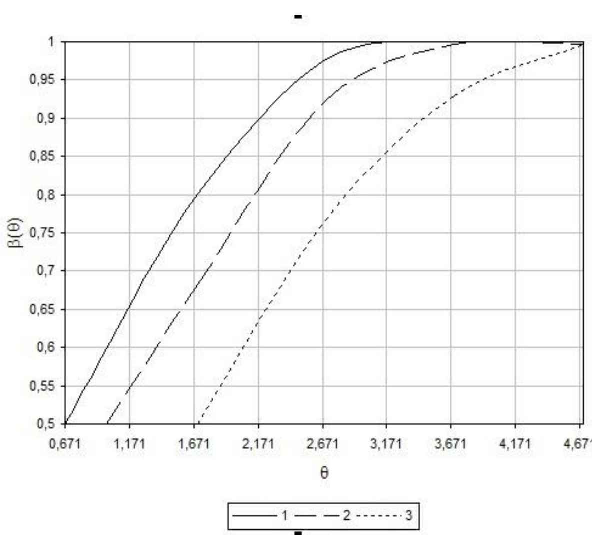


Fig. 2. Curves of the destruction f movement for the damage k $M(t-\tau)=(t-\tau)^{-\alpha}$. $\chi=0{,}2;\alpha=0{,}25$: 1. $g=3{,}9$, $g=4{,}7$, 3. $g=5{,}9$.

Figure 3-4 shows the curves of the destruction front movement for various values of the residual strength parameter depending on the parameter $g$ .

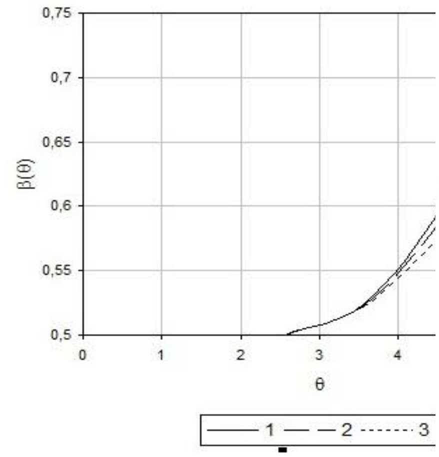


Fig. 3. Curves of the destruction f movement for the damage ke $M(t-\tau)=m=const.$ For $g=4{,}7$ : $\chi=0,$ 2. $\chi=0{,}2,$ 3. $\chi=0{,}6$ .

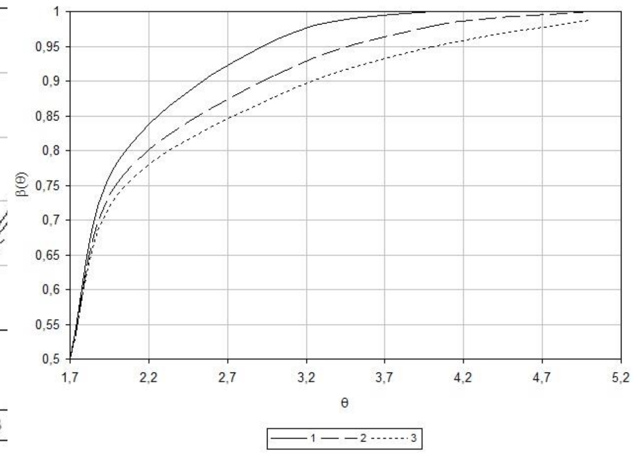


Fig. 4. Curves of the destruction front mover for the damage kernel $M(t-\tau)=(t-\tau)^{-\alpha}$ . For $g=5{,}9;\alpha=0{,}25:$ 1. $\chi=0{,}2,$ $\chi=0{,}4,$ 3. $\chi=0{,}6$

**4. Discussion of results**

As follows from the graph, the movement of the destruction front occurs with a decreasing speed. Calculations also showed that the presence of residual strength behind the destruction front has little effect on the nature of the destruction front movement, but it strongly affects the time of the onset of delamination [8].

As it turns out, an aggressive environment affects only the magnitude of the instantaneous strength. The damage process is described by an integral operator of hereditary type. The problem is solved taking into account the residual strength of the pipe material behind the fracture front.

Numerical calculations are carried out and dependences of the coordinates of the fracture front on time are constructed for various values of the concentration of an aggressive medium and residual strength behind the fracture front.

**5. Conclusions**

1. A kinetic equation is derived for the radial coordinate of the destruction front, taking into account the process of damage to the material of the pipe itself.

2. Explicit formulas are obtained for the contact pressures at the destruction front. An analysis of the relationship between critical situations of delamination at the destruction front and analysis of destruction due to the accumulation of a critical volume of damage are given.

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