ON EQUITABLE EDGE COLORING OF DIFFERENT TYPES OF GRAPHS

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Abstract: An equitable edge coloring of a graph is a proper edge coloring for which the difference between any two-color classes is at most one. Consider a graph G with an edge set E(G) and a set of vertices set V(G) Let Δ(G) and χ(G) represent, respectively, the largest degree and chroma number of G. If there is a proper Δ(G)-coloring of G such that the sizes of any two colour classes differ by no more than one, then we can claim that G is equitably colorable. It follows that if G is fairly "Δ(G)-colorable," then ΔG≥ χ (G). In contrast, we cannot ensure that G must be equally -colorable even if G satisfies ΔG≥ χ (G).The minimum cardinality of G for such coloring is called equitable edge chromatic number. In this article, we determine the theorem on equitable edge coloring for sunlet graph S_n, Wheel graph W_n , Helm graph H_n , fan graph $F_{1,n}$, Gear graph G_n and Double star graph $K_{1,n,n}$ respectively.

KEYWORDS: Sunlet graph, wheel graph, helm graph, gear graph, double star graph, fan graph, equitable edge coloring.

AMS Subject Classification: 05C15

1. Introduction

Let us consider all graphs are finite, simple and undirected graph G. The concept of edge coloring introduced by Tait in 1880. Clearly $\chi^{'}(G) \geq \Delta(G)$, where $\Delta(G)$ is the maximum degree of graph G.In 1916, Konig was proved that every bipartite graph can be edge colored with exactly $\Delta(G)$ colors. Xia Zhang and Guizhen Liu [7] prove that the equitable edge-colorings of simple graphs.

In 1949 Shannon proved that every graph can be edge colored with $\leq \frac{3}{2}\Delta(G)$ colors. In 1964, Vizing [6,8] given the tight bound for edge coloring that $\Delta(G) \leq \chi^{'}(G) \leq \Delta(G)+1$. In 1973, Meyer [4,10] presented the concept of equitable coloring and equitable chromatic number. After few years, as an extension of equitable coloring, the concept of equitable edge coloring was introduced by Hilton and deWerra [1,10] in 1994. K. Kaliraj [12] proved that equitable edge coloring of some join graphs. Veninstinevivik et.al [5] proved the equitable edge coloring of splitting graph of helm and sunlet graph. In 2020, Manikandan et.al [3,13] proved that an equitable edge coloring of strong roduct of P_n and C_n .

2. Preliminaries

Definition 2.1[5]

The *n*- sunlet graph S_n is got by joining *n* pendant edges to all the vertices of the cycle C_n

Definition 2.2[5]

For $n \geq 4$, the wheel W_n is obtained by joining a vertex v_0 to each of the $n-1$ vertices $v_1, v_2, \ldots, v_{n-1}$ of C_{n-1}.

Definition 2.3[5] The Helmgraph H_n is the graph attained by a W_n by adjoining a pendant edge to each vertex of the $n-1$ vertices of the cycle in W_n.

Definition 2.4: TheGeargraph G_n is obtained from a wheel graph W_n by insert a new vertex to each edge of the $n-1$ cycle in W_n.

Lemma 2.5[5]: Let G be a simple graph, then $\chi_e'(G) \geq \Delta(G)$.

3. Main results

Theorem3.1. *Forany* $n \geq 3,$ *the equitable chromatic index for sunlet graph is* $\chi_e'(S_n) = 3.$

Proof.

Let $V(S_n) = \{u_k, v_k : 1 \leq k \leq n\}$ and Let $E(S_n) = \{e_k, s_k : 1 \leq k \leq n\}$, where the edges $\{e_k : 1 \leq k \leq n\}$ represents the edge $\{v_k v_{k+1 (\text{mod } n)} : 1 \leq k \leq n\}$,the edges $\{s_k : 1 \leq k \leq n\}$ represents the edge $\{u_k v_k : 1 \leq k \leq n\}$

Define an edge coloring $c : E(S_n) \to \{1, 2, 3\}$ as follows. Let us partition the edge set of sunlet graph $E(S_n)$ as follows.

Case (i): $n \equiv 0 (\text{mod } 3)$ (i.e) 3, 6, 9...

$$E_1 = \{e_1, e_3, e_7 \ldots, e_{n-2}\} \cup \{s_3, s_6, \ldots, s_n\} \tag{3.1}$$

$$E_2 = \{e_2, e_5, e_8 \ldots, e_{n-1}\} \cup \{s_1, s_4, \ldots, s_{n-2}\} \tag{3.2}$$

$$E_3 = \{e_3, e_6, e_7 \ldots, e_n\} \cup \{s_2, s_5, \ldots, s_{n-1}\} \tag{3.3}$$

From the equation (3.1) to (3.3), clearly the sunlet graph S_n is equitable edge colored with 3 colors. Also we observe that the color classes E_1, E_2 and E_3 are independent sets of S_n and its satisfies the in equiality $||E_i| - |E_j|| \leq 1,$ for $i \neq j$. Hence $\chi_e'(S_n) \leq 3$. Since $\Delta = 3$ and $\chi_e'(S_n) \geq \Delta = 3$ Therefore $\chi_e'(S_n) = 3.$ When

$n = 3, 6, 9, \ldots$, i.e consider $n = 6,$ for which the color classes $|E_1| = |E_2| = |E_3| = 4$ and which implies that $||E_1| - |E_2|| \le 1.$ Thus, it is equitable edge colored with 3 colors. Therefore $\chi_e^{'}(S_6) \le 3.$ The maximum degree of sunlet graph is 3($\Delta = 3$) and by lemma 2.5, $\chi_e^{'}(S_6) \ge \Delta = 3$. Hence $\chi_e^{'}(S_6) = 3.$

Case (ii): $n \equiv 1 \pmod 3$

$$E_1 = \{e_1, e_4, e_7 \ldots, e_{n-3}\} \cup \{s_3, s_6, \ldots, s_{n-1}\} \cup \{s_n\} \tag{3.4}$$

$$E_2 = \{e_2, e_5, e_8 \ldots, e_{n-2}\} \cup \{e_n\} \cup \{s_4, s_7, \ldots, s_{n-3}\} \tag{3.5}$$

$$E_3 = \{e_3, e_6, e_9 \ldots, e_{n-1}\} \cup \{s_1, s_2\} \cup \{s_5, s_8, \ldots, s_{n-2}\} \tag{3.6}$$

From the equation (3.4) to (3.6), clearly the sunlet graph S_n is equitable edge colored with 3 colors. Also we observe that the color classes E_1, E_2 and E_3 are independent sets of S_n and its satisfies the in equiality $||E_i| - |E_j|| \le 1,$ for $i \ne j$. Hence $\chi_e^{'}(S_n) \le 3$. Since $\Delta = 3$ and $\chi_e^{'}(S_n) \ge \Delta = 3$ Therefore $\chi_e^{'}(S_n) = 3.$ For example, in the case(ii) when $n \equiv 1 \pmod 3$, i.e consider $n = 10,$ for which the color classes $|E_1| = |E_3| = 7$ and $|E_2| = 6$, which implies that $||E_i| - |E_j|| \le 1$. Thus, it is equitable edge colored with 3 colors. So that $\chi_e^{'}(S_{10}) \le 3.$ The maximum degree of sunlet graph is 3($\Delta = 3$) and by lemma 2.5, $\chi_e^{'}(S_{10}) \ge \Delta = 3.$ Hence $\chi_e^{'}(S_{10}) = 3.$

Case (iii): $n \equiv 2 \pmod 3$

$$E_1 = \{e_1, e_4, e_7 \ldots, e_{n-1}\} \cup \{s_3, s_6, \ldots, s_{n-2}\} \tag{3.7}$$

$$E_2 = \{e_2, e_5, e_8 \ldots, e_{n-3}\} \cup \{e_n\} \cup \{s_4, s_7, s_{10} \ldots, s_{n-1}\} \tag{3.8}$$

$$E_3 = \{e_3, e_6, e_9 \ldots, e_{n-2}\} \cup \{s_1, s_2\} \cup \{s_5, s_8, \ldots, s_n\} \tag{3.9}$$

From the equation (3.7) to (3.9), clearly the sunlet graph S_n is equitable edge colored with 3 colors. Also we observe that the color classes E_1, E_2 and E_3 are independent sets of S_n and its satisfies the inequiality $||E_i| - |E_j|| \le 1,$ for each (i, j). Hence $\chi_e^{'}(S_n) \le 3$. Since $\Delta = 3$ and $\chi_e^{'}(S_n) \ge \Delta = 3$ Therefore $\chi_e^{'}(S_n) = 3.$ For example, in the case(iii) when $n \equiv 2 \pmod 3$, i.e consider $n = 11,$ for which the color classes $|E_1| = |E_2| = 7$ and $|E_3| = 8,$ which implies that $||E_1| - |E_2|| \le 1.$

Thus $\chi_e^{'}(S_{11}) \leq 3.$ The maximum degree of sunlet graph is 3($\Delta = 3$) and by using the lemma 2.5, $\chi_e^{'}(S_{11}) \geq \Delta = 3$. Hence $\chi_e^{'}(S_{11}) = 3.$

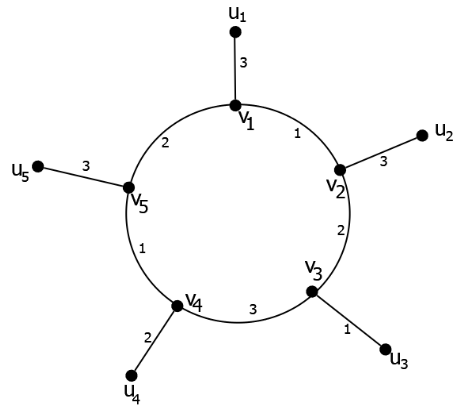

Figure 1: Equitable edge coloring of sunlet graph with 5 vertices

Theorem 3.2 *For any* $n \geq 4,$ *the equitable chromatic index for wheel graph is* $\chi_e^{'}(w_n) = n-1.$

Proof. Let $V(W_n) = \{v_0\} \bigcup \{v_k : 1 \leq k \leq n-1\}$ and

Let $E(W_n) = \{g_k : 1 \leq k \leq n-1\} \bigcup \{s_k : 1 \leq k \leq n-1\}$, where the edges $\{g_k : 1 \leq k \leq n-1\}$ represents the edge $\{v_0 v_k : 1 \leq k \leq n-1\}$ the edges $\{s_k : 1 \leq k \leq n\}$ represents the edge $\{v_k v_{k+1} : 1 \leq k \leq n-1\}$

Construct an edge coloring $c : E(W_n) \rightarrow \{1,2,3,.....,n-1\}$ as follows. Let us partition the edge set for wheel graph $E(W_n)$ as follows.

$$E_1 = \{g_1, s_2\} \tag{3.10}$$

$$E_2 = \{g_2, s_3\} \tag{3.11}$$

$$E_3 = \{g_3, s_4\} \tag{3.12}$$

$$E_4 = \{g_4, s_5\} \tag{3.13}$$

$$E_5 = \{g_5, s_6\} \tag{3.14}$$

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$$E_{n-4} = \{g_{n-4}, s_{n-3}\} \quad (3.15)$$

$$E_{n-3} = \{g_{n-3}, s_{n-2}\} \quad (3.16)$$

$$E_{n-2} = \{g_{n-2}, s_{n-1}\} \quad (3.17)$$

$$E_{n-1} = \{g_{n-1}, s_n\} \quad (3.18)$$

From the equation (3.10) to (3.18), clearly the wheel graph W_n is equitable edge colored with $n-1$ colors. Also observe that color classes $E_1, E_2, ..., E_{n-1}$ are independent sets of W_n, the cardinality of the color classes $|E_1|=|E_2|=|E_3|.....=|E_{n-2}|=|E_{n-1}|=2$ and its satisfies the in-equiality $||E_i|-|E_j|| \leq 1,$ for $i \neq j$. Hence $\chi_e^{'}(W_n) \leq n-1$. Since $\Delta = n-1$ and $\chi_e^{'}(W_n) \geq n-1.$ Therefore $\chi_e^{'}(W_n) = n-1.$ For example, consider $n = 8,$ vertices, such that the color classes $|E_1|=|E_2|=|E_3|.....=|E_7|=2$ and which implies that $||E_1|-|E_2|| \leq 1.$ Thus, an equitable edge colored with 7 colors and so that $\chi_e^{'}(W_8) \leq 7.$ The maximum degree of wheel graph is 7 $(\Delta = 7)$ and by lemma 2.5, $\chi_e^{'}(W_8) \geq \Delta = 7.$ Hence $\chi_e^{'}(W_8) = 7.$

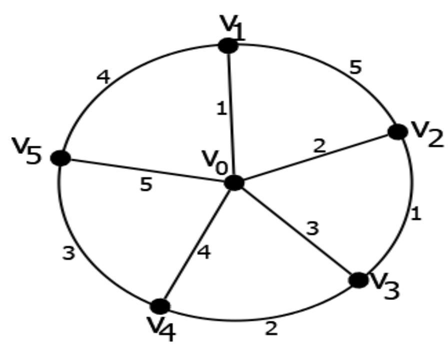

Figure 2: Equitable edge coloring of wheel graph with 6 vertices

Theorem 3.3 *For any* $n \geq 4,$ *the equitable chromatic index for helm graph is* $\chi_e^{'}(H_n) = n-1.$

Proof.

Let $V(H_n) = \{v_0\} \cup \{v_k : 1 \leq k \leq n-1\} \cup \{u_k : 1 \leq k \leq n-1\}$ and

Let

$$E(H_n) = \{e_k : 1 \leq k \leq n-1\} \cup \{f_k : 1 \leq k \leq n-1\} \cup \{s_k : 1 \leq k \leq n-1\}$$

where the edges $\{e_k : 1 \leq k \leq n-1\}$ represents the edge

$\{v_0 v_k : 1 \le k \le n-1\}$,

the edges $\{f_k : 1 \le k \le n-1\}$ represents the edge $\{v_k v_{k+1(\text{mod } n-1)} : 1 \le k \le n-1\}$ and the edges $\{s_k : 1 \le k \le n-1\}$ represents the edge $\{v_k u_k : 1 \le k \le n-1\}$

By construction an edge coloring $c : E(H_n) \to \{1, 2, 3, \ldots, n-1\}$ as follows. Let us partition the edge set for helm graph $E(H_n)$ as follows.

$$E_1 = \{e_1, f_2, s_5\} \tag{3.19}$$

$$E_2 = \{e_2, f_3, s_1\} \tag{3.20}$$

$$E_3 = \{e_3, f_4, s_2\} \tag{3.21}$$

$$E_4 = \{e_4, f_5, s_3\} \tag{3.22}$$

$$E_5 = \{e_5, f_1, s_4\} \tag{3.23}$$

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$$E_{n-5} = \{e_{n-5}, f_{n-4}, s_{n-1}\} \tag{3.24}$$

$$E_{n-4} = \{e_{n-4}, f_{n-3}, s_{n-5}\} \tag{3.25}$$

$$E_{n-3} = \{e_{n-3}, f_{n-2}, s_{n-4}\} \tag{3.26}$$

$$E_{n-2} = \{e_{n-2}, f_{n-1}, s_{n-3}\} \tag{3.27}$$

$$E_{n-1} = \{e_{n-1}, f_{n-5}, s_{n-2}\} \tag{3.28}$$

From the equation (3.19) to (3.28), clearly the helm graph H_n is equitable edge colored with $n-1$ colors. Also observe that the color classes independent sets of H_n, the cardinality of the color classes $|E_1| = |E_2| = |E_3| \ldots\ldots = |E_{n-2}| = |E_{n-1}| = 3$ and its satisfies the inequiality $||E_i| - |E_j|| \le 1,$ for any (i, j). Hence $\chi_e^{'}(H_n) \le n-1$. Since $\Delta = n-1$ and $\chi_e^{'}(H_n) \ge \Delta = n-1.$ Therefore $\chi_e^{'}(H_n) = n-1.$ For example, consider the helm $n=8,$ vertices, the color classes $|E_1|=|E_2|=|E_3|.....=|E_7|=3$ and which implies that $||E_1|-|E_2||\leq 1.$ So that the equitable edge colored with 7 colors. So that $\chi_e^{'}(H_8)\leq 7.$ The maximum degree of helm graph is $7\,(\Delta=7)$ and by lemma 2.5, it follows that $\chi_e^{'}(H_8)\geq\Delta=7$. Hence $\chi_e^{'}(H_8)=7.$

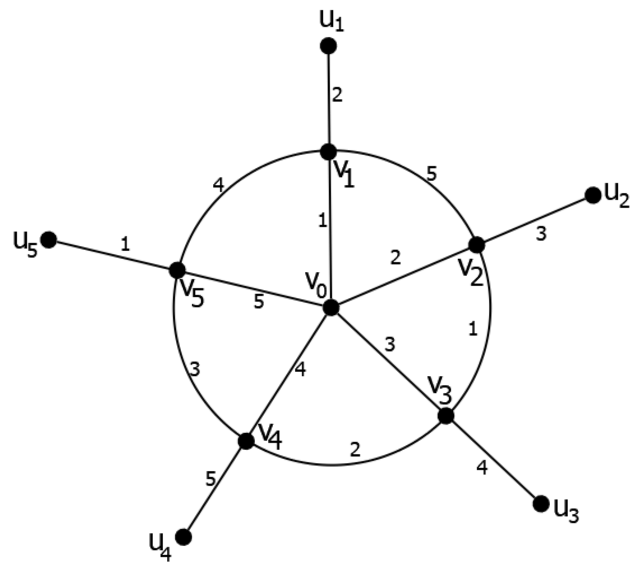

Figure 3: Equitable edge coloring of helm graph with 6 vertices

Theorem 3.4 *For any* $n\geq 4,$ *the equitable chromatic index for gear graph is* $\chi_e^{'}(G_n)=n-1.$

Proof.

Let $V(G_n)=\{v_0\}\bigcup\{u_k:1\leq k\leq n-1\}\bigcup\{v_k:1\leq k\leq n-1\}$ and

Let

$E(G_n)=\{e_k:1\leq k\leq n-1\}\bigcup\{f_k:1\leq k\leq n-1\}\bigcup\{s_k:1\leq k\leq n-1\}$

,where the edges $\{e_k:1\leq k\leq n-1\}$ represents the edge $\{v_0v_k:1\leq k\leq n-1\}$, the edges $\{f_k:1\leq k\leq n-1\}$ represents the edge $\{v_ku_k:1\leq k\leq n-1\}$ and the edges $\{s_k:1\leq k\leq n-1\}$ represents the edge $\{u_kv_{k+1(\mathrm{mod}\ \text{n-1})}:1\leq k\leq n-1\}$

An edge coloring is define as $c:E(G_n)\rightarrow\{1,2,3,...,n-1\}$ as follows. Let us partition the edge set for gear graph $E(S_n)$ as follows.

$$E_1 = \{e_1, f_6, s_1\} \tag{3.28}$$

$$E_2 = \{e_2, f_1, s_2\} \tag{3.29}$$

$$E_3 = \{e_3, f_2, s_3\} \tag{3.30}$$

$$E_4 = \{e_4, f_3, s_4\} \tag{3.31}$$

$$E_5 = \{e_5, f_4, s_5\} \tag{3.32}$$

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$$E_{n-4} = \{e_{n-4}, f_{n-3}, s_{n-4}\} \tag{3.33}$$

$$E_{n-3} = \{e_{n-3}, f_{n-4}, s_{n-3}\} \tag{3.34}$$

$$E_{n-2} = \{e_{n-2}, f_{n-3}, s_{n-2}\} \tag{3.35}$$

$$E_{n-1} = \{e_{n-1}, f_{n-2}, s_{n-1}\} \tag{3.36}$$

From the equation (3.28) to (3.36), clearly the gear graph G_n is equitable edge colored with $n-1$ colors. Also observe that the color classes are independent sets of G_n, the cardinality of the color classes $|E_1|=|E_2|=|E_3|.....=|E_{n-2}|=|E_{n-1}|=3$ and its satisfies the inequiality $||E_i|-|E_j||\leq 1,$ for $i \neq j$. Hence $\chi_e^{'}(G_n) \leq n-1$. Since $\Delta = n-1$ and by lemma 2.5, $\chi_e^{'}(G_n) \geq \Delta = n-1.$ Therefore $\chi_e^{'}(G_n) = n-1.$ For example, consider the gear graph with $n = 8,$ vertices, such that the color classes $|E_1|=|E_2|=|E_3|.....=|E_7|=3$ and which implies that $||E_i|-|E_j||\leq 1,$ for $i \neq j$. Such that the equitable edge colored with 7 colors. Thus $\chi_e^{'}(G_8) \leq 7.$ The maximum degree of gear graph is 7 $(\Delta = 7)$ and $\chi_e^{'}(G_8) \geq \Delta = 7$. Hence $\chi_e^{'}(G_8) = 7.$

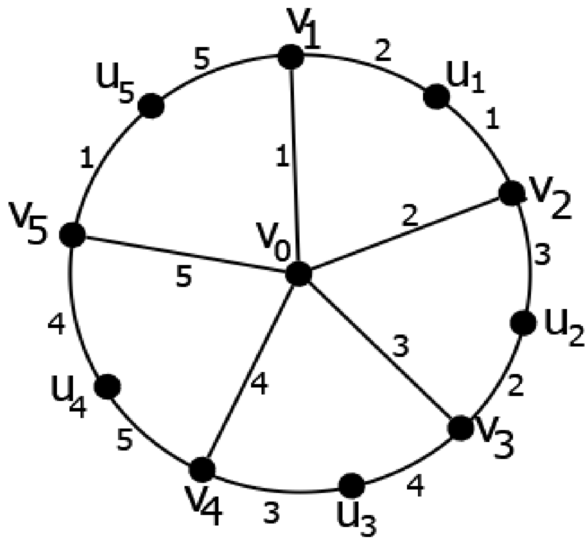

Figure 4: Equitable edge coloring of gear graph

Theorem 3.5 *For any positive integer, the equitable chromatic index for double star graph is* $\chi_e^{'}(K_{1,n,n}) = n.$

Proof. Let $V(K_{1,n,n}) = \{v_0\} \bigcup \{v_k : 1 \le k \le n\} \bigcup \{u_k : 1 \le k \le n\}$ and

Let $E(K_{1,n,n}) = \{e_k : 1 \le k \le n\} \bigcup \{f_k : 1 \le k \le n\}$, where the edges $\{e_k : 1 \le k \le n\}$ represents the edge $\{v_0 v_k : 1 \le k \le n\}$, the edges $\{f_k : 1 \le k \le n\}$ represents the edge $\{v_k u_k : 1 \le k \le n\}$ respectively.

Define an edge coloring $c : E(K_{1,n,n}) \to \{1, 2, 3, ..., 2n\}$ as follows. Let us partition the edge set for double star graph $E(K_{1,n,n})$ as follows.

$$E_1 = \{e_1, f_n\} \tag{3.37}$$

$$E_2 = \{e_2, f_1\} \tag{3.38}$$

$$E_3 = \{e_3, f_2\} \tag{3.39}$$

$$E_4 = \{e_4, f_3\} \tag{3.40}$$

$$E_5 = \{e_5, f_4\} \tag{3.41}$$

$$E_6 = \{e_6, f_5\} \tag{3.42}$$

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$$E_{n-2} = \{e_{n-2}, f_{n-3}\} \tag{3.43}$$

$$E_{n-1} = \{e_{n-1}, f_{n-2}\} \tag{3.44}$$

$$E_n = \{e_n, f_{n-1}\} \tag{3.45}$$

From the equation (3.37) to (3.45), clearly the double star graph $K_{1,n,n}$ is equitable edge colored with n colors. Also we observe that the color classes are independent sets of $K_{1,n,n}$, the cardinality of the color classes $|E_1|=|E_2|=|E_3|.....=|E_{n-1}|=|E_n|=2$ and its satisfies the inequiality $||E_i|-|E_j||\leq 1,$ for $i \neq j$. Therefore $\chi_e^{'}(K_{1,n,n}) \leq n$. We know that $\Delta = n$ and $\chi_e^{'}(K_{1,n,n}) \geq \Delta = n.$ Hence $\chi_e^{'}(K_{1,n,n}) = n.$ For example, Consider $n = 6,$ vertices, the color classes are $|E_1|=|E_2|=|E_3|.....=|E_7|=2$ and which implies that $||E_1|-|E_2||\leq 1.$ Thus, equitable edge colored with 6 colors. Therefore $\chi_e^{'}(K_{1,6,6}) \leq 6$. The maximum degree of double star graph is $6.$ Hence $\chi_e^{'}(K_{1,6,6}) = 6.$

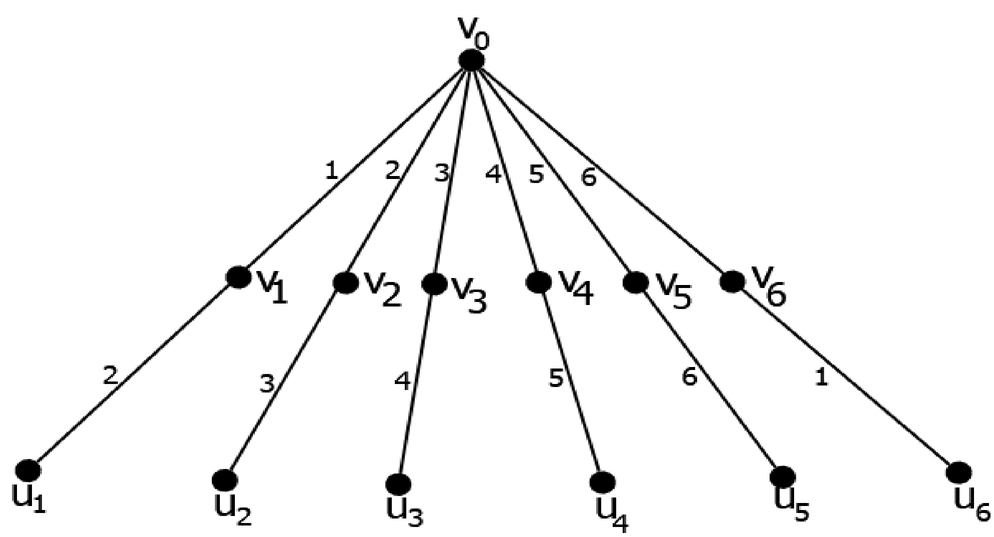

Figure 5: Equitable edge coloring of double star graph

Theorem 3.6 *For any positive integer, the equitable chromatic index for fan graph is* $\chi_e^{'}(F_{1,n}) = n.$

Proof. Let $V(F_{1,n}) = \{v_0\} \cup \{v_k : 1 \leq k \leq n\}$ and

Let $E(F_{1,n})=\{g_k : 1\le k\le n\}\bigcup\{h_k : 1\le k\le n-1\}$, where the edges $\{g_k : 1\le k\le n\}$ represents the edge $\{v_0v_k : 1\le k\le n\}$, the edges $\{h_k : 1\le k\le n-1\}$ represents the edge $\{v_kv_{k+1} : 1\le k\le n-1\}$ respectively.

An edge coloring is define $c: E(F_{1,n})\to\{1,2,3,...,n\}$ as follows. Let us partition the edge set for fan graph $F_{1,n}$ as follows.

$$E_1=\{g_1,h_{n-1}\} \tag{3.46}$$

$$E_2=\{g_2\} \tag{3.47}$$

$$E_3=\{g_3,h_1\} \tag{3.48}$$

$$E_4=\{g_4,h_2\} \tag{3.49}$$

$$E_5=\{g_5,h_3\} \tag{3.50}$$

$$E_6=\{g_6,h_4\} \tag{3.51}$$

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$$E_{n-2}=\{g_{n-2},h_{n-4}\} \tag{3.52}$$

$$E_{n-1}=\{g_{n-1},h_{n-3}\} \tag{3.53}$$

$$E_n=\{g_n,h_{n-1}\} \tag{3.54}$$

From the equation (3.45) to (3.54), clearly the fan graph $F_{1,n}$ is equitable edge colored with n colors. Also we observe that the color classes $E_1,E_2,....,E_n$ are independent sets of $F_{1,n}$, the cardinality of the color classes $|E_1|=|E_3|.....=|E_{n-1}|=|E_n|=2$ and $|E_2|=1$, its satisfies the inequiality $||E_i|-|E_j||\le 1,$ for $i\ne j$. Hence $\chi_e^{'}(F_{1,n})\le n$. We know that $\Delta=n$ and by lemma 2.5 $\chi_e^{'}(F_{1,n})\ge\Delta=n.$ Therefore $\chi_e^{'}(F_{1,n})=n.$ For example, consider the fan graph with $n=6,$ vertices, such that the color classes $|E_1|=|E_3|.....=|E_6|=2$ and $|E_2|=1$, which implies that $||E_1|-|E_2||\le 1.$

Thus, the equitable edge colored with 6 colors. Therefore $\chi_e^{'}(F_{1,6}) \leq 6$ and $\chi_e^{'}(F_{1,6}) \geq \Delta = 6$. Hence $\chi_e^{'}(F_{1,6}) = 6$.

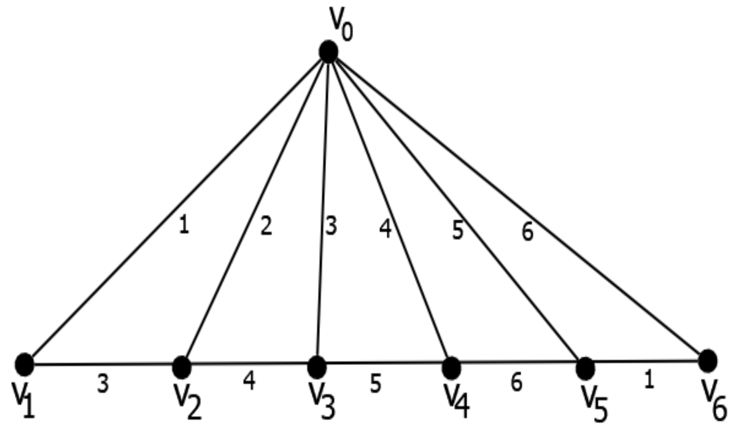

Figure 6: Equitable edge coloring of fan graph

4. Conclusion

In this article, we determined the equitable chromatic index of Sunlet, Wheel, Helm, Gear, Double star and Fan graph. The proofs establish an optimal solution to the equitable edge coloring of these graph families. The field of equitable edge coloring of graphs is broad open. It would be further interesting to determine the bounds of equitable edge coloring of various families of graphs.

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