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A NEW SUBCLASS OF MEROMORPHIC FUNCTIONS WITH POSITIVE COEFFICIENTS DEFINED BY INTEGRAL OPERATOR

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Abstract. The target of this article is acquire coefficient bounds, radii of starlikeness and convexity, convex linear combinations, integral transforms and neighborhood results for the subclass positive coefficient meromorphic functions.

1. Introduction

Let Σ indicate the class of meromorphic functions of the form

$$\eta(w) = \frac{1}{w} + \sum_{n=1}^{\infty} \ell_n w^n \tag{1.1}$$

which are regular in the punctured unit disc

$$U^* = \{w : w \in C \ \text{ and } 0 < |w| < 1\} = U \setminus \{0\}.$$

A function $\eta \in \Sigma$ is given by (1.1) is said to be meromorphically starlike and meromorphically convex of order ϖ if it delights the pursing

$$-\Re\left\{\frac{w\eta'(w)}{\eta(w)}\right\} > \varpi, \ (w \in U) \tag{1.2}$$

$$\text{and} \quad -\Re\left\{1 + \frac{w\eta''(w)}{\eta'(w)}\right\} > \varpi, \ (w \in U) \tag{1.3}$$

for some $\varpi, \ (0 \le \varpi < 1)$ respectively and we say that η is in the class $\Sigma^*(\varpi)$ and $\Sigma_c(\varpi)$ of such functions respectively.

The class $\Sigma^*(\varpi)$ and different subclasses of Σ have been examined broadly by researchers [1, 6, 8, 9]. Over the few years, numerous authors have explored the subclass of positive coefficient meromorphic functions. Juneja and Reddy [3] discussed the Σ_p function of the form

$$\eta(w) = \frac{1}{w} + \sum_{n=1}^{\infty} \ell_n w^n, \ (\ell_n \ge 0), \tag{1.4}$$

which are regular and univalent in U^*. The functions of this class are called to be meromorphic function with a positive coefficients. Jung et al. [4] defined the

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integral operator on the normalized analytic functions and Lashin [5] modified their operator for meromorphic functions in the following manner.

Lemma 1.1. *For $\eta \in \Sigma$ given by (1.1), if the operator $P_{\hbar}^{\wp} : \Sigma \to \Sigma$ is defined by*

$$P_{\hbar}^{\wp}\eta(w) = \frac{\hbar^{\wp}}{\Gamma(\wp)} \frac{1}{w^{\hbar+1}} \int_0^w t^{\hbar} \left(log\frac{w}{t}\right)^{\wp-1} \eta(t)dt, \tag{1.5}$$

($\hbar > 0, \wp > 0$ and $w \in U$) then it can be shown that

$$P_{\hbar}^{\wp}\eta(w) = \frac{1}{w} + \sum_{n=1}^{\infty} L(n,\hbar,\wp)\ell_n w^n$$

where $L(n,\hbar,\wp) = \left(\frac{\hbar}{n+\hbar+1}\right)^{\wp}$ and Γ is the familiar Gamma function.

Now we present the pursuing subclass of Σ_p correlated with the integral operator $P_{\hbar}^{\wp}\eta(w)$.

Definition 1.2. A function $\eta \in \Sigma$ is said to be in the class $\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$ if and only if satisfies the inequality

$$\Re\Big\{\frac{w(P_{\hbar}^{\wp}\eta(w))'}{(\aleph-1)(P_{\hbar}^{\wp}\eta(w)) + \aleph w(P_{\hbar}^{\wp}\eta(w))'}\Big\} > \varpi \tag{1.6}$$

where $0 \leq \hbar < 1, 0 \leq \wp < 1, 0 \leq \varpi < 1$ and $0 \leq \aleph < 1$.

In this article, we get the inequalities of coefficient, distortion theorems, closure theorems radius of starlikeness and convexity for the class $\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. In addition, we discussed integral operators and neighbourhood results for this class as well".

2. Coefficients Inequalities

Our to begin with hypothesis gives a fundamental and adequate condition for a function η to be with in the class $\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$.

Theorem 2.1. *Let $\eta(w) \in \Sigma_p$ be given by (1.1). Then $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$*

$$\Leftrightarrow \sum_{n=1}^{\infty}[n + \varpi - \varpi\aleph(1+n)]L(n,\hbar,\wp)\ell_n \leq 1 - \varpi. \tag{2.1}$$

Proof. If $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$ then

$$\begin{aligned}
&\Re\left\{\frac{w(P_{\hbar}^{\wp}\eta(w))'}{(\aleph-1)(P_{\hbar}^{\wp}\eta(w)) + \aleph w(P_{\hbar}^{\wp}\eta(w))'}\right\} \\
&= \Re\left\{\frac{-1 + \sum_{n=1}^{\infty} n\ L(n,\hbar,\wp)\ell_n w^{n+1}}{-1 + \sum_{n=1}^{\infty}(\aleph - 1 + \aleph n)L(n,\hbar,\wp)\ell_n w^{n+1}}\right\} > \varpi.
\end{aligned}$$

By letting $w \to 1^-$, we have $\left\{\frac{-1+\sum_{n=1}^{\infty} n}{-1+\sum_{n=1}^{\infty} L(n,\hbar,\wp)(\aleph-1+\aleph n)L(n,\hbar,\wp)\ell_n}\right\} > \varpi$.

This shows that (2.1) holds.

Conversely, suppose (2.1) holds. Since

$$\Re(z) > \varpi \;\Leftrightarrow\; |z-1| < |z+1-2\varpi|.$$

It is adequate to show that

$$\left|\frac{w(P_{\hbar}^{\wp}\eta(w))' - [(\aleph-1)P_{\hbar}^{\wp}\eta(w) + \aleph w(P_{\hbar}^{\wp}\eta(w))']}{w(P_{\hbar}^{\wp}\eta(w))' + (1-2\varpi)[(\aleph-1)P_{\hbar}^{\wp}\eta(w) + \aleph w(P_{\hbar}^{\wp}\eta(w))']}\right| \;<\; 1.$$

Using (2.1), we see that

$$\begin{aligned}
&\left|\frac{w(P_{\hbar}^{\wp}\eta(w))' - [(\aleph-1)P_{\hbar}^{\wp}\eta(w) + \aleph w(P_{\hbar}^{\wp}\eta(w))']}{w(P_{\hbar}^{\wp}\eta(w))' + (1-2\varpi)[(\aleph-1)P_{\hbar}^{\wp}\eta(w) + \aleph w(P_{\hbar}^{\wp}\eta(w))']}\right| \\
=&\left|\frac{\sum\limits_{n=1}^{\infty}(1-\aleph)(n+1)L(n,\hbar,\wp)\ell_n w^{n+1}}{-2(1-\varpi) + \sum\limits_{n=1}^{\infty}[n(1+(1-2\varpi)\aleph) + (1-2\varpi)(\aleph-1)]L(n,\hbar,\wp)\ell_n w^{n+1}}\right| \\
\leq&\frac{\sum\limits_{n=1}^{\infty}(1-\aleph)(n+1)L(n,\hbar,\wp)\ell_n}{2(1-\varpi) - \sum\limits_{n=1}^{\infty}[n(1+(1-2\varpi)\aleph) + (1-2\varpi)(\aleph-1)]L(n,\hbar,\wp)\ell_n} \\
\leq&1.
\end{aligned}$$

Thus we have $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. $\square$

Corollary 2.2. *If $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$ then*

$$\ell_n \leq \frac{(1-\varpi)}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}.$$

The estimate is sharp for $F_n(w)$ given by

$$F_n(w) = \frac{1}{w} + \frac{(1-\varpi)}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}w^n,\;\; n = 1,2,3\cdots.$$

3. Growth and Distortion Theorem

Theorem 3.1. *If $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$ then*

$$\frac{1}{r} - \frac{(1-\varpi)}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}r \leq |f(w)| \leq \frac{1}{r} + \frac{(1-\varpi)}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}\;r. \quad (3.1)$$

The estimate is sharp for

$$\eta(w) = \frac{1}{w} + \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}\;w. \quad (3.2)$$

Proof. Since $\eta(w) = \frac{1}{w} + \sum\limits_{n=1}^{\infty} \ell_n w^n$, we have

$$|\eta(w)| \leq \frac{1}{r} + \sum_{n=1}^{\infty} \ell_n r^n \leq \frac{1}{r} + r \sum_{n=1}^{\infty} \ell_n$$

since,

$$\sum_{n=1}^{\infty} \ell_n \leq \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}.$$

Using this, we have

$$|\eta(w)| \leq \frac{1}{r} + \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)} r.$$

Similarly

$$|\eta(w)| \geq \frac{1}{r} - \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)} r.$$

The estimate is sharp for $\eta(w) = \frac{1}{w} + \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)} w$. □

Theorem 3.2. *If* $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$ *then*

$$\frac{1}{r^2} - \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)} \leq |\eta'(w)| \leq \frac{1}{r^2} + \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)} \quad (|w| = r).$$

The estimate is sharp for the function (3.2).

4. Closure Theorems

Let the functions $F_k(w)$ be given by

$$F_k(w) = \frac{1}{w} + \sum_{n=1}^{\infty} \ell_{n,k} w^n, \; k = 1, 2, 3, \cdots. \tag{4.1}$$

We are going to demonstrate the taking after closure theorems for the class $\sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$.

Theorem 4.1. *Let* $F_k(w) \in \sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$, *for every* $k = 1, 2, 3, \cdots$. *Then* $\eta(w)$ *given as*

$$\eta(w) = \frac{1}{w} + \sum_{n=1}^{\infty} \ell_n w^n, \; (\ell_n \geq 0)$$

is in the class $\sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$, *where* $\ell_n = \frac{1}{m} \sum\limits_{n=1}^{m} \ell_{n,k}, \; (n = 1, 2, \cdots)$.

Proof. Since $F_k(w) \in \sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$, it takes after from Hypothesis 2.1 that,

$$\sum_{n=1}^{\infty} [n + \varpi - \varpi\aleph(1+n)] L(n,\hbar,\wp) \ell_{n,k} \leq 1 - \varpi, \text{ for all } k = 1, 2, \cdots, m. \tag{4.2}$$

Hence

$$\begin{aligned}
&\sum_{n=1}^{\infty}[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\ell_n\\
=&\sum_{n=1}^{\infty}[n+\varpi-\varpi\aleph(1+n)]\Big(\frac{1}{m}\sum_{k=1}^{m}L(n,\hbar,\wp)\ell_{n,k}\Big)\\
=&\frac{1}{m}\sum_{n=1}^{m}\left(\sum_{n=1}^{\infty}[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\ell_{n,k}\right)\\
&\leq 1-\varpi.
\end{aligned}$$

We have, from Theorem 2.1, $\eta(w)\in\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. □

Theorem 4.2. *The class $\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$ is closed under convex linear combination.*

Proof. Let $F_k(w)\in\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. t is enough to demonstrate that function.

$$H(w)=\aleph F_1(w)+(1-\aleph)F_2(w),\ (0\leq\aleph\leq 1)$$

is also in the class $\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. Since for $(0\leq\aleph\leq 1)$,

$$H(w)=\frac{1}{w}+\sum_{n=1}^{\infty}[\aleph\ell_{n,1}+(1-\aleph)\ell_{n,2}]w^n.$$

We examined that

$$\sum_{n=1}^{\infty}\{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\}[\aleph\ell_{n,1}+(1-\aleph)\ell_{n,2}]$$

$$\begin{aligned}
=&\aleph\sum_{n=1}^{\infty}\{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\}\ell_{n,1}\\
&+(1-\aleph)\sum_{n=1}^{\infty}\{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\}\ell_{n,2}\\
\leq&1-\varpi.
\end{aligned}$$

With the help of Theorem 2.1, we have $H(w)\in\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$.

Theorem 4.3. *Let $F_0(w)=1$ and*

$$F_n(w)=\frac{1}{w}+\sum_{n=1}^{\infty}\frac{1-\varpi}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}w^n,\ \ for\ n=1,2,\cdots.$$

Then $\eta(w)\in\sigma_{p,\hbar}^{\wp}(\varpi,\aleph)\Leftrightarrow\eta(w)$ may be expressed like
$\eta(w)=\sum\limits_{n=0}^{\infty}\aleph_nF_n(w)$, where $\aleph_n\geq 0$ and $\sum\limits_{n=0}^{\infty}\aleph_n=1$.

Proof. Let

$$\eta(w) = \sum_{n=0}^{\infty} \aleph_n F_n(w)$$
$$= \frac{1}{w} + \sum_{n=1}^{\infty} \frac{\aleph_n (1-\varpi)}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)} w^n.$$

Then

$$\sum_{n=1}^{\infty} \frac{\aleph_n (1-\varpi)}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)} \frac{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}{(1-\varpi)}$$
$$= \sum_{n=1}^{\infty} \aleph_n = 1-\aleph_0 \leq 1.$$

By Theorem 2.1, we have $H(w) \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$.

Conversely, let $\eta(w) \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. By Theorem 2.1, we have

$$\ell_n \leq \frac{1-\varpi}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)} \ for\ n = 1,2,\cdots.$$

Maybe it's written like $\aleph_n = \frac{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}{1-\varpi}\ell_n \ for\ n = 1,2,\cdots$ and $\aleph_0 = 1 - \sum_{n=1}^{\infty} \aleph_n$.

Then $\eta(w) = \sum_{n=0}^{\infty} \aleph_n F_n(w)$. □

5. Radius of meromorphic starlikeness and meromorphic convexity

Theorem 5.1. *Let $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. Then η is meromorphically starlike of order ρ $(0 \leq \rho < 1)$ in $|w| < r_1(\varpi,\aleph,\hbar,\wp,\rho)$, where*

$$r_1(\varpi,\aleph,\hbar,\wp,\rho) = \inf_{n\geq 1} \left[\frac{(1-\rho)(1-\varpi)}{(n+2-\rho)[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}\right]^{\frac{1}{n+1}}. \quad (5.1)$$

Proof. Let $\eta(w) \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. Then with the help of Theorem 2.1, we have

$$[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\ell_n \leq (1-\varpi). \quad (5.2)$$

It's enough to expose that

$$\left|1 + \frac{w\eta'(w)}{\eta(w}\right| \leq 1-\rho \quad (5.3)$$

$$\Rightarrow \left|1 + \frac{w\eta'(w)}{\eta(w)}\right| = \left|\frac{\sum\limits_{n=1}^{\infty}(n+1)\ell_n w^n}{\frac{1}{w} - \sum\limits_{n=1}^{\infty}\ell_n w^n}\right| \leq 1-\rho$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{(n+2-\rho)}{1-\rho}\ell_n |w|^{n+1} \leq 1, \text{ for } 0 \leq \rho < 1 \text{ and } |w| < r_1(\varpi,\aleph,\hbar,\wp,\rho).$$

By Theorem 2.1, (5.3) that's going to be true if

$$\Big(\frac{n+2-\rho}{1-\rho}\Big)|w|^{n+1} \leq \frac{1-\varpi}{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}$$

$$\Rightarrow \ |w| \leq \Big[\frac{1-\varpi}{(n+2-\rho)\{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\}}\Big]^{\frac{1}{n+1}}, \ n \geq 1.$$

This completes Theorem's evidentiary record. $\square$

Theorem 5.2. *Let $\eta \in \sigma_{p,\hbar}^{\wp}(\varpi,\aleph)$. Then η is meromorphically convex of order $\rho(0 \leq \rho < 1)$ in $|w| < r_2(\varpi,\aleph,\hbar,\wp,\rho)$, where*

$$r_2(\varpi,\aleph,\hbar,\wp,\rho) = \inf_{n\geq 1}\Big[\frac{(1-\rho)\{[n+\varpi-\varpi\aleph(n+1)]L(n,\hbar,\wp)\}}{n(n+2-\rho)(1-\varpi)}\Big]^{\frac{1}{n+1}}, \ n \geq 1. \tag{5.4}$$

Proof. Let $\eta(w) \in \sigma_\rho(\varpi,\aleph)$. Then by Theorem 2.1, we have

$$\sum_{n=1}^{\infty}[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\ell_n \leq (1-\varpi). \tag{5.5}$$

Its enough to expose that

$$\left|2+\frac{w\eta''(w)}{\eta'(w}\right| \leq 1-\delta,$$

for $|w| < r_2 = r_2(\varpi,\aleph,\hbar,\wp,\rho)$, where $r_2(\varpi,\aleph,\hbar,\wp,\rho)$ is defined as (5.4). Then

$$\left|2+\frac{w\eta''(w)}{\eta'(w)}\right| = \left|\frac{\sum\limits_{n=1}^{\infty} n(n+1)\ell_n w^{n-1}}{\frac{-1}{w^2}+\sum\limits_{n=1}^{\infty} n\ell_n w^{n-1}}\right| \leq \frac{\sum\limits_{n=1}^{\infty} n(n+1)\ell_n|w|^{n+1}}{1-\sum\limits_{n=1}^{\infty} n\ell_n|w|^{n+1}}.$$

This will be bounded by $(1-\rho)$ if

$$\sum_{n=1}^{\infty}\frac{n(n+2-\delta)}{1-\delta}\ell_n|w|^{n+1} \leq 1, \tag{5.6}$$

By (5.5), it takes afterward (5.6) is true if

$$\frac{n(n+2-\rho)}{1-\rho}|w|^{n+1} \leq \frac{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}{1-\varpi}, \ n \geq 1$$

or

$$|w| \leq \Big[\frac{(1-\rho)\{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)\}}{n(n+2-\rho)(1-\varpi)}\Big]^{\frac{1}{n+1}}, \ n \geq 1. \tag{5.7}$$

TThis completes Theorem's evidentiary record. $\square$

6. Integral Operators

In this section, we look at the integral transform of functions in the class $\sigma^{\wp}_{p,\hbar}(\varpi,\aleph)$.

Theorem 6.1. *Let $\eta \in \sigma^{\wp}_{p,\hbar}(\varpi,\aleph)$. Then the integral operator*

$$F(w) = c\int_0^1 u^c \eta(uw)du, \ \ (0 \le u \le 1, 0 < c < \infty)$$

is in $\sigma^{\wp}_{p,\hbar}(\delta,\aleph)$, where

$$\delta = \frac{(c+2)(1+\varpi-2\varpi\aleph) - c(1-\varpi)}{c(1-\varpi)(1-2\aleph) + (1+\varpi)(1-2\aleph)(c+2)}.$$

The result is sharp for

$$\eta(w) = \frac{1}{w} + \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)}w.$$

Proof. Let $\eta(w) \in \sigma^{\wp}_{p,\hbar}(\varpi,\aleph)$. Then

$$\begin{aligned} F(w) &= c\int_0^1 u^c \eta(uw)du \\ &= \frac{1}{w} + \sum_{n=1}^{\infty} \frac{c}{c+n+1}\ell_n w^n. \end{aligned}$$

It is adequate to show that

$$\sum_{n=1}^{\infty} \frac{c\{[n+\delta-\delta\aleph(1+n)]L(n,\hbar,\wp)\}}{(c+n+1)(1-\delta)}\ell_n \le 1. \tag{6.1}$$

Since $\eta \in \sigma^{\wp}_{p,\hbar}(\varpi,\aleph)$, we have

$$\sum_{n=1}^{\infty} \frac{[n+\varpi-\varpi\aleph(1+n)]L(n,\hbar,\wp)}{(1-\varpi)}\ell_n \le 1. \tag{6.2}$$

Note that (6.1) is fulfilled if

$$\frac{c[n+\delta-\delta\aleph(1+n)]}{(c+n+1)(1-\delta)} \le \frac{[n+\varpi-\varpi\aleph(1+n)]}{(1-\varpi)}.$$

Solving for δ, we have

$$\delta \le \frac{(c+n+1)[n+\varpi-\varpi\aleph(1+n)] - cn(1-\varpi)}{c(1-\varpi)[1-\aleph(1+n)] + [n+\varpi-\varpi\aleph(1+n)](c+n+1)} = G(n).$$

A quick calculation will show that $G(n)$ is increasing and $G(n) \ge G(1)$. Using this, it follows.

□

For the selection of $\aleph = 0$, We achieved the outcome of Uralegaddi and Gangi [11].

Remark 6.2. Let $\eta \in \Sigma_p^*(\varpi)$. Then integral operator

$$F(w) = c\int_0^1 u^c \eta(uw)du, \ (0 < u \leq 1, 0 < c < \infty)$$

is in $\Sigma_p^*(\varpi)$, where $\delta = \frac{1+\varpi+c\varpi}{1+\varpi+c}$.

The result is sharp for

$$\eta(w) = \frac{1}{w} + \frac{1-\varpi}{1+\varpi}w.$$

7. Neighbourhoods for the class $\sigma_p^{\wp}(\varpi, \aleph)$

In this section, we look at the neighbourhood of the class $\sigma_p^{\wp}(\varpi, \aleph)$, which we define here.

Definition 7.1. A function $\eta \in \Sigma_p \in \sigma_p^{\wp}(\varpi, \aleph)$ if there exits a function $g \in \sigma_p^{\wp}(\varpi, \aleph)$ such that

$$\left| \frac{\eta(w)}{g(w)} - 1 \right| < 1 - \wp, \ (w \in U, 0 \leq \wp < 1). \tag{7.1}$$

Resume previous work on areas of regular functions by involving Goodman [2] and Ruscheweyh [10], we define the $\delta-$neighbourhood of function $\eta \in \Sigma_p$ by

$$N_\delta(\eta) = \Big\{ g \in \Sigma_p : g(w) = \frac{1}{w} + \sum_{n=1}^{\infty} b_n w^n \ and \ \sum_{n=1}^{\infty} n|\ell_n - b_n| \leq \delta \Big\}. \tag{7.2}$$

Theorem 7.2. *If $g \in \sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$ and*

$$\wp = 1 - \frac{\delta(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)-(1-\varpi)} \tag{7.3}$$

then $N_\delta(g) \subset \sigma_p^{\wp}(\varpi, \aleph)$.

Proof. Let $\eta \in N_\delta(g)$. Then we find from (7.2) that

$$\sum_{n=1}^{\infty} n|\ell_n - b_n| \leq \delta, \tag{7.4}$$

which yields the coefficient inequality

$$\sum_{n=1}^{\infty} |\ell_n - b_n| \geq \delta, \ (n \in N). \tag{7.5}$$

Since $g \in \sigma_{p,\hbar}^{\wp}(\varpi, \aleph)$, we have

$$\sum_{n=1}^{\infty} b_n \leq \frac{1-\varpi}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}. \tag{7.6}$$

So that

$$\left|\frac{\eta(w)}{g(w)}-1\right| < \frac{\sum\limits_{n=1}^{\infty}|\ell_n-b_n|}{1-\sum\limits_{n=1}^{\infty}b_n}$$

$$=\frac{\delta(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)}{(1+\varpi-2\varpi\aleph)L(1,\hbar,\wp)-(1-\varpi)}$$

$$=1-\wp$$

provided $\wp$ is given by (7.3). As a result by definition, $\eta \in \sigma^{\wp}_{p,\hbar}(\varpi,\aleph)$

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