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Adaptive Interval Type-2 Fuzzy PI Control for Uncertain Nonlinear Time Delay Systems Fast and Large Disturbance Rejection

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ABSTRACT

In this paper, adaptive interval type-2 fuzzy proportional integral (PI) control scheme to attenuate fast and large disturbance for a class of uncertain nonlinear time delay systems is proposed, i.e., the fuzzy collaboration control between time delay interval type-2 FNN and nonlinear time delay system is performed. By incorporating adaptive interval type-2 time delay fuzzy neural network controller (TDFNNC) with PI controller, not only the typical switching law chattering can be significantly attenuated but also the instability resulting from system time delay can be overcome. Based on the Lyapunov theory of stability, the free parameters of the adaptive interval type-2 TDFNNC and PI controller coefficients can be tuned online by output feedback adaptive laws derived from Lyapunov function with time delays. Simulation results show that the chattering phenomena can be attenuated and the prescribed tracking performance can be preserved simultaneously by the advocated control scheme.

Keywords: Adaptive control, interval type-2 fuzzy, nonlinear time delay system, PI control.

1. INTRODUCTION

Owing to time delays are the main source of the instability and lead to unsatisfactory performances, control system design with uncertain time delays has been an active area of research. Over the past years, a number of different researches have been invested in the stability analysis and robust controller design of uncertain systems with delay [3-5, 13, 24]. Moreover, robust H^{∞} control methods for linear systems with time delay [18] and a class of nonlinear time delay systems control [3, 4], [5, 13, 24] have been proposed for many years. Unfortunately, in reality, system uncertainties and external disturbance input are unpredicted, i.e., may be both fast and large. A PI adaptive fuzzy control scheme for a class of uncertain nonlinear systems is introduced in [1, 17, 18, 25] to handle large and fast but bounded external disturbances and uncertainties.

In the past several decades, based on universal approximation theorem [2, 20, 23, 26, 27], a significant adaptive fuzzy neural network (FNN) control structure [1, 8, 18] has been proposed to incorporate with the expert information systematically and the stability can be guaranteed. It is very difficult to control systems with high degree of nonlinear uncertainty, such as chemical process, aircraft, and so on, by using the conventional control theory. However human operators can often successfully control them. A globally stable adaptive FNN controller is defined as an FNN system equipped with an adaptation algorithm, based on the fact that FNN logic systems are capable of uniformly approximating a nonlinear function over a compact set to any degree of accuracy. Adaptive backstepping fuzzy control [21] and observer-based adaptive decentralized fuzzy fault-tolerant control [22] are proposed for nonlinear systems tracking problem. On the other hand, interval type-2 fuzzy logic system (FLS) [15, 16] which is an extension of type-1 FLS is introduced to overcome the limitations because of type-1 FLS cannot fully handle the linguistic and high level uncertainties [9-11, 14].

In this paper, an adaptive interval type-2 TDFNNC is constructed by interval type-2 TDFNN system. In the meantime, a PI controller incorporated into an adaptive interval type-2 TDFNNC to deal with fast and large external disturbance, system uncertainties and system time delay which is a source of instability.

This paper is organized as follows. Problem formulation is given is Section II. A brief description of adaptive interval type-2 time delay fuzzy neural network (TDFNN) is described in Section III. Section IV provides adaptive interval type-2 fuzzy proportional integral (PI) control scheme. Simulation example to illustrate the performance of the proposed control structure is shown in Section V. Section VI concludes the effectiveness of the advocated design methodology.

2. PROBLEM FORMULATION

Consider the nth-order nonlinear dynamical time delay system of the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots \\ \dot{x}_n &= f(\underline{x}, \underline{x}(t-\tau_1)\cdots\underline{x}(t-\tau_r)) \\ y &= x_1 \end{aligned} \tag{1}$$

or equivalently the form

$$x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}) + g(x, \dot{x}, \cdots, x^{(n-1)})u + d, \quad y = x \tag{2}$$

$$x(t) = \Xi(t), t \in [-\varsigma_x, 0]$$

where f and g are unknown but bounded functions, $\Xi(t)$ is the continuous function, $\tau_i (i = 1, 2, ..., r)$ is the time delay, and $\varsigma_x = \max\{\tau_i \,|\, 1 \le i \le r\}$. Moreover $u \in R$ and $y \in R$ are the control input and output of the system, respectively and d is the external bounded disturbance, $|d(t)| \le D$, D is a positive constant. Let $f(\underline{x}, \tau) = f(\underline{x}, \underline{x}(t-\tau_1)\cdots\underline{x}(t-\tau_r))$ and $g(\underline{x}, \tau) = g(\underline{x}(t-\tau_1)\cdots\underline{x}(t-\tau_r))$, (2) can be rewritten in state space representation as

$$\begin{aligned} \dot{\underline{x}} &= \mathrm{A}\underline{x} + \mathrm{B}(f(\underline{x}, \tau) + g(\underline{x}, \tau)u + d) \\ y &= \mathrm{C}^T \underline{x} \end{aligned} \tag{3}$$

where

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad \mathrm{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \mathrm{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

and $\underline{x} = [x_1, x_2, \cdots, x_n]^T = \left[x, \dot{x}, \cdots, x^{(n-1)}\right]^T \in R^n$ is the state vector. In order for (2) to be controllable, it is required that $g(\underline{x},\tau)\neq 0$ for $\underline{x}$ in certain controllability region $U_c \subset R^n$. Without loss of generality, we assume that $g(\underline{x},\tau)>0$ for $\underline{x}\in U_c$. The control object is to force the system output y to follow a given bounded reference signal y_r, under the constraint that all signals involved must be bounded.

To begin with, the reference signal vector $\underline{y}_r$ and the tracking error vector $\underline{e}$ will be defined as

$$\underline{y}_r = \left[y_r, \dot{y}_r, \cdots, y_r^{(n-1)}\right]^T \in R^n ,$$

$$\underline{e} = \underline{y}_r - \underline{x} = \left[e, \dot{e}, \cdots, e^{(n-1)}\right]^T \in R^n$$

Let $\underline{k}_c = \left[k_1^c, k_2^c, \cdots, k_n^c\right]^T \in R^n$ to be chosen such that all roots of the polynomial $p(s) = s^n + k_n^c s^{n-1} + \cdots + k_1^c$ are in the open left half-plane. If the functions $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$ are known and the system is free of external disturbance d, then control law of the certainty equivalent controller is obtained as [9], [10], [11]

$$u^* = \frac{1}{g(\underline{x},\tau)}\left[-f(\underline{x},\tau) + y_r^{(n)} + \underline{k}_c^T \underline{e}\right] \tag{5}$$

Substituting (5) into (2), we have [8]

$$e^{(n)} + k_n^c e^{(n-1)} + \cdots + k_1^c e = 0$$

which is the main objective of control, $\lim_{t\to\infty} e(t) = 0$. Therefore, there exists a positive definite symmetric $n\times n$ matrix P which satisfies the Lyapunov equation

$$(A - B\underline{k}_c^T)^T P + P(A - B\underline{k}_c^T) = -Q \tag{6}$$

where Q is an arbitrary positive definite symmetric $n\times n$ matrix. However, $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$ are unknown, the equivalent controller (5) is unavailable. The interval type-2 adaptive TDFNN system structure described in next section will be developed to approximate $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$.

3. DESCRIPTION OF INTERVAL TYPE-2 TIME DELAY FNN SYSTEM

Time delay FLS (TDFLS) is capable of uniformly approximating any nonlinear function with time delay over a compact set U_c to any degree of accuracy based on the universal approximation theorem [14], [17], [19]. The interval type-2 TDFNN system structure as shown in Fig. 1 [10] is used to approach the system functions $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$. The output of the interval type-2 TDFLS with central average defuzzifier is expressed as

$$y(\underline{x}) = \frac{y_l + y_r}{2} = \frac{1}{2}(\underline{\xi}_r \underline{\Theta}_r + \underline{\xi}_l \underline{\Theta}_l) = \frac{1}{2}\begin{bmatrix}\underline{\xi}_r & \underline{\xi}_l\end{bmatrix}\begin{bmatrix}\underline{\Theta}_r \\ \underline{\Theta}_l\end{bmatrix} = \underline{\xi}\ \underline{\Theta} \tag{7}$$

where $(1/2)[\underline{\xi}_r^T\ \underline{\xi}_l^T] = \underline{\xi}^T$ and $[\underline{\Theta}_r^T\ \underline{\Theta}_l^T] = \underline{\Theta}^T$. The left-most point y_l and the right-most point y_r can be expressed as a fuzzy basis function (FBF) expansion, i.e.,

$$y_l = \frac{\sum_{i=1}^{L} \overline{f}^i y_l^i + \sum_{i=L+1}^{M} \underline{f}^i y_l^i}{\sum_{i=1}^{L} \overline{f}^i + \sum_{i=L+1}^{M} \underline{f}^i} = \sum_{i=1}^{L} \overline{q}_l^i y_l^i + \sum_{i=L+1}^{M} \underline{q}_l^i y_l^i = [\overline{Q}_l\ \underline{Q}^l]\begin{bmatrix}\overline{y}_l \\ \underline{y}^l\end{bmatrix} = \underline{\xi}_l \underline{\Theta}_l$$

and

$$y_r = \frac{\sum_{i=1}^{R} \underline{f}^i y_r^i + \sum_{i=R+1}^{M} \overline{f}^i y_r^i}{\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \overline{f}^i} = \sum_{i=1}^{R} \underline{q}_r^i y_r^i + \sum_{i=R+1}^{M} \overline{q}_r^i y_r^i = [\underline{Q}_r\ \overline{Q}^r]\begin{bmatrix}\underline{y}_r \\ \overline{y}^r\end{bmatrix} = \underline{\xi}_r \underline{\Theta}_r$$

where $\underline{q}_l^i = \underline{f}^i / D_l$, $\overline{q}_l^i = \overline{f}^i / D_l$, $\underline{q}_r^i = \underline{f}^i / D_r$, $\overline{q}_r^i = \overline{f}^i / D_r$, $D_l = (\sum_{i=1}^{L} \overline{f}^i + \sum_{i=L+1}^{M} \underline{f}^i)$, $D_r = (\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \overline{f}^i)$. Meanwhile, we have $\underline{Q}_l = [\underline{q}_l^1, \underline{q}_l^2, \cdots, \underline{q}_l^R]$, $\overline{Q}^l = [\overline{q}_l^1, \overline{q}_l^2, \cdots, \overline{q}_l^R]$, $\underline{Q}_r = [\underline{q}_r^1, \underline{q}_r^2, \cdots, \underline{q}_r^R]$, $\overline{Q}^r = [\overline{q}_r^1, \overline{q}_r^2, \cdots, \overline{q}_r^R]$, $\underline{\xi}_l = [\overline{Q}_l\ \underline{Q}^l]$, $\underline{\xi}_r = [\underline{Q}_r\ \overline{Q}^r]$ and $\underline{\Theta}_l^T = [\overline{y}_l\ \underline{y}^l]$, $\underline{\Theta}_r^T = [\underline{y}_r\ \overline{y}^r]$. In the meantime, an interval type-2 FLS with singleton fuzzification and meet under minimum or product t-norm $\underline{f}^i$ and $\overline{f}^i$ can be obtained as

$$\underline{f}^i = \underline{\mu}_{F_1^i}(x_1) * \cdots * \underline{\mu}_{F_p^i}(x_p)$$

and

$$\overline{f}^i = \overline{\mu}_{F_1^i}(x_1) * \cdots * \overline{\mu}_{F_p^i}(x_p).$$

where

$$\overline{\mu}_{F_i^k}(\underline{x}_i, \tau, \underline{m}_k, \underline{\sigma}_k) = \overline{\mu}_{F_i^k}(\underline{x}_i, \underline{m}_k, \underline{\sigma}_k)\,\overline{\mu}_{F_i^k}\left[\underline{x}_i(t-\tau_1), \underline{m}_k, \underline{\sigma}_k\right] \cdots \overline{\mu}_{F_i^k}\left[\underline{x}_i(t-\tau_r), \underline{m}_k, \underline{\sigma}_k\right]$$

and

$$\underline{\mu}_{F_i^k}(\underline{x}_i, \tau, \underline{m}_k, \underline{\sigma}_k) = \underline{\mu}_{F_i^k}(\underline{x}_i, \underline{m}_k, \underline{\sigma}_k)\,\underline{\mu}_{F_i^k}\left[\underline{x}_i(t-\tau_1), \underline{m}_k, \underline{\sigma}_k\right] \cdots \underline{\mu}_{F_i^k}\left[\underline{x}_i(t-\tau_r), \underline{m}_k, \underline{\sigma}_k\right]$$

Meanwhile, the point to separate two sides by number *R* and *L* can be decided from the K.M algorithm proposed in [6], [7], one side using lower firing strengths $\underline{f}^i$'s and another side using upper firing strengths $\overline{f}^i$'s.

Therefore, we replace the unknown system functions $f(\underline{x}, \tau)$ and $g(\underline{x}, \tau)$ in specific time delay fuzzy logic systems as (7), i.e.,

$$f\left(\underline{x},\tau\middle|\underline{\theta}_f,\underline{m},\underline{\sigma}\right)=\underline{\xi}\left(\underline{x},\tau,\underline{m},\underline{\sigma}\right)\underline{\theta}_f=\frac{1}{2}\underline{\xi}_{fr}\left(\underline{x},\tau,\underline{m}_f,\underline{\sigma}_f\right)\underline{\theta}_{fr}+\frac{1}{2}\underline{\xi}_{fl}\left(\underline{x},\tau,\underline{m}_f,\underline{\sigma}_f\right)\underline{\theta}_{fl} \tag{8}$$

$$g\left(\underline{x},\tau\middle|\underline{\theta}_g,\underline{m},\underline{\sigma}\right)=\underline{\xi}\left(\underline{x},\tau,\underline{m},\underline{\sigma}\right)\underline{\theta}_g=\frac{1}{2}\underline{\xi}_{gr}\left(\underline{x},\tau,\underline{m}_g,\underline{\sigma}_g\right)\underline{\theta}_{gr}+\frac{1}{2}\underline{\xi}_{gl}\left(\underline{x},\tau,\underline{m}_g,\underline{\sigma}_g\right)\underline{\theta}_{gl} \tag{9}$$

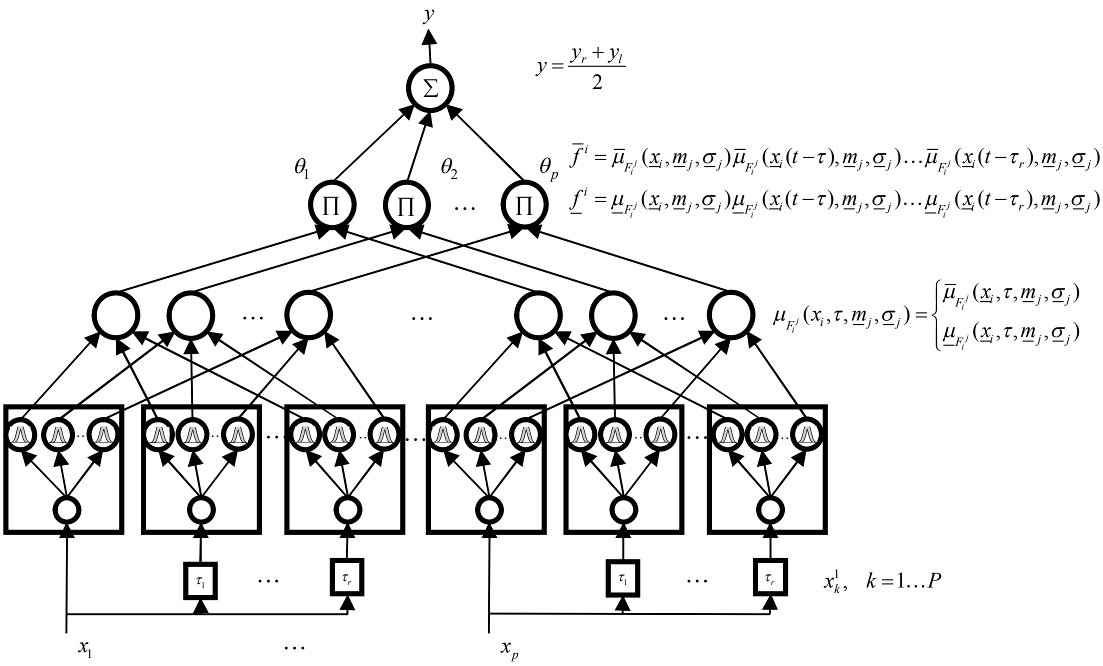


Fig. 1. The interval type-2 TDFNN system structure.

4. ADAPTIVE INTERVAL TYPE-2 FUZZY PI CONTROLLER DESIGN

An indirect adaptive interval type-2 TDFNN controller uses TDFNN system to model the plant and constructs the controller assuming that the TDFNN systems represent the true plant. However, in reality, the knowledge used to construct TDFNN system is often uncertain such as linguistic uncertainty and noisy training data. In the meantime, time delays are always the main source of the instability and lead to unsatisfactory performances. In order to overcome the limitations from type-1 FLS and the instability resulting from system time delay, the interval type-2 TDFNN system is constructed to approximate system functions $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$. Furthermore, PI control structure is developed to handle fast and large but bounded external disturbances. The PI error feedback structure is defined as

$$\rho(\underline{e}^T P\,B|\underline{\theta}_p)=\begin{cases}\underline{\theta}_p^T\zeta\left(\underline{e}^T PB\right), & \left|\underline{e}^T PB\right|\le\psi\\ \hat{D}_m\,\mathrm{sgn}(\underline{e}^T PB), & \left|\underline{e}^T PB\right|>\psi\end{cases} \tag{10}$$

where P is the matrix given in (6), $[K_P, K_I] = \underline{\theta}_p^T$ is the PI parameter vector to be adapted, $[\underline{e}^T PB, \int \underline{e}^T PBdt]^T = \zeta\left(\underline{e}^T PB\right)$. $\hat{D}_m = \hat{D} + \hat{\Omega}$ is an estimate of $D_m = D + \Omega$, D and Ω are the disturbance bound and the minimum approximation error of the interval yupe-2 time delay FNN, respectively. The thickness of the boundary layer Ψ is a compromise between chattering attenuation and accelerating the speed of convergence.

Therefore, the control effort (5) can be rewritten as

$$u = \frac{1}{g\left(\underline{x}, \tau \middle| \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g\right)}\left[-f\left(\underline{x}, \tau \middle| \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f\right) - \rho(\underline{e}^T PB | \underline{\theta}_p) + x_d^{(n)} - \underline{k}_c^T \underline{e}\right] \tag{11}$$

where $f\left(\underline{x}, \tau \middle| \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f\right)$ and $g\left(\underline{x}, \tau \middle| \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g\right)$ are the interval type-2 time delay FNN outputs which are described in (8) and (9) as follows:

$$f\left(\underline{x}, \tau \middle| \underline{\theta}_f, \underline{m}_f, \underline{\sigma}_f\right) = \frac{1}{2}\underline{\xi}_{fr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right)\underline{\theta}_{fr} + \frac{1}{2}\underline{\xi}_{fl}\left(\underline{x}, \tau, \underline{m}_{fl}, \underline{\sigma}_{fl}\right)\underline{\theta}_{fl}$$

$$g\left(\underline{x}, \tau \middle| \underline{\theta}_g, \underline{m}_g, \underline{\sigma}_g\right) = \frac{1}{2}\underline{\xi}_{gr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right)\underline{\theta}_{gr} + \frac{1}{2}\underline{\xi}_{gl}\left(\underline{x}, \tau, \underline{m}_{fl}, \underline{\sigma}_{fl}\right)\underline{\theta}_{gl}$$

By using (11) into (3), the error dynamic equation can be expressed as

$$\begin{aligned}\underline{\dot{e}} &= \left(A - Bk_c^T\right)\underline{e} + B\left(f\left(\underline{x}, \tau\right) - f\left(\underline{x}, \tau \middle| \underline{\theta}_f, \underline{m}_f, \underline{\theta}_f\right)\right) \\ &\quad + B\left(g\left(\underline{x}, \tau\right) - g\left(\underline{x}, \tau \middle| \underline{\theta}_g, \underline{m}_g, \underline{\theta}_g\right)\right)u + B\left(d\left(x, t\right) - \rho\left(\underline{e}^T PB \middle| \theta_p\right)\right)\end{aligned} \tag{12}$$

Applying double Taylor series expansion and let

$\underline{\tilde{\theta}}_{fr} = \underline{\theta}_{fr}^* - \underline{\theta}_{fr}$, $\underline{\tilde{m}}_{fr} = \underline{m}_{fr}^* - \underline{m}_{fr}$, $\underline{\tilde{\sigma}}_{fr} = \underline{\sigma}_{fr}^* - \underline{\sigma}_{fr}$, $\underline{\tilde{\theta}}_{fl} = \underline{\theta}_{fl}^* - \underline{\theta}_{fl}$, $\underline{\tilde{m}}_{fl} = \underline{m}_{fl}^* - \underline{m}_{fl}$, $\underline{\tilde{\sigma}}_{fl} = \underline{\sigma}_{fl}^* - \underline{\sigma}_{fl}$, $\underline{\tilde{\theta}}_{gr} = \underline{\theta}_{gr}^* - \underline{\theta}_{gr}$, $\underline{\tilde{m}}_{gr} = \underline{m}_{gr}^* - \underline{m}_{gr}$, $\underline{\tilde{\sigma}}_{gr} = \underline{\sigma}_{gr}^* - \underline{\sigma}_{gr}$, $\underline{\tilde{\theta}}_{gl} = \underline{\theta}_{gl}^* - \underline{\theta}_{gl}$, $\underline{\tilde{m}}_{gl} = \underline{m}_{gl}^* - \underline{m}_{gl}$, $\underline{\tilde{\sigma}}_{gl} = \underline{\sigma}_{gl}^* - \underline{\sigma}_{gl}$, $\underline{\tilde{\theta}}_p = \underline{\theta}_p^* - \underline{\theta}_p$, $\tilde{D}_m = D_m - \hat{D}_m$, we have

$$\begin{aligned}f(\underline{x}, \tau) - f\left(\underline{x}, \tau \middle| \underline{\sigma}_f, \underline{m}_f, \underline{\sigma}_f\right) &= -\frac{1}{2}\{[\underline{\xi}_{fr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right) - \underline{m}_{fr}\underline{\xi}_{mfr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right) \\ &\quad - \underline{\sigma}_{fr}\underline{\xi}_{\sigma fr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right)]\underline{\tilde{\theta}}_{fr} + (\underline{\tilde{m}}_{fr}\underline{\xi}_{mfr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right) \\ &\quad + \underline{\tilde{\sigma}}_{fr}\underline{\xi}_{\sigma fr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right)\underline{\theta}_{fr} + \underline{m}_{fr}^*\underline{\xi}_{mfr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right)\underline{\tilde{\theta}}_{fr} \\ &\quad + \underline{\sigma}_{fr}^*\underline{\xi}_{sfr}\left(\underline{x}, \tau, \underline{m}_{fr}, \underline{\sigma}_{fr}\right)\underline{\tilde{\theta}}_{fr} + [\underline{\xi}_{fl}\left(\underline{x}, \tau, \underline{m}_{fl}, \underline{\sigma}_{fl}\right) \\ &\quad - \underline{m}_{fl}\underline{\xi}_{mfl}\left(\underline{x}, \tau, \underline{m}_{fl}, \underline{\sigma}_{fl}\right) - \underline{\sigma}_{fl}\underline{\xi}_{\sigma fl}\left(\underline{x}, \tau, \underline{m}_{fl}, \underline{\sigma}_{fl}\right)]\underline{\tilde{\theta}}_{fl}\end{aligned}$$

$$+(\underline{\tilde{m}}_{fl}\underline{\xi}_{mfl}\left(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}\right)+\underline{\tilde{\sigma}}_{fl}\underline{\xi}_{\sigma fl}\left(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}\right))\underline{\theta}_{fl}$$

$$+\underline{m}_{fl}^{*}\underline{\xi}_{mfl}\left(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}\right)\underline{\tilde{\theta}}_{fl}+\underline{\sigma}_{fl}^{*}\underline{\xi}_{\sigma fl}\left(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}\right)\underline{\tilde{\theta}}_{fl}\} \quad (13)$$

and

$$g(\underline{x},\tau)-g\left(\underline{x},\tau\middle|\underline{\theta}_{g},\underline{m}_{g},\underline{\sigma}_{g}\right)=-\frac{1}{2}\{[\underline{\xi}_{gr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right)-\underline{m}_{gr}\underline{\xi}_{mgr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right)$$

$$-\underline{\sigma}_{gr}\underline{\xi}_{\sigma gr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right)]\underline{\tilde{\theta}}_{gr}+(\underline{\tilde{m}}_{gr}\underline{\xi}_{mgr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right)$$

$$+\underline{\tilde{\sigma}}_{gr}\underline{\xi}_{\sigma gr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right))\underline{\theta}_{gr}+\underline{m}_{gr}^{*}\underline{\xi}_{mgr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right)\underline{\tilde{\theta}}_{gr}$$

$$+\underline{\sigma}_{gr}^{*}\underline{\xi}_{\sigma gr}\left(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}\right)\underline{\tilde{\theta}}_{gr}+[\underline{\xi}_{gl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right)$$

$$-\underline{m}_{gl}\underline{\xi}_{mgl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right)-\underline{\sigma}_{gl}\underline{\xi}_{\sigma gl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right)]\underline{\tilde{\theta}}_{gl}$$

$$+(\underline{\tilde{m}}_{gl}\underline{\xi}_{mgl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right)+\underline{\tilde{\sigma}}_{gl}\underline{\xi}_{\sigma gl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right))\underline{\theta}_{gl}$$

$$+\underline{m}_{gl}^{*}\underline{\xi}_{mgl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right)\underline{\tilde{\theta}}_{gl}+\underline{\sigma}_{gl}^{*}\underline{\xi}_{\sigma gl}\left(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}\right)\underline{\tilde{\theta}}_{gl}\} \quad (14)$$

where the optimal parameter vectors are defined as follows:

$$\underline{\theta}_{f}^{*}=\arg\min_{\underline{\theta}_{f}\in R^{M}}\left[\sup_{\underline{x}\in R^{n}}\left|f(\underline{x},\tau\middle|\underline{\theta}_{f},\underline{m}_{f},\underline{\sigma}_{f})-f(\underline{x},\tau)\right|\right] \quad (15)$$

$$\underline{\theta}_{g}^{*}=\arg\min_{\underline{\theta}_{g}\in R^{M}}\left[\sup_{\underline{x}\in R^{n}}\left|g(\underline{x},\tau\middle|\underline{\theta}_{g},\underline{m}_{g},\underline{\sigma}_{g})-g(\underline{x},\tau)\right|\right] \quad (16)$$

$$\underline{m}_{f}^{*}=\arg\min_{\underline{\theta}_{f}\in R^{M}}\left[\sup_{\underline{x}\in R^{n}}\left|f(\underline{x},\tau\middle|\underline{\theta}_{f},\underline{m}_{f},\underline{\sigma}_{f})-f(\underline{x},\tau)\right|\right] \quad (17)$$

$$\underline{m}_{g}^{*}=\arg\min_{\underline{\theta}_{g}\in R^{M}}\left[\sup_{\underline{x}\in R^{n}}\left|g(\underline{x},\tau\middle|\underline{\theta}_{g},\underline{m}_{g},\underline{\sigma}_{g})-g(\underline{x},\tau)\right|\right] \quad (18)$$

$$\underline{\sigma}_{f}^{*}=\arg\min_{\underline{\theta}_{f}\in R^{M}}\left[\sup_{\underline{x}\in R^{n}}\left|f(\underline{x},\tau\middle|\underline{\theta}_{f},\underline{m}_{f},\underline{\sigma}_{f})-f(\underline{x},\tau)\right|\right] \quad (19)$$

$$\underline{\sigma}_{g}^{*}=\arg\min_{\underline{\theta}_{g}\in R^{M}}\left[\sup_{\underline{x}\in R^{n}}\left|g(\underline{x},\tau\middle|\underline{\theta}_{g},\underline{m}_{g},\underline{\sigma}_{g})-g(\underline{x},\tau)\right|\right] \quad (20)$$

In order to simplify all formulas, the following notations are defined as

$$T_{fr1}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})=\left[\underline{\xi}_{fr}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})-\underline{m}_{fr}\underline{\xi}_{m_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})-\underline{\sigma}_{fr}\underline{\xi}_{\sigma_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\right]$$

(21)

$$T_{fr2}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})=\left[\underline{\tilde{m}}_{fr}\underline{\xi}_{m_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})+\underline{\tilde{\sigma}}_{fr}\underline{\xi}_{\sigma_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\right] \quad (22)$$

$$T_{fl1}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})=\left[\underline{\xi}_{fl}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})-\underline{m}_{fl}\underline{\xi}_{m_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})-\underline{\sigma}_{fl}\underline{\xi}_{\sigma_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\right] \quad (23)$$

$$T_{fl2}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})=\left[\tilde{\underline{m}}_{fl}\underline{\xi}_{m_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})+\tilde{\underline{\sigma}}_{fl}\underline{\xi}_{\sigma_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\right] \quad (24)$$

$$T_{gr1}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})=\left[\underline{\xi}_{gr}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})-\underline{m}_{gr}\underline{\xi}_{m_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})-\underline{\sigma}_{gr}\underline{\xi}_{\sigma_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\right]$$

(25)

$$T_{gr2}(\underline{x},\tau,\underline{m}_{gr2},\underline{\sigma}_{gr2})=\left[\tilde{\underline{m}}_{gr}\underline{\xi}_{m_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})+\tilde{\underline{\sigma}}_{gr}\underline{\xi}_{\sigma_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\right] \quad (26)$$

$$+T_{gr2}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\theta}_{gr}+T_{gl1}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\tilde{\underline{\theta}}_{gr}$$

$$T_{gl1}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})=\left[\underline{\xi}_{gl}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})-\underline{m}_{gl}\underline{\xi}_{m_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})-\underline{\sigma}_{gl}\underline{\xi}_{\sigma_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\right] \quad (27)$$

$$T_{gl2}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})=\left[\tilde{\underline{m}}_{gl}\underline{\xi}_{m_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})+\tilde{\underline{\sigma}}_{gl}\underline{\xi}_{\sigma_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\right] \quad (28)$$

By using (21) ~ (28), the error dynamics (12) can be re-expressed as

$$\begin{aligned}
\dot{\underline{e}}&=\left(A-Bk_c^T\right)\underline{e}+B\{-\frac{1}{2}[T_{fr1}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\tilde{\underline{\theta}}_{fr}+T_{fr2}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\theta}_{fr}+T_{fl1}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\tilde{\underline{\theta}}_{fl}\\
&\quad+T_{fl2}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\theta}_{fl}]\}+B\{-\frac{1}{2}[T_{gr1}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\tilde{\underline{\theta}}_{gr}+T_{gr2}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\theta}_{gr}\\
&\quad+T_{gl1}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\tilde{\underline{\theta}}_{gr}+T_{gr2}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\theta}_{gr}]\}u+B\left(\rho\left(\underline{e}^TPB\middle|\underline{\theta}_p^*\right)-\rho\left(\underline{e}^TPB\middle|\underline{\theta}_p\right)\right)\\
&\quad+B\left(d\left(\underline{x},t\right)-\rho\left(\underline{e}^TPB\middle|\underline{\theta}_p^*\right)\right)+B\omega \qquad (29)
\end{aligned}$$

where ω is the bounded minimum approximation error of the interval type-2 time delay FNN, i.e., $|\omega|\le\Omega$

$$\begin{aligned}
\omega=-\frac{1}{2}\{&(\underline{m}_{fr}^*\underline{\xi}_{m_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})+\underline{\sigma}_{fr}^*\underline{\xi}_{\sigma_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr}))\tilde{\underline{\theta}}_{fr}\\
&+(\underline{m}_{fl}^*\underline{\xi}_{m_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})+\underline{\sigma}_{fl}^*\underline{\xi}_{\sigma_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}))\tilde{\underline{\theta}}_{fl}\\
&+(\underline{m}_{gr}^*\underline{\xi}_{m_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})+\underline{\sigma}_{gr}^*\underline{\xi}_{\sigma_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}))\tilde{\underline{\theta}}_{gr}\\
&+(\underline{m}_{gl}^*\underline{\xi}_{m_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})+\underline{\sigma}_{gl}^*\underline{\xi}_{\sigma_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}))\tilde{\underline{\theta}}_{gl}\} \qquad (30)
\end{aligned}$$

Following the preceding consideration, the following theorem is declared to show that the proposed overall control scheme is asymptotically stable.

Theorem 1: Consider the *n*th-order nonlinear dynamical time delay system in the form of (1) with the control law in (11), the all design parameters are adjusted by the adaptive laws (31)-(39)

$$\dot{\underline{\theta}}_{fr} = -r_1\left[\underline{\xi}_{fr}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr}) - \underline{m}_{fr}\underline{\xi}_{m_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr}) - \underline{\sigma}_{fr}\underline{\xi}_{\sigma_{fr}}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\right]^T\left(B^T P\underline{e}\right) \quad (31)$$

$$\dot{\underline{\theta}}_{fl} = -r_2\left[\underline{\xi}_{fl}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}) - \underline{m}_{fl}\underline{\xi}_{m_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl}) - \underline{\sigma}_{fl}\underline{\xi}_{\sigma_{fl}}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\right]^T\left(B^T P\underline{e}\right) \quad (32)$$

$$\dot{\underline{\theta}}_{gr} = -r_3\left[\underline{\xi}_{gr}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}) - \underline{m}_{gr}\underline{\xi}_{m_{gr}}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr}) - \sigma_{gr}\xi_{\sigma_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})\right]^T\left(B^T P\underline{e}\right)u \quad (33)$$

$$\dot{\underline{\theta}}_{gl} = -r_4\left[\underline{\xi}_{gl}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}) - \underline{m}_{gl}\underline{\xi}_{m_{gl}}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl}) - \sigma_{gl}\xi_{\sigma_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})\right]^T\left(B^T P\underline{e}\right)u \quad (34)$$

$$\dot{\underline{m}}_{fr} = -r_5\underline{\theta}_{fr}^T\underline{\xi}_{m_{fr}}^T(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\left(B^T P\underline{e}\right) \quad (35)$$

$$\dot{\underline{m}}_{fl} = -r_6\underline{\theta}_{fl}^T\underline{\xi}_{m_{fl}}^T(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\left(B^T P\underline{e}\right) \quad (36)$$

$$\dot{\underline{m}}_{gr} = -r_7\underline{\theta}_{gr}^T\underline{\xi}_{m_{gr}}^T(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\left(B^T P\underline{e}\right)u \quad (37)$$

$$\dot{\underline{m}}_{gl} = -r_8\underline{\theta}_{gl}^T\underline{\xi}_{m_{gl}}^T(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\left(B^T P\underline{e}\right)u \quad (38)$$

$$\dot{\underline{\sigma}}_{fr} = -r_9\underline{\theta}_{fr}^T\underline{\xi}_{\sigma_{fr}}^T(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\left(B^T P\underline{e}\right)$$

(39)

$$\dot{\underline{\sigma}}_{fl} = -r_{10}\underline{\theta}_{fl}^T\underline{\xi}_{\sigma_{fl}}^T(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\left(B^T P\underline{e}\right) \quad (40)$$

$$\dot{\underline{\sigma}}_{gr} = -r_{11}\underline{\theta}_{gr}^T\underline{\xi}_{\sigma_{gr}}^T(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\left(B^T P\underline{e}\right)u \quad (41)$$

$$\dot{\underline{\sigma}}_{gl} = -r_{12}\underline{\theta}_{gl}^T\underline{\xi}_{\sigma_{gl}}^T(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\left(B^T P\underline{e}\right)u \quad (42)$$

$$\dot{\underline{\theta}}_p = \gamma_{13}\underline{e}^T PB\zeta\left(\underline{e}^T PB\right) \quad (43)$$

$$\dot{D}_m = \gamma_{14}\left|\underline{e}^T PB\right| \quad (44)$$

where $r_i > 0,\ i = 1 \sim 14$. Based on the Barbalat's lemma[11], the tracking error $e(t)$ will asymptotically approaches to zero, i.e., $\lim_{t\to\infty} e(t) = 0$.

Proof:

To begin with, the Lyapunov function with time delay is defined as

$$V = \frac{1}{2}\underline{e}^T P\underline{e} + \frac{1}{2}\sum_{i=1}^{r}\int_{t-\tau_i}^{t}\underline{e}^T(v)\underline{e}(v)dv + \frac{1}{4\gamma_1}\tilde{\underline{\theta}}_{fr}{}^T\tilde{\underline{\theta}}_{fr} + \frac{1}{4\gamma_2}\tilde{\underline{\theta}}_{fl}{}^T\tilde{\underline{\theta}}_{fl} + \frac{1}{4\gamma_3}\tilde{\underline{\theta}}_{gr}{}^T\tilde{\underline{\theta}}_{gr}$$
$$+\frac{1}{4\gamma_4}\tilde{\underline{\theta}}_{gl}{}^T\tilde{\underline{\theta}}_{gl} + \frac{1}{4\gamma_5}\tilde{\underline{m}}_{fr}\tilde{\underline{m}}_{fr}^T + \frac{1}{4\gamma_6}\tilde{\underline{m}}_{fl}\tilde{\underline{m}}_{fl}^T + \frac{1}{4\gamma_7}\tilde{\underline{m}}_{gr}\tilde{\underline{m}}_{gr}^T + \frac{1}{4\gamma_8}\tilde{\underline{m}}_{gl}\tilde{\underline{m}}_{gl}^T + \frac{1}{4\gamma_9}\tilde{\underline{\sigma}}_{fr}\tilde{\underline{\sigma}}_{fr}^T$$
$$+\frac{1}{4\gamma_{10}}\tilde{\underline{\sigma}}_{fl}\tilde{\underline{\sigma}}_{fl}^T + \frac{1}{4\gamma_{11}}\tilde{\underline{\sigma}}_{gr}\tilde{\underline{\sigma}}_{gr}^T + \frac{1}{4\gamma_{12}}\tilde{\underline{\sigma}}_{gl}\tilde{\underline{\sigma}}_{gl}^T + \frac{1}{2\gamma_{13}}\tilde{\underline{\theta}}_p{}^T\tilde{\underline{\theta}}_p + \frac{1}{2\gamma_{14}}\tilde{D}_m^2 \quad (45)$$

Differentiating (45) with respect to time t along the trajectory (29) we obtain

$$\dot{V}=\frac{1}{2}\underline{\dot{e}}^T P\underline{e}+\frac{1}{2}\underline{e}^T P\underline{\dot{e}}+\frac{1}{2\gamma_1}\underline{\tilde{\theta}}_{fr}{}^T\underline{\dot{\tilde{\theta}}}_{fr}+\frac{1}{2\gamma_2}\underline{\tilde{\theta}}_{fl}{}^T\underline{\dot{\tilde{\theta}}}_{fl}+\frac{1}{2\gamma_3}\underline{\tilde{\theta}}_{gr}{}^T\underline{\dot{\tilde{\theta}}}_{gr}+\frac{1}{2\gamma_4}\underline{\tilde{\theta}}_{gl}{}^T\underline{\dot{\tilde{\theta}}}_{gl}$$
$$+\frac{1}{2\gamma_5}\underline{\dot{\tilde{m}}}_{fr}\underline{\tilde{m}}_{fr}^T+\frac{1}{2\gamma_6}\underline{\dot{\tilde{m}}}_{fl}\underline{\tilde{m}}_{fl}^T+\frac{1}{2\gamma_7}\underline{\dot{\tilde{m}}}_{gr}\underline{\tilde{m}}_{gr}^T+\frac{1}{2\gamma_8}\underline{\dot{\tilde{m}}}_{gl}\underline{\tilde{m}}_{gl}^T+\frac{1}{2\gamma_9}\underline{\dot{\tilde{\sigma}}}_{fr}\underline{\tilde{\sigma}}_{fr}^T+\frac{1}{2\gamma_{10}}\underline{\dot{\tilde{\sigma}}}_{fl}\underline{\tilde{\sigma}}_{fl}^T$$
$$+\frac{1}{2\gamma_{11}}\underline{\dot{\tilde{\sigma}}}_{gr}\underline{\tilde{\sigma}}_{gr}^T+\frac{1}{2\gamma_{12}}\underline{\dot{\tilde{\sigma}}}_{gl}\underline{\tilde{\sigma}}_{gl}^T+\frac{1}{\gamma_{13}}\underline{\tilde{\theta}}_p{}^T\underline{\dot{\tilde{\theta}}}_p+\frac{1}{\gamma_{14}}\dot{\tilde{D}}_m+\frac{1}{2}\sum_{i=1}^{r}\underline{e}^T(t)\underline{e}(t)-\frac{1}{2}\sum_{i=1}^{r}\underline{e}^T(t-\tau_i)\underline{e}(t-\tau_i) \tag{46}$$

Substituting (29) into (46), $\dot{V}$ can be rewritten as

$$\dot{V}=\frac{1}{2}\Big\{\left(A-Bk_c^T\right)\underline{e}+B\Big\{-\frac{1}{2}[T_{fr1}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\tilde{\theta}}_{fr}+T_{fr2}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\theta}_{fr}$$
$$+T_{fl1}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\tilde{\theta}}_{fl}+T_{fl2}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\theta}_{fl}]-\frac{1}{2}\Big[T_{gr1}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\tilde{\theta}}_{gr}$$
$$+T_{gr2}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\theta}_{gr}+T_{gl1}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\tilde{\theta}}_{gl}+T_{gl2}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\theta}_{gl}\Big]u\Big\}$$
$$+B\left(\rho\left(\underline{e}^T PB\middle|\underline{\theta}_p^*\right)-\rho\left(\underline{e}^T PB\middle|\underline{\theta}_p\right)\right)+B\omega+B\left(d\left(\underline{x},t\right)-\rho\left(\underline{e}^T PB\middle|\underline{\theta}_p^*\right)\right)\Big\}^T P\underline{e}$$
$$+\frac{1}{2}\underline{e}^T P\Big\{\left(A-Bk_c^T\right)\underline{e}+\frac{1}{2}B\Big\{-\frac{1}{2}[T_{fr1}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\tilde{\theta}}_{fr}+T_{fr2}(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\theta}_{fr}$$
$$+T_{fl1}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\tilde{\theta}}_{fl}+T_{fl2}(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\theta}_{fl}]-\frac{1}{2}[T_{gr1}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\tilde{\theta}}_{gr}$$
$$+T_{gr2}(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\theta}_{gr}+T_{gl1}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\tilde{\theta}}_{gl}+T_{gl2}(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\theta}_{gl}]u\}$$
$$+B\left(\rho\left(\underline{e}^T PB\middle|\underline{\theta}_p^*\right)-\rho\left(\underline{e}^T PB\middle|\underline{\theta}_p\right)\right)+B\omega+B\left(d\left(\underline{x},t\right)-\rho\left(\underline{e}^T PB\middle|\underline{\theta}_p^*\right)\right)\Big\}$$
$$+\frac{1}{2\gamma_1}\underline{\tilde{\theta}}_{fr}{}^T\underline{\dot{\tilde{\theta}}}_{fr}+\frac{1}{2\gamma_2}\underline{\tilde{\theta}}_{fl}{}^T\underline{\dot{\tilde{\theta}}}_{fl}+\frac{1}{2\gamma_3}\underline{\tilde{\theta}}_{gr}{}^T\underline{\dot{\tilde{\theta}}}_{gr}+\frac{1}{2\gamma_4}\underline{\tilde{\theta}}_{gl}{}^T\underline{\dot{\tilde{\theta}}}_{gl}+\frac{1}{2\gamma_5}\underline{\dot{\tilde{m}}}_{fr}\underline{\tilde{m}}_{fr}^T$$
$$+\frac{1}{2\gamma_6}\underline{\dot{\tilde{m}}}_{fl}\underline{\tilde{m}}_{fl}^T+\frac{1}{2\gamma_7}\underline{\dot{\tilde{m}}}_{gr}\underline{\tilde{m}}_{gr}^T+\frac{1}{2\gamma_8}\underline{\dot{\tilde{m}}}_{gl}\underline{\tilde{m}}_{gl}^T+\frac{1}{2\gamma_9}\underline{\dot{\tilde{\sigma}}}_{fr}\underline{\tilde{\sigma}}_{fr}^T+\frac{1}{2\gamma_{10}}\underline{\dot{\tilde{\sigma}}}_{fl}\underline{\tilde{\sigma}}_{fl}^T$$
$$+\frac{1}{2\gamma_{11}}\underline{\dot{\tilde{\sigma}}}_{gr}\underline{\tilde{\sigma}}_{gr}^T+\frac{1}{2\gamma_{12}}\underline{\dot{\tilde{\sigma}}}_{gl}\underline{\tilde{\sigma}}_{gl}^T+\frac{1}{\gamma_{13}}\underline{\tilde{\theta}}_p{}^T\underline{\dot{\tilde{\theta}}}_p+\frac{1}{\gamma_{14}}\dot{\tilde{D}}_m+\frac{1}{2}\sum_{i=1}^{r}\underline{e}^T(t)\underline{e}(t)$$
$$-\frac{1}{2}\sum_{i=1}^{r}\underline{e}^T(t-\tau_i)\underline{e}(t-\tau_i) \tag{47}$$

$$\leq-\frac{1}{2}\{\underline{e}^T(Q-rI)\underline{e}+\underline{\tilde{\theta}}_{fr}^T\left[-T_{fr1}^T(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\left(B^T P\underline{e}\right)-\frac{1}{r_1}\underline{\dot{\theta}}_{fr}\right]$$

$$+\tilde{\underline{\theta}}_{fl}^T\left[-T_{fl1}^T(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\left(B^TP\underline{e}\right)-\frac{1}{r_2}\dot{\underline{\theta}}_{fl}\right]$$

$$+\tilde{\underline{\theta}}_{gr}^T\left[-T_{gr1}^T(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\left(B^TP\underline{e}\right)u-\frac{1}{r_3}\dot{\underline{\theta}}_{gr}\right]$$

$$+\tilde{\underline{\theta}}_{gl}^T\left[-T_{gl1}^T(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\left(B^TP\underline{e}\right)u-\frac{1}{r_4}\dot{\underline{\theta}}_{gl}\right]$$

$$+\left[-\underline{\xi}_{m_{fr}}^T(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\theta}_{fr}\left(B^TP\underline{e}\right)-\frac{1}{r_5}\dot{\underline{m}}_{fr}\right]\tilde{\underline{m}}_{fr}^T$$

$$+\left[-\underline{\xi}_{m_{fl}}^T(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\theta}_{fl}\left(B^TP\underline{e}\right)-\frac{1}{r_6}\dot{\underline{m}}_{fl}\right]\tilde{\underline{m}}_{fl}^T$$

$$+\left[-\underline{\xi}_{m_{gr}}^T(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\theta}_{gr}\left(B^TP\underline{e}\right)u-\frac{1}{r_7}\dot{\underline{m}}_{gr}\right]\tilde{\underline{m}}_{gr}^T$$

$$+\left[-\underline{\xi}_{m_{gl}}^T(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\theta}_{gl}\left(B^TP\underline{e}\right)u-\frac{1}{r_8}\dot{\underline{m}}_{gl}\right]\tilde{\underline{m}}_{gl}^T$$

$$+\left[-\underline{\xi}_{\sigma_{fr}}^T(\underline{x},\tau,\underline{m}_{fr},\underline{\sigma}_{fr})\underline{\theta}_{fr}\left(B^TP\underline{e}\right)-\frac{1}{r9}\dot{\underline{\sigma}}_{fr}\right]\tilde{\underline{\sigma}}_{fr}^T$$

$$+\left[-\underline{\xi}_{\sigma_{fl}}^T(\underline{x},\tau,\underline{m}_{fl},\underline{\sigma}_{fl})\underline{\theta}_{fl}\left(B^TP\underline{e}\right)-\frac{1}{r_{10}}\dot{\underline{\sigma}}_{fl}\right]\tilde{\underline{\sigma}}_{fl}^T$$

$$+\left[-\underline{\xi}_{\sigma_{gr}}^T(\underline{x},\tau,\underline{m}_{gr},\underline{\sigma}_{gr})\underline{\theta}_{gr}\left(B^TP\underline{e}\right)u-\frac{1}{r_1}\dot{\underline{\sigma}}_{gr}\right]\tilde{\underline{\sigma}}_{gr}^T$$

$$+\left[-\underline{\xi}_{\sigma_{gl}}^T(\underline{x},\tau,\underline{m}_{gl},\underline{\sigma}_{gl})\underline{\theta}_{gl}\left(B^TP\underline{e}\right)u-\frac{1}{r_{12}}\dot{\underline{\sigma}}_{gl}\right]\tilde{\underline{\sigma}}_{gl}^T$$

$$+\tilde{\underline{\theta}}_p{}^T\left[\underline{e}^TPB\zeta\left(\underline{e}^TPB\right)-\frac{1}{\gamma_{13}}\dot{\underline{\theta}}_p\right]+\left|\underline{e}^TPB\right|D_m+\left|\underline{e}^TPB\right|\hat{D}_m-\frac{1}{\gamma_{14}}\tilde{D}_m\dot{D}_m+\underline{e}^TPB\omega \tag{48}$$

Substituting adaptive laws (31)-(44) into (48) we have

$$\dot{V}\le-\frac{1}{2}e^T(Q-rI)e+\underline{e}^TPB\omega \tag{49}$$

Since ω is the bounded minimum approximation error, Q and r can be determined such that $\dot{V}\le-\frac{1}{2}e^T(Q-rI)e+\underline{e}^TPB\omega<0$. Therefore, by the Barbalat's lemma[11], the tracking error $e(t)$ will asymptotically approaches to zero, i.e., $\lim_{t\to\infty}e(t)=0$. The proof is completed.

5. SIMULATION EXAMPLE

In this section, we will apply our adaptive interval type-2 fuzzy PI controller for a slave single-machine-infinite-bus (SMIB) power system described by delay differential equations (DDE) to track the reference trajectory which is the output trajectory of the master (SMIB) power system.

The master system of this DDE is given as follows.

$$\dot{x}_{m1} = x_{m2}$$

$$\dot{x}_{m2} = -2x_{m2} - 2\sin x_{m1} + 5\sin 5t + 3\sin(5(x_{m1}(t-\tau)))$$

where $\tau = 0.002\,\mathrm{sec}$ is delay time and training data are corrupted by white Gaussian noise with signal-to-noise ratio (SNR) 20 dB.

The slave system of this DDE is given as follows

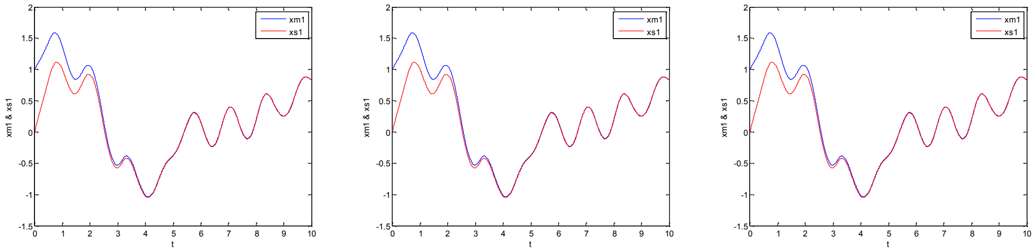
$$\dot{x}_{s1} = x_{s2}$$

$$\dot{x}_{s2} = -2x_{s2} - 2\sin x_{s1} + 5\sin 5t + 3\sin(5(x_{s1}(t-\tau))) + u(t) + d(t)$$

In order to show the advantage of the PI type switching structure, slow and small disturbance $d(t) = \cos(t)$ and fast and large disturbance $d(t) = 4\cos(5t)$ will be simulated.

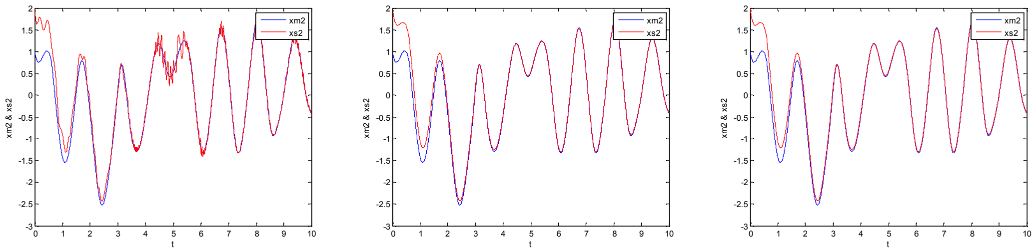
<u>Simulation 1:</u> <u>Slow and small disturbance</u> $d(t) = \cos(t)$

The output and reference trajectories x_{m1} and x_{s1} are given in Fig.2. It shows that the tracking performances of x_{m1} and x_{s1} are almost the same because the unknown system functions are approximated by TDFNN. The output and reference trajectories x_{m2} and x_{s2} are given in Fig.3. We can see that the tracking performance between x_{m2} and x_{s2} of the type-1 fuzzy control is much worse than of the tyepe-1 and the interval type-2 fuzzy PI controls. The control effort obtained by (11) is shown in Fig. 4. We can see that the chattering phenomena can be attenuated and the prescribed tracking performance can be preserved simultaneously. But in order to achieve the tracking performance, type-1 fuzzy control must expend more control than interval type-2 fuzzy control. Also, the 3D tracking performance is shown in Fig. 5 and the PI parameters adaptation K_P and K_I are described in Fig. 6.



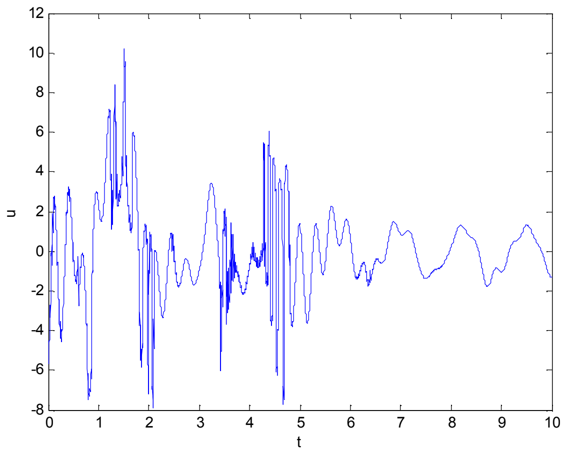
(a)Type-1 fuzzy control (b) Type-1 fuzzy PI control (c) Interval type-2 fuzzy PI control

Fig. 2. The output and reference trajectories x_{m1} and x_{s1}.

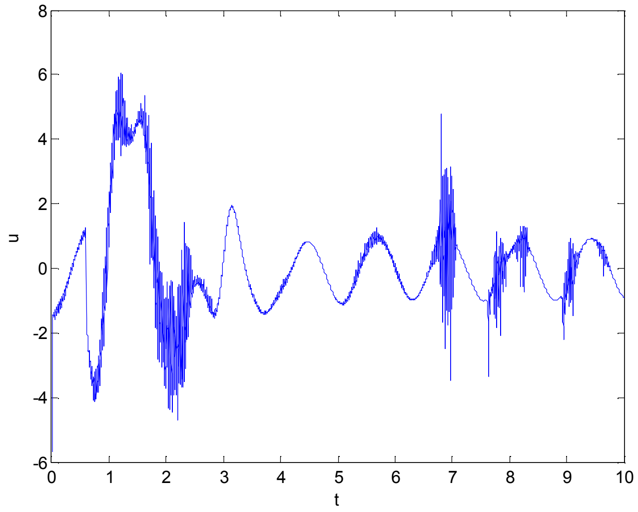


(a)Type-1 fuzzy control (b) Type-1 fuzzy PI control (c) Interval type-2 fuzzy PI control

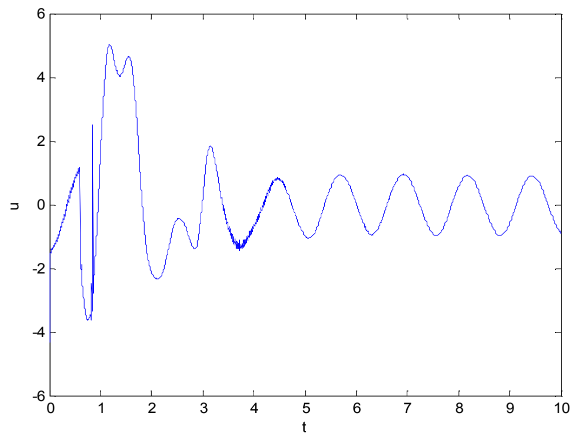
Fig. 3. The output and reference trajectories x_{m2} and x_{s2}.



(a)Type-1 fuzzy control

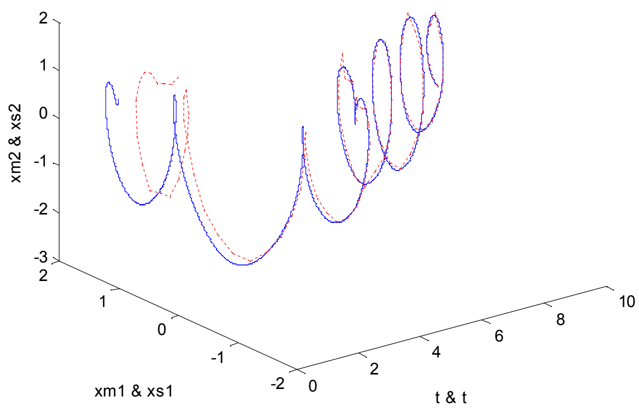


(b) Type-1 fuzzy PI control

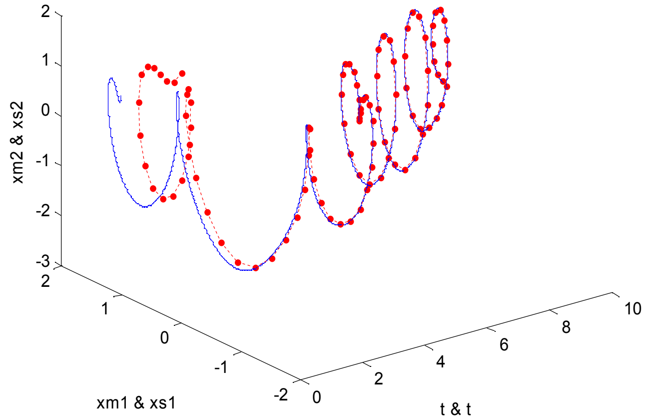


(c) Interval type-2 fuzzy PI control

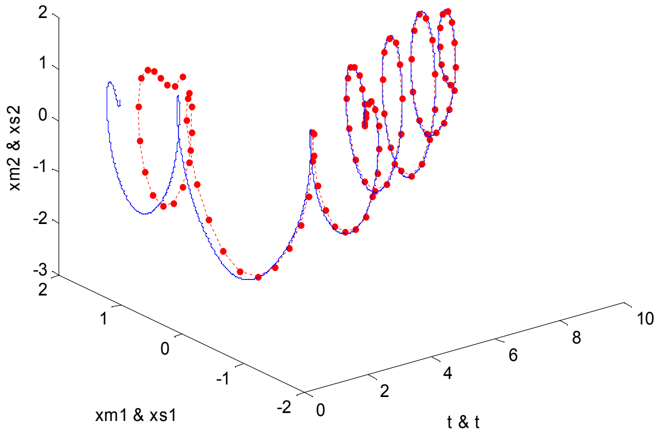
Fig. 4. The control effort.



(a) Type-1 fuzzy control

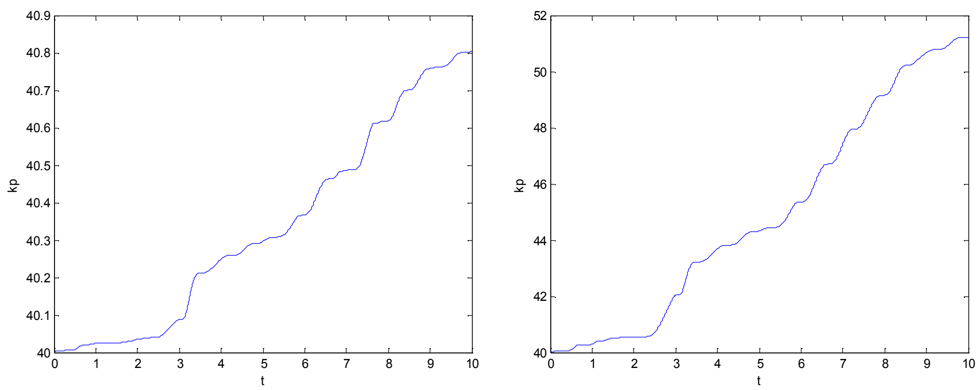


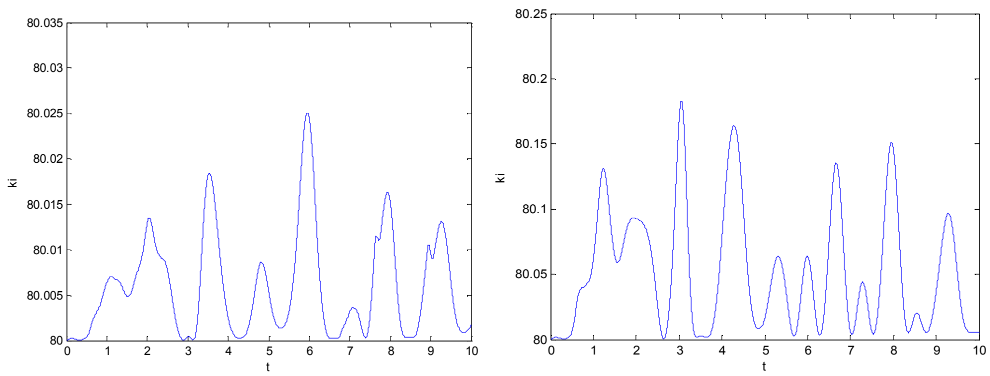
(b) Type-1 fuzzy PI control



(c) Interval type-2 fuzzy PI control

Fig. 5. The 3D tracking performances.



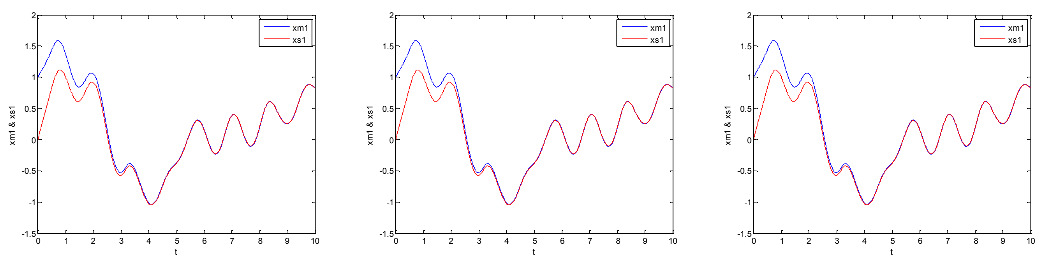


(a) Type-1 fuzzy PI control (b) Interval type-2 fuzzy PI control

Fig. 6. The PI parameters adaption

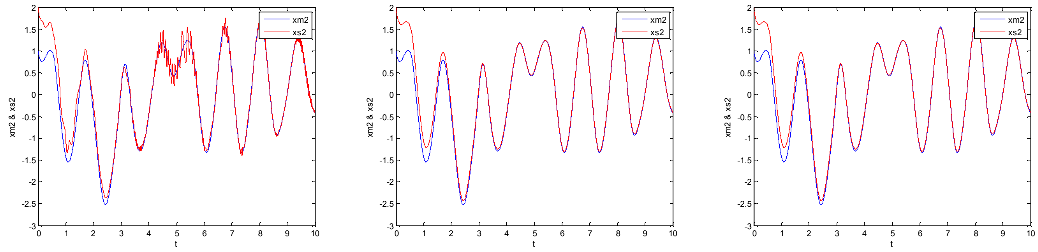
Simulation 2: **Large and small disturbance**: $d = 4\cos(5t)$

The output and reference trajectories x_{m1} and x_{s1} are given in Fig.7. It shows that the tracking performances of x_{m1} and x_{s1} are almost the same because the unknown system functions are approximated by TDFNN. The output and reference trajectories x_{m2} and x_{s2} are given in Fig.8. We can see that the tracking performance between x_{m2} and x_{s2} of the type-1 fuzzy control without PI is much worse than of the tyepe-1 and the interval type-2 fuzzy controls with PI. The control effort obtained by (11) is shown in Fig. 9. We can see that the chattering phenomena can be attenuated and the prescribed tracking performance can be preserved simultaneously. But in order to achieve the tracking performance, type-1 fuzzy control must expend more control effort than interval type-2 fuzzy control. Also, the 3D tracking performance is shown in Fig. 10 and the PI parameters adaptation K_P and K_I are described in Fig. 11.



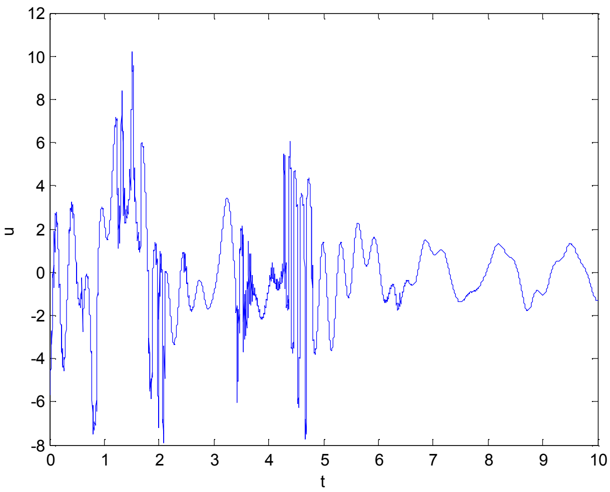
(a)Type-1 fuzzy control (b) Type-1 fuzzy PI control (c) Interval type-2 fuzzy PI control

Fig. 7. The output and reference trajectories x_{m1} and x_{s1}.

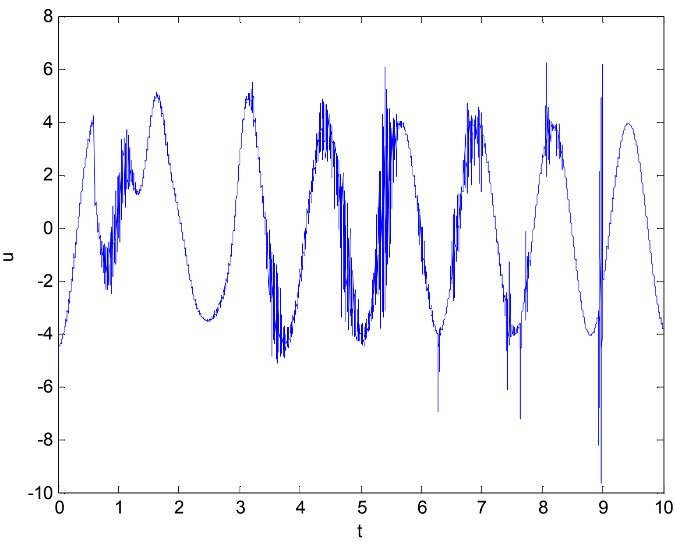


(a)Type-1 fuzzy control (b) Type-1 fuzzy PI control (c) Interval type-2 fuzzy PI control

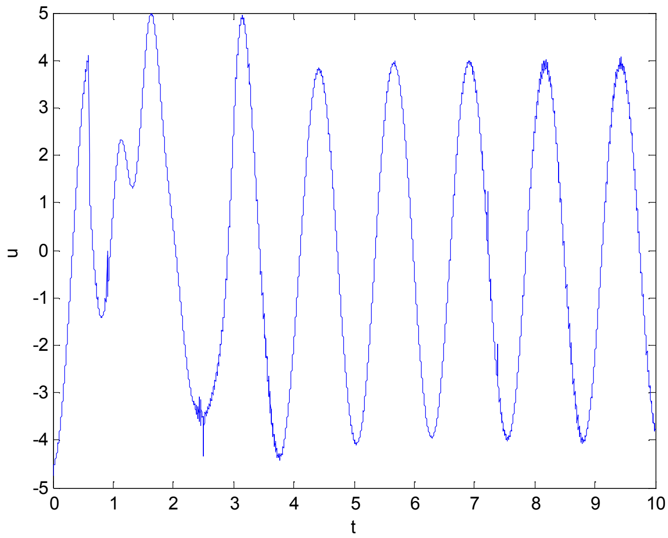
Fig. 8. The output and reference trajectories x_{m2} and x_{s2}.



(a)Type-1 fuzzy control

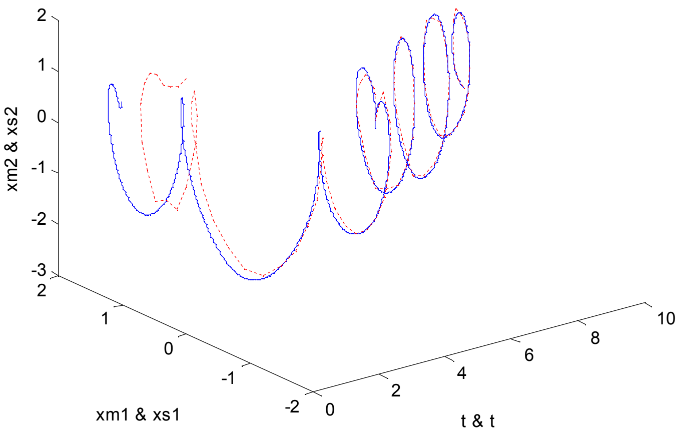


(b) Type-1 fuzzy PI control

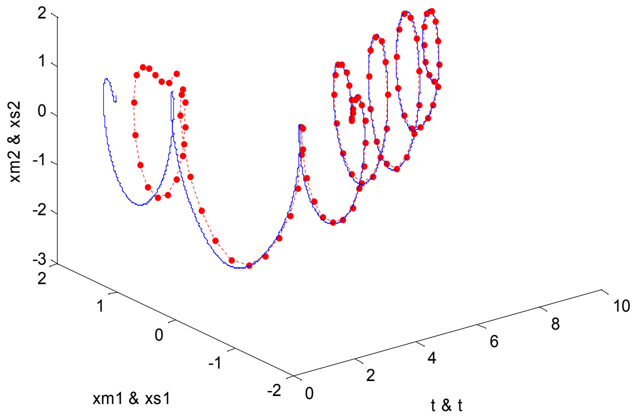


(c) Interval type-2 fuzzy PI control

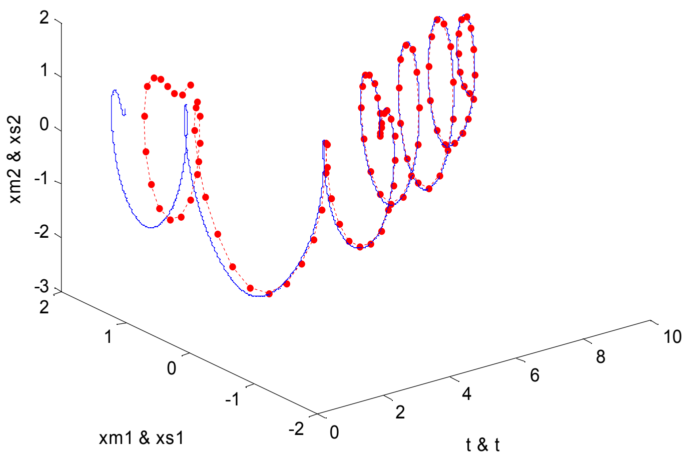
Fig. 9. The control effort.



(a)Type-1 fuzzy control

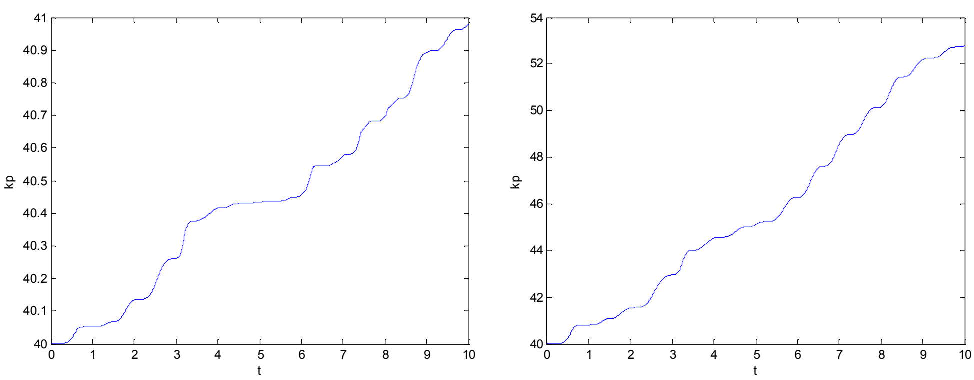


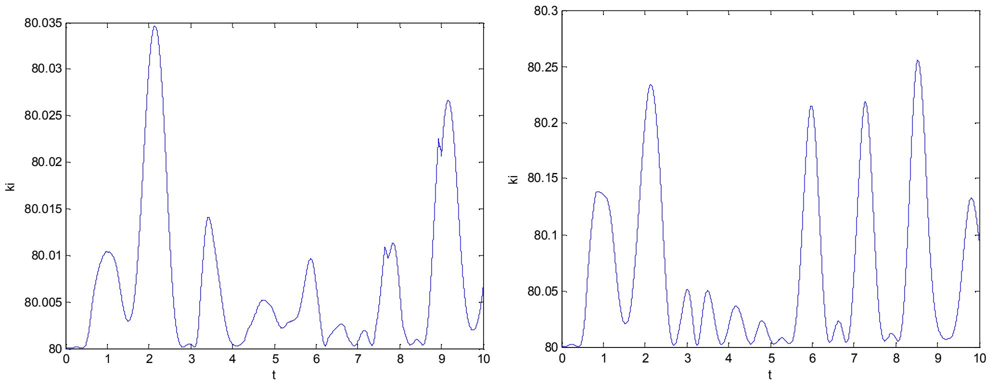
(b) Type-1 fuzzy PI control



(c) Interval type-2 fuzzy PI control

Fig. 10. The 3D tracking performances.





(a) Type-1 fuzzy PI control (b) Interval type-2 fuzzy PI control

Fig. 11. The PI parameters adaption

Remarks: From the above two example, the following remarks are make.

1. The tracking performance between x_{m2} and x_{s2} of type-1 fuzzy control is much worse than of the tyepe-1PI and the interval type-2 fuzzy PI controls.
2. In order to achieve the tracking performance, the type-1 fuzzy control must expend more control effort than the type-1 fuzzy PI control. Meanwhile, the type-1 fuzzy PI control must expend more control effort than the interval type-2 fuzzy PI control.

6. CONCLUSIONS

In order to provide robustness in the presence of fast and large disturbance and to eliminate the instability resulting from system time delay, adaptive interval type-2 fuzzy PI control scheme is proposed by incorporating AT2DFLC with PI controller. Interval type-2 time delay FNN is constructed so as to fully handle the linguistic and high level uncertainties and to estimate the behaviors of the system functions. Based on the Lyapunov theory of stability, the adaptive laws are derived for asymptotic stability of the closed loop system. In the meantime, the chattering phenomena can be significantly attenuated under various different disturbance conditions by using the PI control scheme. Simulation results show that the prescribed tracking performance can be preserved by the advocated control scheme. In the future, the proposed methodology will be extended to sampled-data control of nonlinear time delay systems and be applied to control 3-D overhead crane systems.

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