SOLUTION TO CHALLENGES OF SELF-ASSESSMENT IN LEARNING USING SOFT FUZZY NUMBER VALUED INFORMATION SYSTEM

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ABSTRACT. Quantification of qualitative responses play an important role in survey analysis. The authors [2] introduced the concept of Soft fuzzy number valued information system $\widetilde{IS}$. In the present paper, we have formulated a model using $\widetilde{IS}$ for any self-assessment problem in the field of social science and a procedure is proposed to solve the same. The problem of measuring self-regulated learning at the individual level or at a group level are considered and studied in detail.

1. Introduction

L. A. Zadeh [16] in 1975 expounded linguistic variable and following Zadeh several mathematicians and statisticians interpreted linguistic concepts in various real life problems using different membership functions either linear or non-linear. In most of the questionnaire based survey analysis, the opinions, assessments, perception, etc., of the individuals or experts are linguistic in nature. In particular to express the situation more realistically, the linguistic responses in survey methods are identified as fuzzy numbers by several researchers [6, 7, 4]. The fuzzy hyperbolic inequality associated with single attribute was studied by N. Corral, M. A. Gil and H.L Garcia [11, 5] and the same for multi dimensional attributes is yet to be explored.

In this paper an approach through adequate parametrization of soft fuzzy numbers is considered in evaluating the fuzzy hyperbolic inequality index associated with hierarchical attributes. As a case study the various challenges of self assessment in learning with multi dimensional indicators is handled using $\widetilde{IS}$ and to obtain the solution a three step procedure is presented.

The paper is organized as follows : Section 2 recalls basic definitions related to fuzzy numbers and soft fuzzy numbers. Section 3 deals with the mathematical formulation and procedure to evaluate the fuzzy hyperbolic inequality index of any aspect associated with soft fuzzy numbers. Section 4, discusses in detail a secondary data to support the mathematical theory developed in Section 3. Section 5 gives the conclusion and future work.

2. Preliminaries

In this section for the sake of completeness, we consider Fuzzy numbers $\mathcal{F}(\mathbb{R})$ and Soft Fuzzy number valued information system $\widetilde{IS}$.

2000 *Mathematics Subject Classification.* Primary 03; Secondary E72.
Key words and phrases. Level of self -regulated learning; Soft fuzzy set-up of self-assessment problems.

Definition 2.1. [12] A fuzzy number denoted by $\tilde{A}$ was defined as a mapping $\tilde{A} : \mathbb{R} \to [0, 1]$ that is upper semi continuous, convex and normal. The collection of such fuzzy numbers is denoted $\mathcal{F}(\mathbb{R})$

Definition 2.2. [17] The α cut of a fuzzy number $\tilde{A}$ was given by
$[\tilde{A}]_\alpha = \{t : \mu_{\tilde{A}}(t) \geq \alpha\}$ for $0 < \alpha \leq 1$

Definition 2.3. [12] A fuzzy number $\tilde{A}$ is said to non-negative if $\tilde{A}(t) = 0$ for $t < 0$. The collection of all non-negative fuzzy number is denoted as $\mathcal{F}^*(\mathbb{R})$

For rest of the paper we consider only $\mathcal{F}^*(\mathbb{R})$

Definition 2.4. [8] For any two fuzzy numbers $\tilde{A}_1$, $\tilde{A}_2$ in $\mathcal{F}^*(\mathbb{R})$ with $[\tilde{A}_1]_\alpha = [a_1^\alpha, b_1^\alpha]$ and $[\tilde{A}_2]_\alpha = [a_2^\alpha, b_2^\alpha]$, the arithmetic operations $\oplus$ and $\oslash$, on collection of fuzzy numbers $\mathcal{F}^*(\mathbb{R})$ were expressed using resolution identity due to [17] as follows:
$\tilde{A}_1 \oplus \tilde{A}_2 = \cup_{\alpha\in[0,1]} \alpha[\tilde{A}_1 \oplus \tilde{A}_2]_\alpha$ where $[\tilde{A}_1 \oplus \tilde{A}_2]_\alpha = [a_1^\alpha + a_2^\alpha, b_1^\alpha + b_2^\alpha]$ and $\tilde{A}_1 \oslash \tilde{A}_2 = \cup_{\alpha\in(0,1]} \alpha[\tilde{A}_1 \oslash \tilde{A}_2]_\alpha$ where $\left[\tilde{A}_1 \oslash \tilde{A}_2\right]_\alpha = \left[\frac{a_1^\alpha}{b_2^\alpha}, \frac{b_1^\alpha}{a_2^\alpha}\right]$, if $a_2^\alpha > 0$. $\tilde{A}_1 \oplus \tilde{A}_2$ and $\tilde{A}_1 \oslash \tilde{A}_2 \in \mathcal{F}^*(\mathbb{R})$

Definition 2.5. [1] The scalar multiplication of any $\tilde{A} \in \mathcal{F}^*(\mathbb{R})$ by a non negative real number λ was defined as $\lambda(\tilde{A}) = \cup_{\alpha\in[0,1]} \alpha[\lambda(\tilde{A})]_\alpha$ with
$[\lambda(\tilde{A})]_\alpha = [\lambda a_1^\alpha, \lambda b_1^\alpha]$ and $\lambda(\tilde{A}) \in \mathcal{F}^*(\mathbb{R})$

In 2013, the concept of octagonal fuzzy number was introduced by Malini S.U and Felbin C. Kennedy [9] and used some special class of octagonal fuzzy numbers for solving real life problems [9, 3, 15].

Definition 2.6. A fuzzy number $\tilde{A}$ is said to be a *linear octagonal fuzzy number* denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k)$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8 \in \mathbb{R}$ with membership function $\tilde{A}(x)$ given by

$$\tilde{A}(x) = \begin{cases} k(\frac{x-a_1}{a_2-a_1}) & a_1 \leq x \leq a_2 \\ k & a_2 \leq x \leq a_3 \\ k + (1-k)(\dfrac{x-a_3}{a_4-a_3}) & a_3 \leq x \leq a_4 \\ 1 & a_4 \leq x \leq a_5 \\ k + (1-k)(\dfrac{a_6-x}{a_6-a_5}) & a_5 \leq x \leq a_6 \\ k & a_6 \leq x \leq a_7 \\ k(\dfrac{a_8-x}{a_8-a_7}) & a_7 \leq x \leq a_8 \\ 0 & \text{otherwise} \end{cases}$$

where $0 \leq k \leq 1$

Example: A linear octagonal fuzzy number would look like
The $\alpha-$ cut of a linear octagonal fuzzy number was computed as follows:

$$[\tilde{A}_\alpha] = \begin{cases} \left[a_1 + \frac{\alpha}{k}(a_2 - a_1), a_8 - \frac{\alpha}{k}(a_8 - a_7)\right] & \alpha \in [0, k] \\ \left[a_3 + \frac{\alpha-k}{1-k}(a_4 - a_3), a_6 - \frac{\alpha-1}{1-k}(a_6 - a_5)\right] & \alpha \in (k, 1] \end{cases}$$

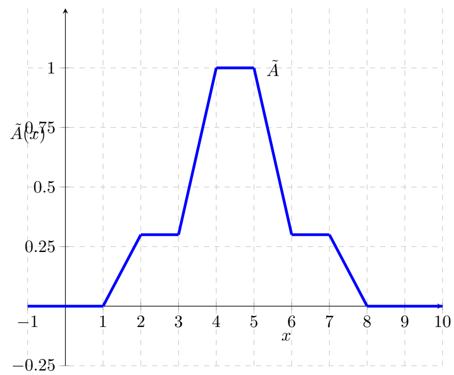

FIGURE 1. $\tilde{A} = (1, 2, 3, 4, 5, 6, 7, 8; 0.3)$

Definition 2.7. z[9] Let $\tilde{A}$ be an octagonal fuzzy number. The *measure* on $\tilde{A}$ is defined by $M^{Oct}(\tilde{A}) = \frac{1}{4}[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1 - k)]$

Remark 2.8. [9] Any two linear octagonal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are compared using the following:
1. $\tilde{A} \preceq \tilde{B} \iff M^{Oct}(\tilde{A}) \leq M^{Oct}(\tilde{B})$
2. $\tilde{A} \approx \tilde{B} \iff M^{Oct}(\tilde{A}) = M^{Oct}(\tilde{B})$
3. $\tilde{A} \succeq \tilde{B} \iff M^{Oct}(\tilde{A}) \geq M^{Oct}(\tilde{B})$

Remark 2.9. Linear octagonal fuzzy numbers yield better results for the choices of $k < 0.5$ ([**?**, 15]) in solving transportation and decision making problems. For our study (in a decision making problem) we consider linear octagonal fuzzy numbers with $k = 0.3$.

Remark 2.10. [9] Triangular fuzzy numbers and Trapezoidal fuzzy numbers can be obtained as a particular case of octagonal fuzzy numbers by considering the following:
- **•:** If $k = 0$ an octagonal fuzzy number reduces to a trapezoidal fuzzy number (a_3, a_4, a_5, a_6)
- **•:** If $k = 1$ it reduces to the trapezoidal fuzzy number (a_1, a_4, a_5, a_8)
- **•:** A degenerate form of an octagonal fuzzy number will be given by $\tilde{r} = (r, r, r, r, r, r, r, r)$

Definition 2.11. [2] A soft fuzzy number was defined as a mapping $\tilde{\tilde{f}} : E \to \mathcal{F}^*(\mathbb{R})$, E the parameter set. The collection of soft fuzzy numbers was denoted as $\tilde{\mathcal{F}}^*(\mathbb{R})(E)$.

Remark 2.12. If the fuzzy number associated with the parameter set E are trapezoidal, then the corresponding soft fuzzy number will be called as soft trapezoidal fuzzy number. Similarly if the fuzzy number associated with E are linear octagonal fuzzy number, then the soft fuzzy number will be called as soft linear octagonal fuzzy number.

Definition 2.13. [2] The sum of any two soft fuzzy numbers $(\tilde{\tilde{f}}, E), (\tilde{\tilde{g}}, E) \in \tilde{\mathcal{F}}^*(\mathbb{R})(E)$ was defined by the function

$$\tilde{\tilde{f}} \tilde{+} \tilde{\tilde{g}} : E \to \tilde{\mathcal{F}}^*(\mathbb{R})$$

where $(\tilde{\tilde{f}} \tilde{+} \tilde{\tilde{g}})(e) = \tilde{\tilde{f}}(e) \oplus \tilde{\tilde{g}}(e), \ \ e \in E$
and $\oplus$ represents the operation such as sum two fuzzy numbers respectively. The scalar

multiplication was defined by $\lambda(\tilde{\tilde{f}}, E) = \{\lambda\tilde{\tilde{f}}(e),\ e \in E\}$ any non-negative real number λ

Definition 2.14. [2] Let $(\tilde{\tilde{f}}, E) \in \tilde{\mathcal{F}}^*(\mathbb{R})(E)$ with $E = \{e_j\}_{j=1}^{l}$. A fuzzy number valued measure on $(\tilde{\tilde{f}}, E)$ was defined by
$\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{f}}, E)\right] = \sum_{j=1}^{l}\left[w_j\tilde{\tilde{f}}(e_j)\right]$ where $w_j \geq 0$ are weights of the parameters in E with $\sum_{j=1}^{l} w_j = 1$

Definition 2.15. [2] Let $\boldsymbol{M} : \mathcal{F}(\mathbb{R}) \to \mathbb{R}$, $\boldsymbol{M}(\tilde{A})$ denote the defuzzified value of a fuzzy number $\tilde{A} \in \mathcal{F}(\mathbb{R})$ based on any suitable defuzzification method under consideration. Then any two soft fuzzy numbers $(\tilde{\tilde{f}}, E), (\tilde{\tilde{g}}, E) \in \tilde{\mathcal{F}}^*(\mathbb{R}(E))$ were compared using the fuzzy number valued measure as follows:
$(\tilde{\tilde{f}}, E)\tilde{\prec}(\tilde{\approx}, \tilde{\preceq}\text{or}\tilde{\succ})(\tilde{\tilde{g}}, E)$ if $(\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{f}}, E)\right] \prec (\approx, \preceq \text{ or } \succ)\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{g}}, E)\right]$ and
$(\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{f}}, E)\right] \prec (\approx, \preceq \text{ or } \succ)\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{g}}, E)\right]$ if
$\boldsymbol{M}\left(\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{f}}, E)\right]\right) < (=, \leq \text{ or } >)\boldsymbol{M}\left(\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{g}}, E)\right]\right)$

Definition 2.16. [2] A soft fuzzy number valued information system is a quadruple
$\widetilde{IS} = (U, A, \tilde{\mathcal{F}}^*(\mathbb{R})(\mathcal{E}), \tilde{I})$ where
$U = \{u_i\}_{i=1}^{m}$ is the set of objects under consideration,
$A = \{a_j\}_{j=1}^{n}$ is the attribute set,
$\mathcal{E} = \{E_1, E_2, ..., E_n\}$ and $E_j = \{e_{jk}\}_{k=1}^{l_j}$ is the parameter set associated with attribute a_j, l_j representing the number of parameters in E_j and
if $\tilde{I} : U \times A \to \tilde{\mathcal{F}}^*(\mathbb{R})(\mathcal{E})$ is a mapping such that $\tilde{I}(u_i, a_j) = (\tilde{\tilde{f}}_{ij}, E_j) \in \tilde{\mathcal{F}}^*(\mathbb{R})(\mathcal{E})$ for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ where $(\tilde{\tilde{f}}_{ij}, E_j)$ is a soft fuzzy number.

Definition 2.17. [2] A soft fuzzy number valued hierarchical information system is a quintuple
$\widetilde{IS}_H = (U, A, H_A, \tilde{\mathcal{F}}^*(\mathbb{R})(\mathcal{E}_A), \tilde{I}_H)$ where $H_A = \{H_{a_j}/a_j \in A\}$, H_{a_j}, denote the concept hierarchy trzzee of attribute a_j for $j = 1, 2 \ldots, n$. $\mathcal{E}_A = \{\mathcal{E}_{a_j}\}_{j=1}^{n}$, with $\mathcal{E}_{a_j} = \{E_{js}\}_{s=1}^{l_j}$ the collection of parameter sets associated with l_j leaf nodes of the concept hierarchy tree and and $\tilde{I}_{a_j} : U \times A \to \tilde{\mathcal{F}}^*(\mathbb{R})(\mathcal{E}_A)$ is a function such that $\tilde{I}_{a_j}(u_i, a_j)$ consists of corresponding collection of soft fuzzy numbers in all levels of concept hierarchy tree.

3. Mathematical Model and Methodology

In this section we consider the problem of measuring any aspect $\mathscr{Q}$ (say) for an individual or a group of individuals, wherein the responses corresponding to several interactive indicators are captured using soft fuzzy numbers. Also the relative strength of the indicators among the individuals and the fuzzy hyperbolic inequality index of the aspect in such a scenario are measured. A mathematical formulation and procedure is developed to obtain the same.

Formulation :
Let U be a set of finite objects. Let $A = \{a_j\}_{j=1}^{n}$ be the set of attributes that are concerned in examining the inequality of the aspect of various sub-collections of U. Suppose we consider collection of s groups $\{U_t\}_{t=1}^{s}$ of size m_t (say), the qualitative responses of the individuals in U_t are captured as soft fuzzy numbers associated with A wherein the questions corresponding to the attributes are considered as parameters and modeled as collection of soft fuzzy number valued information systems $\widetilde{IS}/U_t$ (see definition 2.16), denoted $\widetilde{IS}_t = (U_t, A, \tilde{\mathcal{F}}^*(\mathbb{R})(\mathcal{E}), \tilde{I}_t)$ where $U_t = \{u_1^t, u_2^t, \ldots, u_{m_t}^t\}$ and $\tilde{I}/U_t$, denoted $\tilde{I}_t$. for $t = 1, 2, \ldots, s$.
Suppose sub-attributes are involved in the problem, then the situation is modeled as $\widetilde{IS}_{H_t}$ (see Definition **??**).
The problem consists of three parts:
Part A : Measure the value of the aspect of an individual $\mathscr{Q}_h^t$ in the t^{th} group and $\mathscr{Q}^t$ of a group.
Part B : Measure the relative strength of the attributes of an individual with respect to the group.
Part C : Measure fuzzy hyperbolic inequality $\widetilde{FH}^t$ of each group.

Definition 3.1. Fuzzy hyperbolic inequality $\widetilde{FH}$ of a group with respect to the aspect $\mathscr{Q}$ is measured by $\widetilde{FH} = \frac{\sum_h^m (\tilde{\boldsymbol{M}}_{avg}) \oslash \tilde{\boldsymbol{M}}_h}{m} - \tilde{1}$, where $\tilde{\boldsymbol{M}}_h$ and $\tilde{\boldsymbol{M}}_{avg}$ are the value of the aspect corresponding to each entity in a group and average value of the group ($\oslash$, See Definition 2.4).

We propose the following procedure to solve the problem.

- Step1. For each t perform the following steps to calculate the inequality index $\widetilde{FH}^t$.
  - Step1.1. If the information is represented as $\widetilde{IS}_t$ go to Step1.3
  - Step1.2. If the information is represented as $\widetilde{IS}_{H_t}$ perform the following:
    - Step1.2.1. Input the soft fuzzy number for each $u_h^t, h = 1, \ldots, m_t$ corresponding to each sub-attribute in the leaf nodes
    - Step1.2.2. Compute the fuzzy number valued measure for the inputs
    - Step1.2.3. Perform Step1.2.2 recursively back tracking till we reach $\widetilde{IS}_t$ in the form $(U_t, \tilde{\mathcal{F}}^*(\mathbb{R})(A), \tilde{I}_t))$ where $\mathcal{E} = A$, $\tilde{I}_t(u_h^t) = (\tilde{\tilde{f}}_h^t, A)$ and goto Step1.4
  - Step1.3. For each $a_j \in A$ obtain fuzzy number valued measure as follows:
    - Step1.3.1. Input the soft fuzzy number for each entity $u_h^t, h = 1, \ldots, m_t$
    - Step1.3.2. Determine $\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{f}}_{hj}^t, E_j)\right]$ and call it $\tilde{\boldsymbol{M}}_{hj}^t$ and goto Step1.4
  - Step1.4. Compute $\tilde{\boldsymbol{M}}\left[(\tilde{\tilde{f}}_h^t, A)\right]$, denoted $\tilde{\boldsymbol{M}}_h^t$
  - Step1.5. Compute the average level given by $\frac{\sum_{h=1}^{m_t} \tilde{\boldsymbol{M}}_h^t}{m_t}$, denote it $\tilde{\boldsymbol{M}}_{avg}^t$
- Step2. Compute fuzzy hyperbolic inequality index
  $\widetilde{FH}^t = \frac{\sum_h^{m_t} (\tilde{\boldsymbol{M}}_{avg}^t) \oslash \tilde{\boldsymbol{M}}_h^t}{m_t} - \tilde{1}$
- Step3. Compute $\boldsymbol{M}(\widetilde{FH}^t)$ to compare the degree of inequality among the groups.

Remark 3.2. Solution to part A of the problem is obtained by using any suitable defuzzification method given by $\mathscr{Q}_h^t = \boldsymbol{M}\left(\tilde{\boldsymbol{M}}_h^t\right)$ and $\mathscr{Q}^t = \boldsymbol{M}\left(\tilde{\boldsymbol{M}}_{avg}^t\right)$

Remark 3.3. Solution to part B of the problem is obtained by performing the following steps

step **I :** Perform the procedure till Step1.5 and continue the following

step **II :** Evaluate the mean $\frac{\sum_{h=1}^{m_t} \tilde{\boldsymbol{M}}_{hj}^t}{m_t}$ and call it $\overline{\boldsymbol{M}}_j^t$

step **III :** Using any suitable defuzzification method compute $\boldsymbol{M}\left(\tilde{\boldsymbol{M}}_{hj}^t\right)$ for each h and $\boldsymbol{M}\left(\overline{\boldsymbol{M}}_j^t\right)$ for each j

step **IV :** If $\boldsymbol{M}\left(\tilde{\boldsymbol{M}}_{hj}^t\right) < \boldsymbol{M}\left(\overline{\boldsymbol{M}}_j^t\right)$ then u_h^t needs to improve the attribute a_j

Remark 3.4. From step **III** we can identify those attributes for which the relative strength of u_h^t is less than that of the group average. One can work towards the improvement of these attributes.

Remark 3.5. If the information is modeled as $\widetilde{IS}_H$, step **II** to step **IV** can be performed for each sub-attribute involved, to identify those entities that needs furtherance.

4. A Case Study on Two Groups of School Students in a Soft Fuzzy Environment

Pintrich P.R., Smith D.A.F., Garcia T., and Mckeachie W. J. [13] defined self-regulated learning as motivational, cognitive and meta cognitive components of learning and developed a manual for use of motivated strategies for learning questionnaire (MSLQ). This is a standardized self-report questionnaire used to assess level of self-regulated learning of a person in learning a course. The pupil rate themselves to 81 questions on seven point scale starting from " not at all true of me" to "very true of me" as crisp numbers 1 to 7. We have taken the data from [10], wherein self-regulated learning was measured using the manual. We consider the same in a fuzzy set up.

According to the manual, self regulated learning are classified into two sections each with various components and related questions. The students self assess themselves based on 81 questions on seven point scale starting from " not at all true of me" to "very true of me" as crisp numbers 0 to 7.

We describe MSLQ in a soft fuzzy environment by considering the 7 point scale described through linguistic terms as Octagonal fuzzy numbers (See Definition 2.6) given in Table 1.

The self assessment given by the individuals to the questions corresponding to the various components (See Appendix) being modeled as hierarchical soft fuzzy number valued information system (see Definition 2.17). We present them as follows:

Here the two sections in the manual are considered as the attribute set given by $A = \{a_1, a_2\}$, $a_1 =$ Motivation scales; $a_2 =$ Learning strategies with $H_A = \{H_{a_1}, H_{a_2}\}$ its corresponding hierarchical tree consisting of the sub-characteristic features (value components) $\{a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}$ of a_1 and $\{a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}\}$ of a_2 where $a_{11} =$ Intrinsic goal orientation; $a_{12} =$ Extrinsic goal orientation; $a_{13} =$ Task value;
$a_{14} =$ Control of Learning Beliefs; $a_{15} =$ Self Efficacy for Learning and Performance;
$a_{16} =$ Test Anxiety; $a_{21} =$ Rehearsal; $a_{22} =$ Elaboration; $a_{23} =$ Organisation;

TABLE 1. Fuzzy numbers corresponding to linguistic terms

Linguistic terms	Linear octagonal fuzzy numbers
very true of me (VT)	(0.75, 0.80, 0.85, 0.90, 1.00, 1.00, 1.00, 1.00;0.3)
true of me (T)	(0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95;0.3)
slightly true of me (ST)	(0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85;0.3)
neutral (NE)	(0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70;0.3)
slightly not true of me (NST)	(0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55;0.3)
not true of me (NT)	(0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40;0.3)
not at all true of me (NVT)	(0, 0, 0, 0, 0.05, 0.10, 0.15, 0.20;0.3)

a_{24} = Critical Thinking; a_{25} = Meta cognitive Self-regulation; a_{26} = Time/ Study Environmental Management; a_{27} = Effort Regulation; a_{28} = Peer Learning; a_{29} = Help Seeking; and $\mathcal{E}_A = \{\mathcal{E}_{a_1}, \mathcal{E}_{a_2}\}$ the collection of parameter sets associated with leaf nodes, where $\mathcal{E}_{a_1} = \{E_{11}^{(1)}, E_{12}^{(1)}, E_{13}^{(1)}, E_{14}^{(1)}, E_{15}^{(1)}, E_{16}^{(1)}\}$,
$\mathcal{E}_{a_2} = \{E_{21}^{(1)}, E_{22}^{(1)}, E_{23}^{(1)}, E_{24}^{(1)}, E_{25}^{(1)}, E_{26}^{(1)}, E_{27}^{(1)}, E_{28}^{(1)}, E_{29}^{(1)}\}$
with $E_{11}^{(1)} = \{e_{11,1}, e_{11,2}, e_{11,3}, e_{11,4}\}$; $E_{12}^{(1)} = \{e_{12,1}, e_{12,2}, e_{12,3}, e_{12,4}\}$;
$E_{13}^{(1)} = \{e_{13,1}, e_{13,2}, e_{13,3}, e_{13,4}, e_{13,5}, e_{13,6}\}$; $E_{14}^{(1)} = \{e_{14,1}, e_{14,2}, e_{14,3}, e_{14,4}\}$;
$E_{15}^{(1)} = \{e_{15,1}, e_{15,2}, e_{15,3}, e_{15,4}, e_{15,5}, e_{15,6}, e_{15,7}, e_{15,8}\}$; $E_{16}^{(1)} = \{e_{16,1}, e_{16,2}, e_{16,3}, e_{16,4}, e_{16,5}\}$;
$E_{21}^{(1)} = \{e_{21,1}, e_{21,2}, e_{21,3}, e_{21,4}\}$; $E_{22}^{(1)} = \{e_{22,1}, e_{22,2}, e_{22,3}, e_{22,4}, e_{22,5}, e_{22,6}\}$;
$E_{23}^{(1)} = \{e_{23,1}, e_{23,2}, e_{23,3}, e_{23,4}\}$; $E_{24}^{(1)} = \{e_{24,1}, e_{24,2}, e_{24,3}, e_{24,4}, e_{24,5}\}$;
$E_{25}^{(1)} = \{e_{25,1}, e_{25,2}, e_{25,3}, e_{25,4}, e_{25,5}, e_{25,6}, e_{25,7}, e_{25,8}, e_{25,9}, e_{25,10}, e_{25,11}, e_{25,12}\}$;
$E_{26}^{(1)} = \{e_{26,1}, e_{26,2}, e_{26,3}, e_{26,4}, e_{26,5}, e_{26,6}, e_{26,7}, e_{26,8}\}$; $E_{27}^{(1)} = \{e_{27,1}, e_{27,2}, e_{27,3}, e_{27,4}\}$;
$E_{28}^{(1)} = \{e_{28,1}, e_{28,2}, e_{28,3}\}$; $E_{29}^{(1)} = \{e_{29,1}, e_{29,2}, e_{29,3}, e_{29,4}\}$; and
$e_{11,1} = Q_1; e_{11,2} = Q_{16}; e_{11,3} = Q_{22}; e_{11,4} = Q_{24}; e_{12,1} = Q_7; e_{12,2} = Q_{11};$
$e_{12,3} = Q_{13}; e_{12,4} = Q_{30}; e_{13,1} = Q_4; e_{13,2} = Q_{10}; e_{13,3} = Q_{17}; e_{13,4} = Q_{23};$
$e_{13,5} = Q_{26}; e_{13,6} = Q_{27}; e_{14,1} = Q_2; e_{14,2} = Q_9; e_{14,3} = Q_{18}; e_{14,4} = Q_{25};$
$e_{15,1} = Q_5; e_{15,2} = Q_6; e_{15,3} = Q_{12}; e_{15,4} = Q_{15}; e_{15,5} = Q_{20}; e_{15,6} = Q_{21};$
$e_{15,7} = Q_{29}; e_{15,8} = Q_{31}; e_{16,1} = Q_3; e_{16,2} = Q_8; e_{16,3} = Q_{14}; e_{16,4} = Q_{19};$
$e_{16,5} = Q_{28}; e_{21,1} = Q_{39}; e_{21,2} = Q_{46}; e_{21,3} = Q_{59}; e_{21,4} = Q_{72}; e_{22,1} = Q_{53};$
$e_{22,2} = Q_{62}; e_{22,3} = Q_{64}; e_{22,4} = Q_{67}; e_{22,5} = Q_{69}; e_{22,6} = Q_{81}; e_{23,1} = Q_{32};$
$e_{23,2} = Q_{42}; e_{23,3} = Q_{49}; e_{23,4} = Q_{63}; e_{24,1} = Q_{38}; e_{24,2} = Q_{47}; e_{24,3} = Q_1;$
$e_{24,4} = Q_{66}; e_{24,5} = Q_{71}; e_{25,1} = Q_{33}; e_{25,2} = Q_{36}; e_{25,3} = Q_{41}; e_{25,4} = Q_{44};$
$e_{25,5} = Q_{54}; e_{25,6} = Q_{55}; e_{25,7} = Q_{56}; e_{25,8} = Q_{57}; e_{25,9} = Q_{61}; e_{25,10} = Q_{76};$
$e_{25,11} = Q_{78}; e_{25,12} = Q_{79}; e_{26,1} = Q_{35}; e_{26,2} = Q_{43}; e_{26,3} = Q_{52}; e_{26,4} = Q_{65};$
$e_{26,5} = Q_{70}; e_{26,6} = Q_{73}; e_{26,7} = Q_{77}; e_{26,8} = Q_{80}; e_{27,1} = Q_{37}; e_{27,2} = Q_{48};$
$e_{27,3} = Q_{60}; e_{27,4} = Q_{74}; e_{28,1} = Q_{34}; e_{28,2} = Q_{45}; e_{28,3} = Q_{50}; e_{29,1} = Q_{40};$
$e_{29,2} = Q_{58}; e_{29,3} = Q_{68}; e_{29,4} = Q_{75};$

Problem Description :

The problem under consideration consists of the following three parts:

Part A : Determining the level of self regulated learning and that of various strategies for each individual in t groups (say).

Part B : Comparative study of self regulated learning at individual level with that of the group average.

Part C : Evaluation of fuzzy hyperbolic inequality index of self regulated learning level among the groups.

For our study we have considered 2 groups. The information obtained from the self assessment through MSLQ for a group of 65 students from 2 different schools (say) U_1 and U_2 each consisting of 33 and 32 students are considered as hierarchical soft fuzzy number valued information system $\{\widetilde{IS}_{H_t}\}_{t=1}^{2}$. The level of self regulated learning of each student and that of the group are obtained by executing the procedure given in section 3. Equal weights are assigned to the parameters and the attributes involved in the problem.

Solution :

A Matlab 2016a program is developed for the procedure. Here for each group the information is modeled as $\widetilde{IS}_{H_t}$. Hence the self assessment of the students at each sub-attributes are considered as soft fuzzy numbers and entered as input in matrix format (See Appendix).

Part A :

By performing the steps : Step1.2.1 to Step1.2.2 and Step1.4, $\tilde{\boldsymbol{M}}_h^t$ $\mathscr{Q}_h^t$ (using measure on linear octagonal fuzzy number)are computed. We infer the following:

The level of self regulated learning of each student in group 1 is tabulated in table 2.

Remark 4.1. Even though ranking of individuals are not considered in the manual, we note that one can rank the individuals based on the deffuzified values of their level of self regulated learning and also give the interpretation in percentage. For example: the self regulated learning in group 1 for the student with the highest rank is 89%

Remark 4.2. In working with the crisp data of group 1, it was identified that 9^{th}, 13^{th} and 28^{th} student had the same level of self regulated learning. Also 23^{rd} and 22^{nd} student have the same level. But with quantification of responses involving soft linear octagonal fuzzy numbers, we infer from table 2 we infer that each student's self regulated learning level is unique.

TABLE 2. Self Regulated learning of individual Students in group 1

u_h^1	$\tilde{\boldsymbol{M}}_h^1$	$\mathscr{Q}_h^1$	Rank
u_1^1	(0.5592, 0.6006, 0.6420, 0.6834, 0.7614, 0.7834, 0.8054, 0.8274;0.3)	0.7117	22
u_2^1	(0.5723, 0.6212, 0.6702, 0.7191, 0.7832, 0.8191, 0.8550, 0.8908;0.3)	0.7440	18
u_3^1	(0.5442, 0.5942, 0.6442, 0.6942, 0.7447, 0.7942, 0.8437, 0.8933;0.3)	0.7192	21
u_4^1	(0.5047, 0.5547, 0.6047, 0.6547, 0.7095, 0.7547, 0.8000, 0.8453;0.3)	0.6795	28
u_5^1	(0.6715, 0.7194, 0.7674, 0.8153, 0.8991, 0.9153, 0.9316, 0.9478;0.3)	0.8398	7
u_6^1	(0.6890, 0.7390, 0.7890, 0.8390, 0.9188, 0.9390, 0.9582, 0.9774;0.3)	0.8625	5
u_7^1	(0.5556, 0.6056, 0.6556, 0.7056, 0.7560, 0.8056, 0.8553, 0.9049;0.3)	0.7306	20
u_8^1	(0.4467, 0.4961, 0.5456, 0.5950, 0.6477, 0.6950, 0.7423, 0.7896;0.3)	0.6202	32
u_9^1	(0.5159, 0.5659, 0.6159, 0.6659, 0.7184, 0.7659, 0.8133, 0.8607;0.3)	0.6907	25
u_{10}^1	(0.6592, 0.7092, 0.7592, 0.8092, 0.8832, 0.9092, 0.9353, 0.9613;0.3)	0.8330	8
u_{11}^1	(0.5875, 0.6343, 0.6810, 0.7277, 0.7990, 0.8277, 0.8564, 0.8852;0.3)	0.7534	17
u_{12}^1	(0.5119, 0.5619, 0.6119, 0.6619, 0.7176, 0.7619, 0.8062, 0.8505;0.3)	0.6866	26
u_{13}^1	(0.5127, 0.5619, 0.6110, 0.6602, 0.7145, 0.7602, 0.8059, 0.8517;0.3)	0.6855	27
u_{14}^1	(0.6606, 0.7097, 0.7589, 0.8080, 0.8932, 0.9080, 0.9229, 0.9378;0.3)	0.8317	9
u_{15}^1	(0.3638, 0.4119, 0.4600, 0.5081, 0.5616, 0.6081, 0.6546, 0.7011;0.3)	0.5340	33
u_{16}^1	(0.5309, 0.5809, 0.6309, 0.6809, 0.7499, 0.7809, 0.8119, 0.8429;0.3)	0.7049	23
u_{17}^1	(0.4979, 0.5465, 0.5950, 0.6436, 0.7027, 0.7436, 0.7844, 0.8253;0.3)	0.6689	30
u_{18}^1	(0.6743, 0.7235, 0.7726, 0.8218, 0.9005, 0.9218, 0.9430, 0.9642;0.3)	0.8458	6
u_{19}^1	(0.5690, 0.6186, 0.6681, 0.7177, 0.7881, 0.8177, 0.8473, 0.8769;0.3)	0.7419	19
u_{20}^1	(0.6491, 0.6980, 0.7470, 0.7959, 0.8719, 0.8959, 0.9199, 0.9438;0.3)	0.8202	11
u_{21}^1	(0.5248, 0.5725, 0.6202, 0.6679, 0.7292, 0.7679, 0.8065, 0.8452;0.3)	0.6936	24
u_{22}^1	(0.6088, 0.6580, 0.7071, 0.7563, 0.8231, 0.8563, 0.8894, 0.9226;0.3)	0.7809	14
u_{23}^1	(0.6366, 0.6858, 0.7349, 0.7841, 0.8554, 0.8841, 0.9128, 0.9416;0.3)	0.8085	12
u_{24}^1	(0.6538, 0.7034, 0.7531, 0.8027, 0.8783, 0.9027, 0.9272, 0.9516;0.3)	0.8266	10
u_{25}^1	(0.6171, 0.6671, 0.7171, 0.7671, 0.8411, 0.8671, 0.8930, 0.9190;0.3)	0.7909	13
u_{26}^1	(0.7080, 0.7580, 0.8080, 0.8580, 0.9449, 0.9580, 0.9711, 0.9842;0.3)	0.8811	3
u_{27}^1	(0.4668, 0.5168, 0.5668, 0.6168, 0.6736, 0.7168, 0.7601, 0.8033;0.3)	0.6415	31
u_{28}^1	(0.5112, 0.5577, 0.6041, 0.6506, 0.7110, 0.7506, 0.7902, 0.8298;0.3)	0.6770	29
u_{29}^1	(0.6953, 0.7453, 0.7953, 0.8453, 0.9302, 0.9453, 0.9603, 0.9754;0.3)	0.8685	4
u_{30}^1	(0.7124, 0.7624, 0.8124, 0.8624, 0.9502, 0.9624, 0.9746, 0.9868;0.3)	0.8855	2
u_{31}^1	(0.7157, 0.7657, 0.8157, 0.8657, 0.9542, 0.9657, 0.9771, 0.9886;0.3)	0.8887	1
u_{32}^1	(0.6065, 0.6522, 0.6980, 0.7437, 0.8158, 0.8437, 0.8716, 0.8996;0.3)	0.7700	15
u_{33}^1	(0.5894, 0.6390, 0.6887, 0.7384, 0.7986, 0.8384, 0.8781, 0.9179;0.3)	0.7630	16

Part B :
Performing the steps : step **II** to step **IV**, the average level of motivation (a_1), level of learning strategies (a_2) and average level of self regulated learning for the groups, along with the number of students whose corresponding individual level less than that of the group are recorded in the table 3.

TABLE 3. Comparative Analysis

Group	a_1		a_2		Self-regulated learning	
	A.L	N.S	A.L	N.S	A.L	N.S
1(33)	0.77	17	0.74	17	0.77	17
2(32)	0.76	13	0.70	16	0.73	14

where A.L and N.S refers to Average Level and Number of Students respectively

Along the lines for the sub-attributes associated with a_1 and a_2 are given in the tables 4 and 5 respectively.

TABLE 4. No.of Students with less than average level in each of the 2 groups : sub- attributes of a_1

sub	Average level		No.of Students	
	1	2	1	2
a_{11}	0.76	0.72	12	12
a_{12}	0.79	0.79	13	12
a_{13}	0.78	0.77	12	13
a_{14}	0.79	0.80	14	12
a_{15}	0.79	0.75	15	11
a_{16}	0.69	0.73	17	15

TABLE 5. No.of Students with less than average level in each of the 2 groups: sub- attributes of a_2

sub	Average level		No.of Students	
	1	2	1	2
a_{21}	0.76	0.73	16	6
a_{22}	0.75	0.70	16	18
a_{23}	0.75	0.70	14	16
a_{24}	0.74	0.67	16	16
a_{25}	0.74	0.68	15	17
a_{26}	0.76	0.70	17	16
a_{27}	0.76	0.73	15	13
a_{28}	0.70	0.66	14	15
a_{29}	0.76	0.74	14	14

Part C :

Evaluating the Step2 and Step3, the inequality associated with the two school are shown in the figure 2 and table 6.

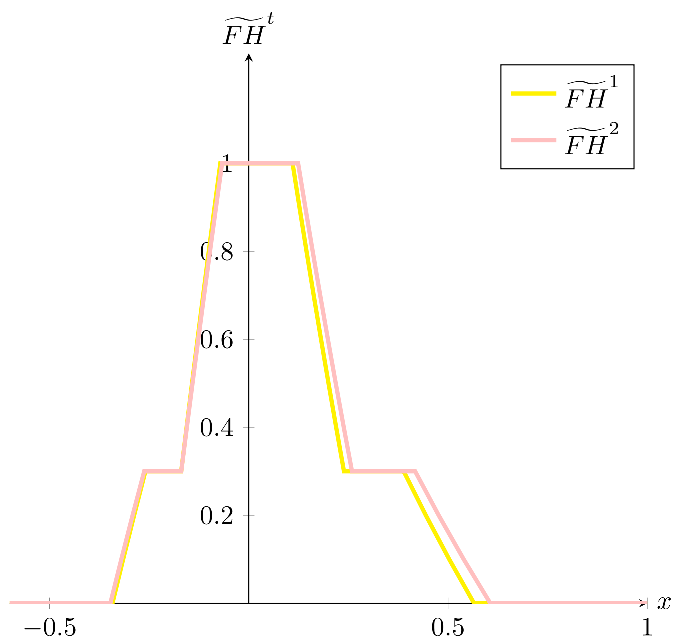

FIGURE 2. Self Regulated Learning inequality index of two schools

TABLE 6. Inequality index of 2 schools

Group	$\widetilde{FH}^t$
1	0.044076
2	0.055264

Inference:

(1) For each individual their level of self- regulated learning is calculate by successive application of fuzzy number valued measure. Even though ranking of individuals are not considered in the MSLQ manual, representation of MSLQ as soft fuzzy number valued information system facilitate to rank the individuals.
(2) The uniqueness in ranking is attained due to the quantification of the information at base level.
(3) The comparative study of individual's level at each sub-attribute (indicator) with that of the group helps oneself to identify those attributes that he or she needs to improve.
(4) From figure 2 and table 6 we infer that the inequality associated with all the school shows small variations and the self regulated learning is at medium level in all the schools.

Remark 4.3. For the sake of comparison we had used equal weights, however it is possible to vary the weights. But un equal weights can be assigned to the parameters and attributes. Along the lines by performing the procedure for group 1 with different weights opted for parameters and attributes, the output for group 1 is given in table 7 and in comparison with table 2 we infer few change in the ranking order of some individuals.

Remark 4.4. From Remark 4.2 and 4.3, we infer that modeling the problem as $\widetilde{IS_H}$ allows one to incorporate different weights to the parameters, attributes and obtain a better ranking of the students based on their self regulated learning level.

TABLE 7. Self Regulated learning of individual Students in group 1 (With different weights assigned)

u_h^1	$\tilde{\boldsymbol{M}}_h^1$	$\mathscr{Q}_h^1$	Rank
u_1^1	(0.5591, 0.6008, 0.6425, 0.6842, 0.7598, 0.7842, 0.8087, 0.8332;0.3)	0.7125	23
u_2^1	(0.5733, 0.6214, 0.6695, 0.7177, 0.7852, 0.8177, 0.8501, 0.8825;0.3)	0.7428	17
u_3^1	(0.5554, 0.6054, 0.6554, 0.7054, 0.7560, 0.8054, 0.8548, 0.9042;0.3)	0.7304	20
u_4^1	(0.4786, 0.5286, 0.5786, 0.6286, 0.6821, 0.7286, 0.7750, 0.8215;0.3)	0.6534	29
u_5^1	(0.6806, 0.7292, 0.7779, 0.8265, 0.9109, 0.9265, 0.9422, 0.9578;0.3)	0.8505	6
u_6^1	(0.6881, 0.7381, 0.7881, 0.8381, 0.9180, 0.9381, 0.9582, 0.9782;0.3)	0.8616	4
u_7^1	(0.5588, 0.6088, 0.6588, 0.7088, 0.7590, 0.8088, 0.8585, 0.9083;0.3)	0.7337	19
u_8^1	(0.4752, 0.5250, 0.5748, 0.6247, 0.6782, 0.7247, 0.7711, 0.8176;0.3)	0.6496	30
u_9^1	(0.5052, 0.5552, 0.6052, 0.6552, 0.7082, 0.7552, 0.8021, 0.8491;0.3)	0.6800	25
u_{10}^1	(0.6540, 0.7040, 0.7540, 0.8040, 0.8757, 0.9040, 0.9324, 0.9607;0.3)	0.8279	9
u_{11}^1	(0.5735, 0.6197, 0.6658, 0.7120, 0.7832, 0.8120, 0.8408, 0.8697;0.3)	0.7381	18
u_{12}^1	(0.4930, 0.5430, 0.5930, 0.6430, 0.6989, 0.7430, 0.7871, 0.8312;0.3)	0.6677	28
u_{13}^1	(0.5064, 0.5555, 0.6046, 0.6537, 0.7075, 0.7537, 0.8000, 0.8463;0.3)	0.6790	26
u_{14}^1	(0.6467, 0.6936, 0.7404, 0.7873, 0.8716, 0.8873, 0.9029, 0.9185;0.3)	0.8123	10
u_{15}^1	(0.3690, 0.4176, 0.4662, 0.5148, 0.5679, 0.6148, 0.6617, 0.7086;0.3)	0.5404	33
u_{16}^1	(0.5086, 0.5586, 0.6086, 0.6586, 0.7259, 0.7586, 0.7914, 0.8241;0.3)	0.6828	24
u_{17}^1	(0.5058, 0.5542, 0.6026, 0.6510, 0.7086, 0.7510, 0.7933, 0.8357;0.3)	0.6765	27
u_{18}^1	(0.6726, 0.7208, 0.7690, 0.8172, 0.8991, 0.9172, 0.9353, 0.9534;0.3)	0.8416	7
u_{19}^1	(0.5950, 0.6447, 0.6944, 0.7442, 0.8192, 0.8442, 0.8691, 0.8941;0.3)	0.7681	15
u_{20}^1	(0.6199, 0.6690, 0.7180, 0.7670, 0.8407, 0.8670, 0.8934, 0.9197;0.3)	0.7914	12
u_{21}^1	(0.5566, 0.6053, 0.6540, 0.7027, 0.7652, 0.8027, 0.8401, 0.8776;0.3)	0.7277	21
u_{22}^1	(0.5811, 0.6280, 0.6748, 0.7217, 0.7862, 0.8217, 0.8572, 0.8927;0.3)	0.7477	16
u_{23}^1	(0.6145, 0.6613, 0.7082, 0.7550, 0.8273, 0.8550, 0.8827, 0.9104;0.3)	0.7806	14
u_{24}^1	(0.6624, 0.7124, 0.7623, 0.8123, 0.8896, 0.9123, 0.9349, 0.9576;0.3)	0.8359	8
u_{25}^1	(0.6317, 0.6817, 0.7317, 0.7817, 0.8590, 0.8817, 0.9044, 0.9271;0.3)	0.8054	11
u_{26}^1	(0.7080, 0.7580, 0.8080, 0.8580, 0.9459, 0.9580, 0.9702, 0.9823;0.3)	0.8811	3
u_{27}^1	(0.4596, 0.5096, 0.5596, 0.6096, 0.6670, 0.7096, 0.7521, 0.7947;0.3)	0.6342	32
u_{28}^1	(0.4802, 0.5231, 0.5661, 0.6090, 0.6706, 0.7090, 0.7474, 0.7859;0.3)	0.6373	31
u_{29}^1	(0.6785, 0.7285, 0.7785, 0.8285, 0.9086, 0.9285, 0.9483, 0.9681;0.3)	0.8520	5
u_{30}^1	(0.7177, 0.7677, 0.8177, 0.8677, 0.9570, 0.9677, 0.9783, 0.9889;0.3)	0.8907	1
u_{31}^1	(0.7167, 0.7667, 0.8167, 0.8667, 0.9556, 0.9667, 0.9778, 0.9889;0.3)	0.8898	2
u_{32}^1	(0.5653, 0.6073, 0.6494, 0.6914, 0.7627, 0.7914, 0.8201, 0.8488;0.3)	0.7197	22
u_{33}^1	(0.6107, 0.6606, 0.7106, 0.7605, 0.8270, 0.8605, 0.8941, 0.9276;0.3)	0.7848	13

Remark 4.5. If in the problem we consider computation with soft trapezoidal fuzzy number as in Remark 2.10, in the place of soft linear octagonal fuzzy numbers, the value of self regulated learning of each student in all the four groups is carried out by executing the proposed algorithm. We verified that the value of self regulated learning for few individuals in their respective groups are very close when computed using trapezoidal fuzzy numbers than octagonal fuzzy numbers. This small variations among the individuals, leads to some change in the ranking order of few individuals in group 2 (the level of 7^{th} student is very close to students $24^{th}, 26^{th}$ who have same level), group 3 (5^{th} and $16^{th}, 10th$ and $15^{th}, 22^{nd}$ and 34^{th} share same level of learning) and group 4 ($20^{th}, 35^{th}$). Using octagonal fuzzy numbers with $0 \leq k < 0.5$ and $0.5 < k < 1$ the value of self regulated learning of an individual at maximum level and minimum level are obtained respectively. In such cases we are able to accurately rank the students. Thus appropriate choice for the value of k yield better results.

Remark 4.6. Various varieties of fuzzy numbers are available in the literature. Comparing the problem for all possible fuzzy numbers to give a comparative study will be unwieldy and out of direction with respect to the paper.

5. Conclusion and Future Work

In this paper we have used $\widetilde{IS}$ as a tool to model self assessment problems with multi dimensional indicators involving imprecise information. A solution to various challenges of self assessment in learning were obtained through the procedure proposed.This new model and methodology has given way to consider the weights for the attributes and parameters involved in MSLQ, which has an impact on measuring the self regulated learning at an individual level. Also the new model and fuzzy number valued measure on soft fuzzy numbers has shown a new direction in computation of fuzzy hyperbolic inequality index of any aspect associated with hierarchical structured attributes. More accuracy in ranking position of students with very close value for self regulated learning is obtained by considering the problem using soft linear octagonal fuzzy numbers than soft trapezoidal fuzzy numbers.

Acknowledgment.

- **•:** We thank DIST (FIST 2015) MATLAB which is used for computational purpose
- **•:** This work is supported by Maulana Azad National Fellowship F1-17.1/2017-18/MANF-2017-18-TAM-77243 of the University Grants commission (UGC), New Delhi, India

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Appendix

Parameters and Data - Self Regulating Response from School Students

The formated responses of the 33 students in group 1:

```
Enter the number of entities for the group 1 : 33
Enter the number of attributes : 2
Enter the weight vector for the attributes of size 2[.5 .5]
enter the number of sub-attributes for attribute 1 : 6
Enter the matrix for 1 sub-attribute of attribute 1 :
{VT VT NST NE VT VT T NST ST T T ST T ST ST NE ST T T T T ST
T ST VT VT T T ST T VT VT ST;VT VT NE NE VT T T T ST T VT T
T VT NE NST ST T VT VT NT VT ST VT T VT NE NST VT VT T T T;
T T T ST VT ST T NE ST VT T T T VT NE VT ST VT T VT T VT VT
T ST VT T VT VT VT VT VT T;T ST ST ST T VT T ST NE T T T T
NE NST NE NST T T VT ST VT VT VT VT T NE NST VT T VT VT ST}
Enter the weight vector of size 4 [.25 .25 .25 .25]
Enter the matrix for 2 sub-attribute of attribute 1 :
{VT NT T T NE VT ST ST NE T VT VT T VT NE NE VT VT VT ST T
ST T VT ST VT T T VT T VT T T ;VT VT ST T VT VT ST NE ST VT
VT VT VT VT NE VT T VT VT T T NE VT VT T VT VT T VT T T T
T;NST VT ST NST NVT VT T VT VT T VT T VT VT NE NE ST VT VT
T T VT VT VT VT VT T T VT T VT T T;T NT ST NST VT VT ST T
ST VT VT VT NST T NE NST T VT VT VT VT VT VT T T VT VT T VT
VT VT T VT}
Enter the weight vector of size 4 [.25 .25 .25 .25]
Enter the matrix for 3 sub-attribute of attribute 1 :
{VT VT T NE T VT ST NST ST VT VT T VT T NE VT T T VT T T T
T T T VT T VT T VT T ST T;VT T T NE VT T NE NST ST T T VT T
VT NST VT NE VT VT VT T VT VT VT VT VT NE NE VT VT VT T VT;
```

```
NE VT T ST VT ST T ST ST VT T NST T VT NE NST ST T VT T NT
T T VT VT VT T T VT VT VT T ST;VT ST T ST VT VT ST ST ST VT
VT VT T NE NST NE NE T T VT T T VT VT VT VT VT ST VT VT VT
T T;VT T T ST VT VT T ST ST T VT ST T VT NST NE T T ST VT
NE T VT T ST T NE T VT VT VT VT T;VT T T ST T VT T ST ST VT
T T T VT NE NE T VT VT VT T T T VT VT VT NE T VT VT VT VT
ST}
Enter the matrix for 4 sub-attribute of attribute 1 :
{NVT VT T VT VT VT T NT VT VT VT T VT VT NE VT VT VT VT VT
VT T VT VT VT VT T ST VT VT T T VT;VT T T NE NVT T ST ST ST
VT ST T T T NVT NE ST VT VT VT VT T T VT T VT T T VT VT VT
T T;VT VT T ST VT VT T ST ST VT VT T T VT NE NE NST T VT VT
T VT VT ST T VT VT VT VT VT T T T;VT T T ST T T ST T NST VT
VT NST  NST VT  NST NE ST VT ST T NST VT VT VT VT VT VT T
VT VT VT VT VT}
Enter the weight vector of size 4 [.25 .25 .25 .25]
Enter the matrix for 5 sub-attribute of attribute 1 :
{VT VT T VT VT VT ST T VT VT VT T VT VT ST VT VT VT VT VT
VT T VT VT T VT VT VT T VT VT T VT;VT ST T ST T T ST NST T
T VT T T VT NST NE T T T T NVT ST ST T T VT T VT VT VT VT
T;VT VT ST ST VT T T ST ST VT T T T VT ST VT  ST VT VT VT
T T T T VT VT ST VT T T VT T VT;T T NE T VT T T ST ST ST VT
NST NST VT ST NE ST VT VT VT NVT T NE T VT VT VT T T ST VT
T ST;T VT T T VT VT T ST ST VT VT T T VT NE NST VT VT VT VT
VT VT VT VT T T T T T T VT T T ;VT VT T ST VT VT T T T VT
VT T T VT NE NE VT VT VT T NE T T VT VT VT T T T VT T VT
T;T VT T T T VT T ST T VT VT T T VT NE NE T VT T T T T T T
VT VT T T ST VT VT T T;VT T T NST VT VT ST NE ST T T T NST
VT NE VT ST T VT T ST ST T VT ST T NE T T VT VT T T}
Enter the weight vector of size 8
[.125 .125 .125 .125 .125 .125 .125 .125]
Enter the matrix for 6 sub-attribute of attribute 1 :
{NE NVT T NST VT T T VT NE T ST NST ST T ST VT T NVT VT T T
T T VT ST T NE T VT VT VT VT VT;NVT ST T ST VT VT NST VT T
ST VT NST NVT T NVT NE T VT T VT T NE VT VT VT VT T NST VT
VT VT T ST;VT VT ST NE VT VT T T T T NVT T T T NE NE ST VT
VT VT T T VT VT VT VT ST NVT VT VT T NVT T;T T T ST VT VT T
T ST T NE NT T NVT NE NE ST VT VT NT T NVT NVT VT VT VT NT
NVT T VT VT NVT VT;NVT T ST ST VT T ST T ST VT NT NST T VT
NE NST VT T VT NVT T ST T T T T VT NT VT VT T NVT VT}
Enter the weight vector of size 5 [.2 .2 .2 .2 .2]
Enter the matrix for 1 sub-attribute of attribute 2 :
{NVT T T NE VT VT T NT T T VT NE T T NT VT NST VT T NE NVT
T T T NT VT T VT  VT VT VT T T;VT T ST T VT VT T ST ST T T
T T VT ST VT VT T NE VT ST T T VT VT VT NST T ST VT VT VT
ST;VT ST ST T T T T T ST VT VT T NST VT NE T VT T ST T T VT
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T VT VT T T T VT T VT NT ST;VT ST ST T VT T T NE T ST VT
NST T NE VT VT VT T ST  T T VT T VT ST VT NT  T VT VT VT VT
T}
Enter the weight vector of size 4 [.25 .25 .25 .25]
Enter the matrix for 2 sub-attribute of attribute 2 :
{T ST ST VT VT VT ST NE ST VT NE T T VT ST VT T VT NST T ST
ST ST T VT VT NST T VT VT T VT T;ST T ST ST VT VT T ST ST
VT T NE T T NE VT VT VT NVT VT T ST VT ST NT VT NT NST VT T
T T T;T ST ST T VT T T ST ST T T NST T VT ST VT ST VT NE VT
VT VT T T T T NT ST VT T VT VT T;VT ST ST T T T ST T ST NST
T T T NE NE VT ST VT NE VT T VT VT ST VT VT NT T T VT VT VT
NE;VT T ST T T T ST NE ST ST T T NT VT NE VT ST VT NT T T
VT T VT T VT NE ST VT VT VT VT ST;VT ST VT T VT VT T T ST T
T NST T VT VT NT NT VT VT T T VT VT ST  VT VT T NVT VT VT
VT VT VT}
Enter the weight vector of size 6 [.16 .16 .17 .17 .17 .17]
Enter the matrix for 3 sub-attribute of attribute 2 :
{VT VT T NST VT T ST NE ST T T T NST VT NE NE T VT VT VT VT
VT VT VT VT VT T VT T VT VT T ST;VT T ST NE VT VT ST NE ST
T T T T VT NT T T ST NE VT VT ST T T ST VT NST T VT VT VT T
T;T VT ST ST VT T ST T ST VT T T NST VT NE VT NVT VT T T NT
T VT ST NT VT NT VT T VT VT VT NE;NVT T T T T T T ST NE T
VT T T VT ST VT T T NE T T T T ST VT VT NE T ST VT VT VT ST}
Enter the weight vector of size 4 [.25 .25 .25 .25]
Enter the matrix for 4 sub-attribute of attribute 2 :
{NVT ST ST ST T T ST NVT ST T NVT T T VT NT VT ST T ST T
NVT T T VT ST T T T VT VT VT T T;VT ST ST T  VT T ST ST ST
VT NE T NST VT NE VT T VT NST T ST T T T VT T NST T VT VT
VT VT ST;VT ST ST ST VT VT ST T ST  T T NST NST VT NE VT
NST T VT VT VT T VT VT  ST VT NST NE T T VT VT VT;VT ST T T
T VT ST ST T T NE T T VT NE  T NE T ST T T T ST ST ST VT NT
T VT T VT VT T;NE T NE ST VT VT T T ST VT VT T T VT VT VT
VT VT VT T NST VT VT T T T ST NT VT T VT T T}
Enter the weight vector of size 5 [.2 .2 .2 .2 .2]
Enter the matrix for 5 sub-attribute of attribute 2 :
{NVT NT ST T T VT T T T VT NVT ST NST NE NE NST VT ST NST T
T VT T VT VT T T NVT T T T NVT ST;VT T T T VT VT T ST NE T
VT T T VT  NST NE T VT NST VT ST T T VT VT VT NST VT T T VT
T ST;VT ST T NE VT VT T ST ST T T T T T NT  VT ST VT NST T
VT T T T VT VT NST T T VT T T ST;VT T ST ST VT VT T ST T T
VT ST T VT ST VT ST T T T NE T T VT ST T T T ST VT VT VT
VT;T T ST T T T T NST T T T NST T VT ST T ST T NE VT T T ST
T T T NE NT NE VT VT NVT ST;NVT ST  T T VT VT T ST ST T VT
T T ST ST VT ST VT NST VT T VT T VT VT VT T VT VT VT VT T
T;VT ST T ST VT T ST NE ST VT  NE NT T VT NE  VT T VT VT
VT ST ST T T ST VT NE T VT VT T T T;NVT NVT ST ST T VT T ST
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T NST NVT T NST ST NE VT ST T ST NVT NT ST T ST NT VT NT
NVT T T VT NVT VT;VT ST ST ST VT VT T T ST VT T T NST NE NE
T T T T T ST ST VT VT VT VT ST VT VT VT VT T T;VT T T T T
VT T NE ST T NE VT T VT ST T NT T ST VT T VT T T T T T VT
VT VT VT T;T T T T VT T T NE ST T VT T T VT ST VT ST T T VT
T VT VT T VT VT ST VT VT T T VT ST;T T ST ST T VT ST T T VT
VT T T NE ST T T VT NE T T VT T NE ST T NE VT VT VT VT VT T}
Enter the weight vector of size 12
[.08 .08 .08 .08 .08 .08 .08 .08 .09 .09 .09 .09]
Enter the matrix for 6 sub-attribute of attribute 2 :
{VT T ST VT T VT T NE T T VT T T VT NST NST NE T ST VT VT
VT VT T VT VT T T T VT VT T T;VT ST T T T VT T NE ST VT T
NST NST VT ST T NST VT ST VT T T VT T VT VT NST T ST VT VT
VT T;T ST ST VT VT VT ST NE ST VT NE T T VT ST VT T VT NST
T ST ST ST T VT VT NST T VT VT T VT T;NVT NT ST VT VT T T
ST ST VT T T T NT NE T NST VT VT ST NT NT VT NVT VT VT NT
NT VT VT T NVT NVT;NE T T T VT VT T T NE T T VT T VT T T VT
T T VT NE VT T T VT T NE ST  T VT VT VT T;NVT VT NE ST VT
VT VT ST ST ST VT T T VT VT VT T  VT T ST T ST VT VT VT T T
VT VT VT VT VT VT;VT T ST T VT VT T T T T NE T T VT ST ST
NE VT  NT T T T VT VT VT VT T ST VT VT VT VT VT;VT T T T T
T T NE ST VT VT T T NE ST T VT T ST VT VT T T NE T VT T VT
T VT T VT T}
Enter the weight vector of size 8
[.125 .125 .125 .125 .125 .125 .125 .125]
Enter the matrix for 7 sub-attribute of attribute 2 :
{VT T T ST VT VT T NST ST VT ST T T VT NT VT NE VT NE VT VT
T NE VT T ST T NT VT VT VT T VT;VT VT T ST T T T ST T T T NST
 NST VT NE T VT T ST VT T VT VT VT VT VT T VT VT VT VT
VT T;T NST T ST T VT ST NE T VT NVT T NST VT T T T VT T T
VT VT ST ST NE VT NST NT VT VT T NVT T;T VT ST ST VT T T ST
T ST VT VT T VT VT T NE T NE T VT ST VT T T ST T VT T T VT
VT T}
Enter the weight vector of size 4 [.25 .25 .25 .25]
Enter the matrix for 8 sub-attribute of attribute 2 :
{NVT T T VT VT T T NT ST VT T ST T NE NST NE ST VT T VT VT
T ST T ST VT T ST T VT VT T NE;VT ST T T VT VT T NST ST VT
T ST T VT ST T T VT NST VT VT ST T ST T T NT NVT VT VT T
VT T;NVT T T NE T VT T NE T T NE NST NST VT NE T NT T NE T
NT T T T NT T NST T VT VT VT VT NST}
Enter the weight vector of size 3 [.33 .33 .34]
Enter the matrix for 9 sub-attribute of attribute 2 :
{NVT T ST ST VT T ST NE ST T T T T VT NT VT NT T ST VT T VT
T ST VT VT ST VT ST VT VT T T;VT T T T VT VT ST NE ST VT VT
T NST VT T VT NE VT NST VT NT ST T VT VT VT ST T VT T VT VT
T;VT T T T T VT T ST T T T T T NE VT T ST T T VT T T VT T
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VT T T T VT VT VT VT T;VT ST ST VT VT VT T ST ST VT NVT NST
T VT ST NE NVT VT NE T T T ST VT ST VT T T VT VT VT VT T}
Enter the weight vector of size 4 [.25 .25 .25 .25]
```

Value Component: Intrinsic Goal Orientation

Item

1. In a class like this, I prefer course material that really challenges me so I can learn new things.

16. In a class like this, I prefer course material that arouses my curiosity, even if it is difficult to learn.

22. The most satisfying thing for me in this course is trying to understand the content as thoroughly as possible.

24. When I have the opportunity in this class, I choose course assignments that I can learn from even if they don't guarantee a good grade.

Value Component: Extrinsic Goal Orientation

Item

7. Getting a good grade in this class is the most satisfying thing for me right now.

11. The most important thing for me right now is improving my overall grade point average, so my main concern in this class is getting a good grade.

13. If I can, I want to get better grades in this class than most of the other students.

30. I want to do well in this class because it is important to show my ability to my family, friends, employer, or others.

Value Component: Task Value

Item

4. I think I will be able to use what I learn in this course in other courses.

10. It is important for me to learn the course material in this class.

17. I am very interested in the content area of this course.

23. I think the course material in this class is useful for me to learn.

26. I like the subject matter of this course.

27. Understanding the subject matter of this course is very important to me.

Expectancy Component: Control of Learning Beliefs

Item

2. If I study in appropriate ways, then I will be able to learn the

material in this course.
9. It is my own fault if I don't learn the material in this course.
18. If I try hard enough, then I will understand the course material.
25. If I don't understand the course material, it is because I didn't try hard enough

Expectancy Component: Self-Efficacy for Learning and Performance

Item
5. I believe I will receive an excellent grade in this class.
6. I'm certain I can understand the most difficult material presented in the readings for this course.
12. I'm confident I can understand the basic concepts taught in this course.
15. I'm confident I can understand the most complex material presented by the instructor in this course.
20. I'm confident I can do an excellent job on the assignments and tests in this course.
21. I expect to do well in this class.
29. I'm certain I can master the skills being taught in this class.
31. Considering the difficulty of this course, the teacher, and my skills, I think I will do well in this class.

Affective Component: Test Anxiety

Item
3. When I take a test I think about how poorly I am doing compared with other students.
8. When I take a test I think about items on other parts of the test I can't answer.
14. When I take tests I think of the consequences of failing.
19. I have an uneasy, upset feeling when I take an exam.
28. I feel my heart beating fast when I take an exam.

LEARNING STRATEGIES SCALES

Cognitive and Metacognitive Strategies: Rehearsal
Item
39. When I study for this class, I practice saying the material to myself over and over.
46. When studying for this class, I read my class notes and the course readings over and over again.
59. I memorize key words to remind me of important concepts in this class.

72. I make lists of important terms for this course and memorize the lists.

Cognitive and Metacognitive Strategies: Elaboration

Item
53. When I study for this class, I pull together information from different sources, such as lectures, readings, and discussions.
62. I try to relate ideas in this subject to those in other courses whenever possible.
64. When reading for this class, I try to relate the material to what I already know.
67. When I study for this course, I write brief summaries of the main ideas from the readings and the concepts from the lectures.
69. I try to understand the material in this class by making connections between the readings and the concepts from the lectures.
81. I try to apply ideas from course readings in other class activities such as lecture and discussion

Cognitive and Metacognitive Strategies: Organization

Item
32. When I study the readings for this course, I outline the material to help me organize my thoughts.
42. When I study for this course, I go through the readings and my class notes and try to find the most important ideas.
49. I make simple charts, diagrams, or tables to help me organize course material.
63. When I study for this course, I go over my class notes and make an outline of important concepts.

Cognitive and Metacognitive Strategies: Critical Thinking
Item
38. I often find myself questioning things I hear or read in this course to decide if I find them convincing.
47. When a theory, interpretation, or conclusion is presented in class or in the readings, I try to decide if there is good supporting evidence.
51. I treat the course material as a starting point and try to develop my own ideas about it.
66. I try to play around with ideas of my own related to what I am learning in this course.
71. Whenever I read or hear an assertion or conclusion in this class,

I think about possible alternatives.

Cognitive and Metacognitive Strategies: Metacognitive Self-Regulation
Item
33. During class time I often miss important points because I'm thinking of other things. (REVERSED)
36. When reading for this course, I make up questions to help focus my reading.
41. When I become confused about something I'm reading for this class, I go back and try to figure it out.
44. If course materials are difficult to understand, I change the way I read the material.
54. Before I study new course material thoroughly, I often skim it to see how it is organized.
55. I ask myself questions to make sure I understand the material I have been studying in this class.
56. I try to change the way I study in order to fit the course requirements and instructor's teaching style.
57. I often find that I have been reading for class but don't know what it was all about. (REVERSED)
61. I try to think through a topic and decide what I am supposed to learn from it rather than just reading it over when studying.
76. When studying for this course I try to determine which concepts I don't understand well.
78. When I study for this class, I set goals for myself in order to direct my activities in each study period.
79. If I get confused taking notes in class, I make sure I sort it out afterwards.

Resource Management Strategies: Time and Study Environment
Item
35. I usually study in a place where I can concentrate on my course work.
43. I make good use of my study time for this course.
52. I find it hard to stick to a study schedule. (REVERSED)
65. I have a regular place set aside for studying.
70. I make sure I keep up with the weekly readings and assignments for this course.
73. I attend class regularly.
77. I often find that I don't spend very much time on this course because of other activities. (REVERSED)
80. I rarely find time to review my notes or readings before an exam. (REVERSED)

Resource Management Strategies: Effort Regulation

Item

37. I often feel so lazy or bored when I study for this class that I quit before I finish what I planned to do. (REVERSED)
48. I work hard to do well in this class even if I don't like what we are doing.
60. When course work is difficult, I give up or only study the easy parts. (REVERSED)
74. Even when course materials are dull and uninteresting, I manage to keep working until I finish.

Resource Management: Peer Learning

Item

34. When studying for this course, I often try to explain the material to a classmate or a friend.
45. I try to work with other students from this class to complete the course assignments.
50. When studying for this course, I often set aside time to discuss the course material with a group of students from the class.

Resource Management: Help Seeking

Item

40. Even if I have trouble learning the material in this class, I try to do the work on my own, without help from anyone.
(REVERSED)
58. I ask the instructor to clarify concepts I don't understand well.
68. When I can't understand the material in this course, I ask another student in this class for help.
75. I try to identify students in this class whom I can ask for help if necessary

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