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SOME CONTRIBUTIONS TO THE DOMINATION NUMBER IN PICTURE FUZZY GRAPHS BASED ON THE STRONG EDGE

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Abstract. The Picture fuzzy graph is a useful mathematical tool for dealing with uncertain real world problems when fuzzy graphs and intuitionistic fuzzy graphs are ineffective. It is especially helpful in cases where there are numerous options of the same type, such as yes, no, abstain, and refusal. The main objective of this study is to define the dominating set and domination number in a picture fuzzy graph by using strong edges. The picture fuzzy dominating set is introduced based on the importance of the notion of domination and its applications in several instances. Furthermore, some important properties are discussed. Some theorems are proved with examples. Finally, an application of Picture fuzzy domination is provided to place the minimum number of fire stations with adequate infrastructure and equipment for keeping pace with advancement of technology in order to reduce massive loss of life and property due to fire accidents in the high rise buildings which are located in metropolitan city.

1. Introduction

L.A. Zadeh [20] first proposed the concept of fuzzy sets in 1965, and it has been successfully applied to a range of uncertain real-life scenarios. A fuzzy set is an extended version of a crisp set in which members have varying degree of membership functions. This crisp set cannot handle uncertain real-world problems because it just has two values: 0 and 1 (no or yes). Instead of considering 0 or 1, a fuzzy gives its elements with membership values between 0 and 1 for a better outcome. In other situations, however, such single membership degree values are unable to cope with the uncertainty. To deal with this type of unknown scenario, Atanassov [2] introduced the intuitionistic fuzzy set, which includes an extended membership value known as the hesitation margin. The intuitionistic fuzzy set is an advanced version of Zadeh's fuzzy set. It is more accessible and effective to work with uncertainty than a standard fuzzy set because of the presence of hesitation margin. When human perception and knowledge are completely unexpected and unclear, the intuitionistic fuzzy set is implemented in real-world scenarios. In recent years, scientists and analysts have successfully applied the concept of intuitionistic fuzzy set-in image processing, social networks, machine learning, decision making, and medical diagnosis among other fields. But the concept of neutrality degree, however, is not included in the intuitionistic fuzzy set theory. However, the degree of neutrality must be

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addressed in many common scenarios, such as democratic election stations, medical diagnosis recognition, social networks, decision making and so on.

Cuong and Kreinovich [4] introduced the picture fuzzy set as an improved kind of intuitionistic fuzzy set to satisfy the neutrality degree. The degree of positive membership value $\mu : X \to [0,1]$, neutral membership value $\eta : X \to [0,1]$ and negative membership value $\gamma : X \to [0,1]$ build up the picture fuzzy set under the condition $0 \leq \mu(\mathrm{x}) + \eta(\mathrm{x}) + \gamma(\mathrm{x}) \leq 1$, where $\pi(\mathrm{x}) = 1 - (\mu(\mathrm{x}) + \eta(\mathrm{x}) + \gamma(\mathrm{x}))$ is the degree of refusal membership values of a vertex. The notion of picture fuzzy graph was suggested by Cen Zuo et al. [3], which is based on picture fuzzy relations for the effective way of expressing ambiguity. Phong et al. [15] have proposed a variety of Picture fuzzy relation compositions. Xiao wei [19] investigated the regular picture fuzzy graphs and its properties. This motivated us to introduce the concept of the picture fuzzy dominating set.

This paper is constructed as follows. Section 2 provides the primary definitions of picture fuzzy graphs, whereas Section 3 provides definitions of picture fuzzy dominating set and picture fuzzy domination number. Some propositions and theorems of this domination parameter are discussed. In section 4, an algorithm is provided to compute the picture fuzzy dominating set and its domination number. Section 5 gives an application of domination in the picture fuzzy graph.

2. Preliminaries

In this section, some basic definitions which are used to construct theorems and properties related to the picture fuzzy graph are given.

Definition 2.1. A fuzzy graph $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \vee$ and μ is a symmetric fuzzy relation on σ.

Definition 2.2. An Intuitionistic Fuzzy Graph is of the form $G = (V, E)$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that the mapping $\mu_1 : V \to [0,1]$ is the degree of membership and the mapping $\gamma_1 : V \to [0,1]$ is the degree of non-membership of the element $v_i \in V$ respectively and $O \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V(i = 1, 2 \ldots n)$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \min((v_i), \mu_1(v_j))$$
$$\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_1(v_j))$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1, \forall (v_i, v_j)\, E.(i, j = 1, 2 \ldots\ldots, n)$

Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of nonmembership of the Vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

In an Intuitionistic Fuzzy Graph G, when $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.

Definition 2.3. A pair $G = (V, E)$ is known as Picture Fuzzy Graph (PFG) if

(i) $V = \{v_1, v_2 \ldots v_n\}$ such that $\mu_1 : V \to [0,1]\eta_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ degree of Positive, neutral and negative membership function of the vertex $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \eta_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, \quad i = 1, 2 \ldots n$

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1], \eta_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le \min(\mu_1(v_i), \mu_1(v_j))$$
$$\eta_2(v_i, v_j) \le \min(\eta_1(v_i), \eta_1(v_j))$$
$$\gamma_2(v_i, v_j) \le \max(\gamma_1(v_i), \gamma_2(v_j))$$

Where $0 \le \mu_2(v_i, v_j) + \eta_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E, \quad i, j = 1, 2 \ldots n$. Here the 4-tuple $(v_i, \mu_{1i}, \eta_{1i}, \gamma_{1i})$ denotes the degree of positive membership, neutral membership and negative membership of the vertex v_i and the 4 -tuple $(e_{ij}, \mu_{2ij}, \eta_{2ij}, \gamma_{2ij})$ denotes the degree of positive membership, neutral membership and negative membership of the edge relation $e_{ij} = (v_i, v_j)$.

Example 2.4.

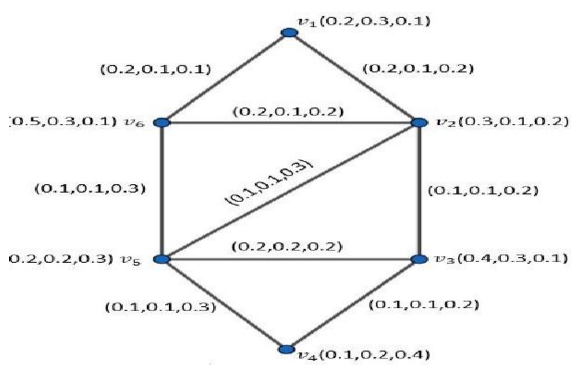

FIGURE 1.

Definition 2.5. A Picture fuzzy graph $G = (V, E)$ is said to be complete, if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$, $\eta_2(v_i, v_j) = \min(\eta_1(v_i), \eta_1(v_j))$ and $\gamma_2(v_i, v_j) = \max(\gamma_1(v_i), \gamma_1(v_j))$ for every $v_i \ \mathcal{V} \in V$

Definition 2.6. An edge (v_i, v_j) is called a strong edge, if $\mu_2(v_i, v_j) \ge \mu'^{\infty}(v_i, v_j)$, $\eta_2(v_i, v_j) \ge \eta_2^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) \le \gamma_2'^{\infty}(v_i, v_j)$ For every $v_i, v_j \in V$, where $\mu_2'^{\infty}(v_i, v_j), \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2'^{\infty}(v_i, v_j)$ is the strength of the connectedness between v_i and v_j in the picture fuzzy graph obtained from G by deleting the edge (v_i, v_j)

Definition 2.7. The strong degree of a vertex v_i in the Picture fuzzy graph $G = (V, E)$ is defined to be the addition of the weights of the strong edges incident at v_i. It is denoted by $d_s(v_i)$. Then the minimum strong degree of PFG G is defined as

$$\delta_s(G) = \min\{d_s(v_i)/v_i \in V\}$$

The maximum strong degree of PFG G is defined as $\Delta_s(G) = \max\{d_s(v_i)/v_i \in V\}$

Definition 2.8. Two vertices v_i and v_j are said to be neighbors in PFG if either one of the following conditions hold.

i) $\mu_2(v_i, v_j) > 0, \eta_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) > 0$
ii) $\mu_2(v_i, v_j) = 0, \eta_2(v_i, v_j) \geq 0, \gamma_2(v_i, v_j) > 0$
iii) $\mu_2(v_i, v_j) > 0, \eta_2(v_i, v_j) = 0, \gamma_2(v_i, v_j) \geq 0$
iv) $\mu 2(v_i, v_j) \geq 0, \eta_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) = 0, \quad \forall v_i, v_j \in V$

Definition 2.9. Let v_i be a vertex in a Picture fuzzy graph $G = (V, E)$ then $N_s(V_i) = \{V_j \in V : (V_i, V_j) \text{ is a strong edge }\}$ is called strong neighborhood of v_i. $N_s[v_i] = N_s(v_i) \cup \{v_i\}$ is called the closed strong neighborhood of v_i

Definition 2.10. A vertex $v_i \in V$ of the PFG $G = (V, E)$ is said to be an isolated vertex if $\mu_2(v_i, v_j) = 0, \eta_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for all $v_i \in V$ i. $e)\, N(u) = \varphi$. Thus, an isolated vertex does not dominate any other vertex in G.

3. Properties on Picture Fuzzy Dominating Set and its Domination Number

In this section, the picture fuzzy dominating set and its domination number are defined. Some propositions and theorems are stated and proved.

Definition 3.1. Let $G = (V, E)$ be a Picture fuzzy graph. Let $v_i, v_j \in V$. Then v_i dominates v_j in G if there exists a strong edge between them.

Definition 3.2. A dominating set D_{pf} of the $PFGG$ is said to be minimal picture fuzzy dominating set if there is no proper subset of D_{pf} is a picture fuzzy dominating set.

Definition 3.3. A dominating set D_{pf} of the $PFGG$ is said to be minimal picture fuzzy dominating set if there is no proper subset of D_{pf} is a picture fuzzy dominating set.

Definition 3.4. The minimum cardinality among all picture fuzzy dominating set is called domination number or lower domination number of G and it is denoted by $\gamma_{pf}(G)$.

The maximum cardinality among all dominating set is called upper domination number of G and is denoted by $\Gamma_{pf}(G)$.

Definition 3.5. Any two vertices in a PFG $G = (V, E)$ is said to be independent if there is no strong edge between them.

Definition 3.6. A subset D of V is said to be an independent set of G if $\mu_2(v_i, v_j) < \mu'^{\infty}(v_i, v_j), \eta_2(v_i, v_j) < \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) > \gamma_2'^{\infty}(v_i, v_j)$ for all $v_i, v_j \in V$

Definition 3.7. An independent set D of G in a Picture fuzzy graph $G = (V, E)$ is said to be maximal independent, if for every vertex $v_i \in V - D$, the set $D \cup \{v_i\}$ is not independent.

Definition 3.8. The minimum cardinality among all maximal independent set is called lower independence number of G and it is denoted by $i_{pf}(G)$.

Definition 3.9. The maximum cardinality among all maximal independent set is called upper independence number of G and it is denoted by $I_{pf}(G)$.

Definition 3.10. The weight of a picture fuzzy dominating set D_{pf} is defined as

$$W\left(D_{pf}\right)=\sum_{v_i\in D_{pf}}\left(\mu_2\left(v_i,v_j\right),\eta_2\left(v_i,v_j\right),\gamma_2\left(v_i,v_j\right)\right)$$

Where $\mu_2\left(v_i,v_j\right),\eta_2\left(v_i,v_j\right)$ and $\gamma_2\left(v_i,v_j\right)$ are the minimum of the positive, neutral and the maximum of the negative membership values of the strong edges incidents at v_i respectively.

The picture fuzzy domination number of a PFG G is defined as the minimum value of picture fuzzy dominating sets of G and is denoted by $\gamma_w(G)$

Example 3.11.

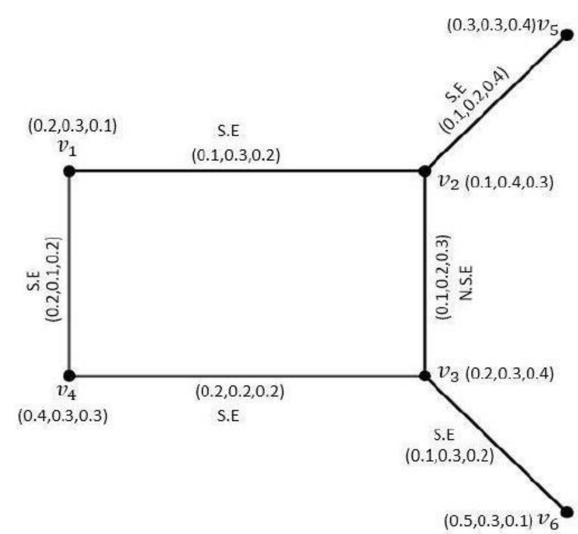

FIGURE 2.

Here $\left(v_1,v_2\right),\left(v_3,v_4\right),\left(v_4,v_1\right),\left(v_2,v_5\right)$ and $\left(v_3,v_6\right)$ are strong edges. The minimum picture fuzzy dominating set $D_{pf}=\{v_2,v_3\}$

$$\begin{aligned}W_{pf}&=\sum_{v_i\in D_{pf}}\left(\mu_2\left(v_i,v_j\right),\eta_2\left(v_i,v_j\right),\gamma_2\left(v_i,v_j\right)\right)\\&=(0.1,0.2,0.4)+(0.1,0.2,0.2)\\&=0.45+0.55=1.00\\W_{pf}&=1.00\end{aligned}$$

Here $\left(v_1,v_2\right)\left(v_2,v_3\right)\left(v_4,v_5\right)\left(v_5,v_1\right)$ are strong edges.

The picture fuzzy dominating sets are

$$D_{pf_1} = \{v_2, v_5\}, \quad D_{pf_2} = \{v_4, v_2\}, \quad D_{pf_3} = \{v_3, v_5\}$$

$$W_{pf_1}(v_1) = (0.1, 0.2, 0.2) = 0.55$$
$$W_{pf_2}(v_2) = (0.1, 0.2, 0.2) = 0.55$$
$$W_{pf3}(v_3) = (0.2, 0.3, 0.1) = 0.7$$
$$W_{pf_4}(v_4) = (0.2, 0.1, 0.2) = 1.1$$
$$W_{pf_5}(v_5) = (0.1, 0.1, 0.2) = 1$$
$$|D_{pf_1}| = 1.55, \quad |D_{pf_2}| = 1.65, \quad |D_{pf_3}| = 1.7$$

The minimum weight of a picture fuzzy dominating set $D_{pf_1} = \{v_2, v_5\}$.
The weight of a picture fuzzy domination number $\gamma_w(G) = 0.55 + 1 = 1.55$

Theorem 3.12. *Let $G = (V, E)$ be a picture fuzzy graph. A picture fuzzy dominating set D_{pf} is a minimal picture fuzzy dominating set iff for every $v_i \in D_{pf}$, one of the following conditions must hold.*

(i) v_i is not a strong neighbor of any vertex in D_{pf}
(ii) $\exists$ a vertex $v_j \in V - D_{pf} \ni N_s(v_j) \cap D_{pf} = \{v_i\}$

Proof. Let $G = (V, E)$ be a picture fuzzy graph. Let D_{pf} be a picture fuzzy dominating set. Assume that D_{pf} is a minimal picture fuzzy dominating set of G. Then $\forall v_i \in D_{pf}$, $D_{pf-}\ \{v_i\}$ need not be a picture fuzzy dominating set. Therefore, any vertex in $D_{pf}\ -\{vi\}$ doesn't dominate the vertex $v_j \in V - (D_{pf} - \{v_j\})$

If $v_j = v_i$, then v_j is not a strong neighbor of the vertex in D_{pf}. If $v_j \neq v_i$, then any vertex in $D_{pf} - \{v_j\}$ does not dominate v_j, but any vertex in D_{pf} dominates v_j. Therefore, the vertex v_j is a strong neighbor of $v_i \in D_{pf}$ i.e $N_s(v_j) \cap D_{pf} = \{v_i\}$

Conversely, let D_{pf} is a picture fuzzy dominating set and $\forall v_i \in D_{pf}$ with one of the conditions hold. Suppose let us assume that D_{pf} is not a minimal picture fuzzy dominating set, then $\exists$ a vertex $v_i \in D_{pf}, D_{pf} - \{v_i\}$ is a picture fuzzy dominating set. Therefore the strong edge exists between v_i and any of the vertices in $D_{pf} - \{v_i\}$. This implies that the condition(i) is not satisfied. If $D_{pf} - \{v_i\}$ is a picture fuzzy dominating set, then every vertex in $V - D_{pf}$ is a strong neighbor to at least one vertex in $D - \{v_i\}$. This implies that the condition (ii) is not satisfied. which is a contradiction to our assumption. Hence D_{pf} is a minimal picture fuzzy dominating set. □

Theorem 3.13. *Let $G = (V, E)$ be a complete bipartite picture fuzzy graph. If D_{pf} be a picture fuzzy dominating set,then the picture fuzzy domination number $\gamma_{pf} = \min |V_i| + \min |v_j|$ where $v_i \in V_1$ and $v_j \in V_2$*

Proof. Let $G = (V, E)$ be a complete bipartite fuzzy graph. By the definition of the complete bipartite, every edge in G is a strong edge. Therefore $D_{pf} = \{v_i, v_j\}$ is a picture fuzzy dominating set of G with minimum picture fuzzy cardinality for $v_i \in V_1$ and $v_j \in V_2$. Hence the picture fuzzy domination number

$$\gamma_{pf} = \ min\left\{\sum_{v_i \in V} \frac{1 + \mu_1(v_i) + y_1(v_i) - \gamma_1(v_i)}{2}\right\}$$

$$+\min\left\{\sum_{v_j\in V}\frac{1+\mu_1(v_j)+y_1(v_j)-\gamma_1(v_j)}{2}\right\}$$

Example 3.14.

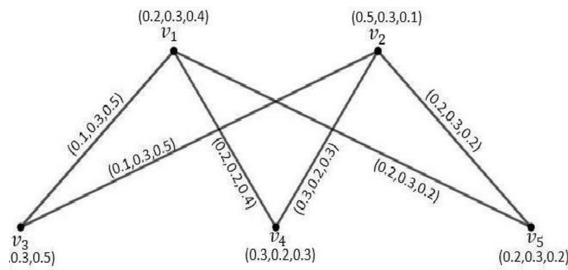

FIGURE 3.

Here $V_1=\{v_1,v_2\}$ and $V_2=\{v_3,v_4,v_5\}$.
The minimal picture fuzzy dominating sets are $\{v_1,v_3\},\{v_1,v_4\},\{v_1,v_5\}$, $\{v_2,v_3\},\{v_2,v_4\}$ and $\{v_2,v_5\}$
The minimal picture fuzzy dominating set with minimum cardinality is $D_{pf}=\{v_1,v_3\}$

$$\begin{aligned}\gamma_{pf}&=\min\{0.55,0.85\}+\min\{0.45,0.6,0.65\}\\&=0.55+0.45\\\gamma_{pf}&=1.0\end{aligned}$$

Theorem 3.15. *Let $G=(V,E)$ be a picture fuzzy graph of order p. If D_{pf} be the picture fuzzy dominating set with domination number $\gamma_{pf}(G)$, then the following inequalities hold.*

(i) $\gamma pf(G)\le p-\Delta_N(G)\le p-\Delta s(G)$
(ii) $\gamma pf(G)\le p-\delta_N(G)\le p-\delta_s(G)$

Proof. Let $v_i,v_j\in V$ of the picture fuzzy graph $G=(V,E)$. Then $p-\Delta_N(G)$ be the difference between the order p and the maximum neighborhood degree. It is clear that

$$\gamma_{pf}(G)\le p-\Delta_N(G) \tag{3.1}$$

Similarly, $p-\delta_s(G)$ is the difference between the order P and the minimum neighborhood degree. It is clear that

$$\gamma_{pf}(G)\le p-\delta_s(G) \tag{3.2}$$

$\Delta_S(G)\le\Delta_N(G)$ and $\delta_S(G)\le\delta_N(G)$ gives that

$$p-\Delta_s(G)\ge p-\Delta_N(G)\quad\text{and} \tag{3.3}$$
$$p-\delta_s(G)\ge p-\delta_N(G) \tag{3.4}$$

From inequalities (3.1) and (3.3), we have

$$\gamma_{pf}(G) \leq p - \Delta_N(G) \leq p - \Delta_s(G) \tag{3.5}$$

From inequalities (3.2) and (3.4), we have

$$\gamma pf(G) \leq p - \delta_N(G) \leq p - \delta_s(G) \tag{3.6}$$

Hence inequalities are proved. □

Example 3.16. For the given PFG G, the order is calculated as

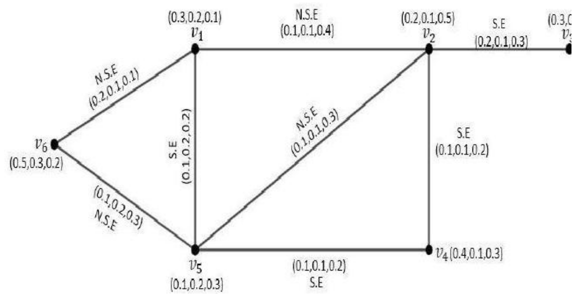

FIGURE 4.

$$\begin{aligned} p &= |v_1| + |v_2| + |v_3| + |v_4| + |v_5| + |v_6| \\ &= 0.7 + 0.4 + 0.45 + 0.6 + 0.5 + 0.8 \\ &= 3.48 \end{aligned}$$

Picture fuzzy dominating set is $D_{pf} = \{v_2, v_5, v_6\}$

$$\begin{aligned} \gamma_{pf} &= 0.4 + 0.5 + 0.8 = 1.7 \\ \Delta_N(G) &= 1.05, \Delta_s(G) = 1.0 \\ \delta_N(G) &= 0.5, \delta_s(G) = 0 \end{aligned}$$

Therefore, the inequality (3.5) becomes $1.7 \leq 3.48 - 1.05 \leq 3.48 - 1.0$

i.e) $1.7 \leq 2.43 \leq 2.48$

Similarly, the inequality (3.6) becomes $1.7 \leq 2.98 \leq 3.48$. Hence inequalities are verified.

Theorem 3.17. *Let $G = (V, E)$ be a PFG. If $D_{ipf} \subseteq V$ be a picture fuzzy independent set of G, then the subset $V - D_{ipf}$ is a vertex covering of G.*

Proof. By the Definition 3.5, if D_{ipf} is a picture fuzzy independent set, then there is no two vertices of D_{ipf} are adjacent and each vertex of D_{ipf} is incident with at least one vertex of $V - D_{ipf}$. Hence $V - D_{ipf}$ is a vertex covering of the picture fuzzy graph G. □

Theorem 3.18. *Let $G = (V, E)$ be a picture fuzzy graph without independent vertices. Let p be the order of G. If S be the vertex covering and D_{ipf} be the picture fuzzy independent set of G. Then $\alpha_{pf}(G) + \beta_{pf}(G) = p$, where $\alpha_{pf}(G)$ and $\beta_{pf}(G)$ are cardinalities of vertex cover and picture fuzzy independent set of G.*

Proof. Let D_{ipf} be the independent picture fuzzy set of G and S be the vertex covering of G such that $|D_{ipf}| = \beta_{pf}(G)$ and $|S| = \alpha_{pf}(G)$. By theorem 3.12, D_{ipf} is an picture fuzzy independent set. Then $V - D_{ipf}$ is a vertex covering of G. Hence $|S| \leq |V - D_{ipf}|$. It gives $\alpha_{pf}(G) \leq p - \beta_{pf}(G)$. This implies

$$\alpha_{pf}(G) + \beta_{pf}(G) \leq p \tag{3.7}$$

Also, $V-(V-S) = S$ is a vertex cover, Since $V-S$ is an picture fuzzy independent set of G. Hence $|D_{ipf}| \geq |V - S|$. This implies $\beta_{pf}(G) \geq p - \alpha_{pf}(G)$. It gives

$$\alpha_{pf}(G) + \beta_{pf}(G) \geq p \tag{3.8}$$

From (3.7) and (3.8), $\alpha_{pf}(G) + \beta_{pf}(G) = p$ □

Theorem 3.19. *Let $G = (V, E)$ be a picture fuzzy graph without isolated vertices. If D_{pf} be a minimal picture fuzzy dominating of G. Then $V - D_{pf}$ is a picture fuzzy dominating set.*

Proof. Let $v_i \in D_{pf}$. Since there is no isolated vertices in $G, \exists v_j \in N_s(v_i)$ which implies that $v_j \in V - D_{pf}$. Therefore every vertex of D_{pf} is dominated by some vertices of $V - D_{pf}$. Hence $V - D_{pf}$ is a picture fuzzy dominating set. □

Theorem 3.20. *Let $G = (V, E)$ be a picture fuzzy graph. A picture fuzzy independent set D_{ipf} of G is a maximal picture fuzzy independent set iff the set D_{ipf} is a picture fuzzy independent set as well as picture fuzzy dominating set.*

Proof. Let $G = (V, E)$ be a Picture fuzzy graph. Let D_{ipf} be the maximal picture fuzzy independent set. Then $\forall v_i \in V - D_{ipf}$ such that $D_{ipf}U\{v_i\}$ is not an independent picture fuzzy set. $\forall v_i \in V - D_{ipf}$ such that (v_i, v_j) is a strong edge. Therefore, D_{ipf} is a picture fuzzy dominating set. Thus, D_{ipf} is picture fuzzy independent as well as picture fuzzy dominating set.

Conversely, let us assume that D_{ipf} is a picture fuzzy independent set as well as picture fuzzy dominating set. Suppose D_{ipf} is not a maximal picture fuzzy independent set, then $\exists$ a vertex $v_i \in V - D_{ipf}, D_{ipf} \cup \{v_i\}$ is a picture fuzzy independent set. This implies that there is no vertex in D_{ipf} is a strong neighbor to v_i. It gives that D_{ipf} need not be a picture fuzzy dominating set which is a contradiction to our assumption. Therefore, D_{ipf} is a maximal picture fuzzy independent set. □

Theorem 3.21. *Let $G = (V, E)$ be a Picture fuzzy graph. Then every maximal picture fuzzy independent set D_{ipf} is a minimal picture fuzzy dominating set.*

Proof. Let D_{ipf} be the picture fuzzy independent set which is also maximal in a PFG $G = (V, E)$. Then by theorem 3.15, D_{ipf} is a picture fuzzy dominating set. Suppose let us assume that D_{ipf} is a minimal picture fuzzy dominating set, then there exists $v_i \in D_{ipf}, D_{ipf} - \{v_i\}$ is a picture fuzzy dominating set. If $V - \{D_{ipf} - \{v_i\}\}$ is dominated by $D_{ipf} - \{v_i\}$, then at least one vertex in $D_{ipf} - \{v_i\}$ consists of a strong neighbor to v_i which is a contradiction to our assumption that D_{ipf} is a picture fuzzy independent set of G. Hence D_{ipf} is a picture fuzzy dominating set of G □

4. Application of Domination Number in Picture Fuzzy Graph

In metropolitan cities, high-rise apartments are currently the most popular option for the people as the growing population and congested city spaces created the demand for high-rise apartments. They can generally be found in prime city locations such as shopping mall, hospitals, and restaurants or in busy commercial areas.

Even though modern high-rise residential buildings are constructed with superior technology, they are still prone to fire and other accidents. Considering the high number of residential apartments, such an accident can cause a huge lossof life and property. These are a harsh reminder that fire safety is of utmost significance, especially in a residential area. In case of a high-rise building, this assumes even greater significance. Over the past years, India had 46 reported deaths due to fire accidents every day in 2016 , the highest in residential areas (50%) but buildings often do not follow fire safety standards leaving them susceptible to fires, according to experts and an analysis of the latest available data from the National Crime Records Bureau (NCRB). But the Fire services are not well organized in India according to the website of the Directorate General of Fire Services.

In recent years, the requirements for fire safety have increased whereas the development of Fire Service has not made much advancement. The fire services need to be organized properly with adequate infrastructure and equipment for keeping pace with advancement of technology and economic growth. This motivates us to provide the application of placing the minimum number of fire stations with adequate infrastructure and equipment for keeping pace with advancement of technology by using the concept of the dominating set in Picture fuzzy graph. Let us assume that the structure of the metropolitan city as a Picture fuzzy graph $G = (V, E)$. Prime locations of the city be considered as vertices be V_1, V_2, V_3, V_4, V_5, V_6 of G and the connection between prime locations are considered as edges of G. Therefore, in a picture fuzzy graph, the degree of positive membership function denotes the knowledge in selecting the prime location which has to minimize the response time and maximize the coverage area. However, the degree of neutral membership function denotes unbiased position in selecting the prime location with traffic congestion and densely populated area in city and the degree of negative membership function denotes the less knowledge to know about the suitable environment and geological hazard while selecting the prime location.

Here $(V_1, V_2), (V_2, V_3), (V_3, V_4), (V_3, v_5), (V_5, v_6)$ are strong edges.
The possible picture fuzzy dominating sets are

$$D_{p_{f_1}} = \{v_2, v_4, v_5\} = 0.65 + 0.65 + 0.6 = 1.9$$
$$D_{p_2} = \{v_1, v_3, v_6\} = 0.45 + 0.8 + 0.5 = 1.75$$
$$D_{p_3} = \{v_1, v_3, v_5\} = 0.45 + 0.8 + 0.6 = 1.85$$
$$D_{p_3} = \{v_2, v_3, v_6\} = 0.65 + 0.8 + 0.5 = 1.95$$

Since the Picture fuzzy domination number $\gamma_{pf} = 0.45 + 0.8 + 0.5 = 1.75$, the minimum cardinality of the picture fuzzy dominating set is $D_{pf} = \{v_1, v_3, v_6\}$.

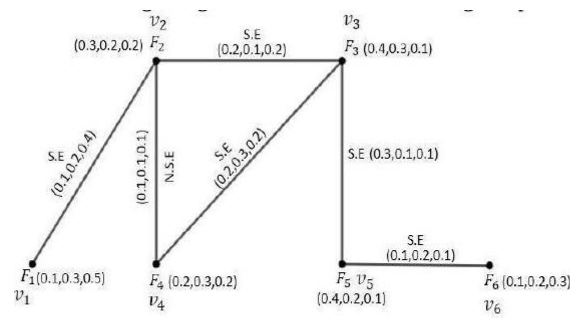

FIGURE 5.

Hence, in order to reduce massive loss of life and property due to fire accidents in the city's high-rise buildings, we have identified the prime locations with minimum of three fire stations which will have updated infrastructure to keep up with technical advancements.

5. Conclusion

In this paper, the dominating set and the domination number have been defined in picture fuzzy graph by using strong edges. Bounds of the domination number in some picture fuzzy graphs have been determined. Theorems and propositions of the picture fuzzy dominating set and its domination number have been proved with examples.An application of the domination in the picture fuzzy graph has been discussed to place the minimum number of fire stations with adequate infrastructure and equipment for keeping pace with advancement of technology in order to reduce massive loss of life and property due to fire accidents in the high rise buildings which are located in metropolitan city.

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