

## PERFECT G-ECCENTRIC DOMINATION IN FUZZY GRAPHS

S. MUTHUPANDIYAN AND A. MOHAMED ISMAYIL

**Abstract:** A dominating set  $D \subseteq V$  of a fuzzy graph  $G(\sigma, \mu)$  is said to be a  $g$ -eccentric dominating set if for every vertex  $v \in V - D$ , there exists at least one  $g$ -eccentric vertex  $u$  of  $v$  in  $D$ . A dominating set  $D \subseteq V(G)$  of a fuzzy graph  $G(\sigma, \mu)$  is said to be a perfect  $g$ -eccentric dominating set, if for each vertex  $v \in V - D$ , there exists exactly one  $g$ -eccentric vertex  $u$  of  $v$  in  $D$ . The minimum cardinality among all the perfect  $g$ -eccentric dominating set is called perfect  $g$ -eccentric domination number. In this article, perfect  $g$ -eccentric dominating set and its number are introduced. Perfect  $g$ -eccentric domination number for some standard fuzzy graphs are obtained. Theorem related to perfect  $g$ -eccentric dominating set and its number for fuzzy graphs are stated and proved. Bounds on perfect  $g$ -eccentric domination number for some standard fuzzy graphs are obtained.

**Keywords:** Fuzzy graph,  $g$ -Eccentric dominating set, Perfect  $g$ -eccentric dominating set, Perfect  $g$ -eccentric domination number.

**AMS Subject Classification 2020:** 05C05, 05C12, 05C72.

### 1. Introduction

L. A. Zadeh [12] introduced the concept of fuzzy sets in 1965. Rosenfeld [8] developed the concept of Fuzzy Graphs (FG) in 1975. E. J. Cockayne and S. T. Hedetniemi [2] introduced the theory of domination in graph in 1977. Paul M. Weichsel [11] introduced the perfect domination in graphs in 1994. A. Somasundaram and S. Somasundaram [9] were introduced the concept of domination in FG independently in 1998. Bhuttani and A. Rosenfeld [1] introduced the concept of geodesies distance in FG in 2003. Janakiraman et al. [5], initiated the eccentric domination in graph in 2010. Linda.J.P and Sunitha M.S [6] introduced on the  $g$ -eccentric nodes and related concepts

in FG in 2012. A. Mohamed Ismayil and S. Muthupandiyar[4] introduced the complementary nil g-eccentric domination in fuzzy graphs in the year 2020.

In this article, the perfect g-eccentric point set, perfect g-eccentric dominating set and their number in FG are introduced. Also obtained the bounds on the perfect g-eccentric domination number for some standard FG as well as some theorems related to perfect g-eccentric domination in FG are stated and proved.

The terms graph and fuzzy graph theoretic terminologies can refer Harary[3] and Rosenfeld and L.A. Zadeh[8, 12] respectively. In this paper,  $G$  is a connected fuzzy graph unless otherwise stated.

## 2. Basic Definitions

In this section the following definitions will help the readers comprehend the core concepts.

**Definition 2.1.**[4, 10] A FG  $G = (\sigma, \mu)$  is characterized with two functions  $\sigma$  on  $V$  and  $\mu$  on  $E \subseteq V \times V$ , where  $\sigma: V \rightarrow [0,1]$  and  $\mu: E \rightarrow [0,1]$  such that  $\mu(x, y) \leq \sigma(x) \wedge \sigma(y), \forall x, y \in V$ . We indicate the crisp graph  $G^* = (\sigma^*, \mu^*)$  of the fuzzy graph  $G = (\sigma, \mu)$  where  $\sigma^* = \{x \in V: \sigma(x) > 0\}$  and  $\mu^* = \{(x, y) \in E : \mu(x, y) > 0\}$ . The order and size of a FG  $G$  are defined by  $p = \sum_{u \in V} \sigma(u)$  and  $q = \sum_{uv \in E} \mu(u, v)$  respectively.

**Definition 2.2.** [1,4, 10] Let  $G$  be a FG. A path  $P$  of length  $n$  is a sequence of distinct nodes  $u_0, u_1, \dots, u_n$  such that  $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \dots, n$  and strength of the path  $P$  is  $(P) = \min\{\mu(u_{i-1}, u_i), i = 1, 2, \dots, n\}$ . The strength of connectedness between nodes  $u$  and  $v$  is defined as the maximum strengths of all paths between  $u$  and  $v$  and is denoted by  $CONN_G(u, v)$ . An  $u - v$  path  $P$  is called a Strong path if it contains only strong arcs. An  $u - v$  path is called strongest  $u - v$  path if its strength is equals  $CONN_G(u, v)$ . An arc is strong iff its weight is equal to the strength of connectedness of its end nodes. i.e.,  $\mu(u, v) \geq CONN_{G-(u,v)}(u, v)$ . If an arc  $(u, v)$  is strong, then  $u$  dominates  $v$  or  $v$  dominates  $u$ .

**Definition 2.3.**[1, 4] A strong path  $P$  from  $u$  to  $v$  is called geodesics if there is no shorter strong path from  $u$  to  $v$  and a length of a  $u - v$  geodesic is the geodesic distance (gdistance) from  $u$  to  $v$  denoted by  $d_g(u, v)$ . The geodesic eccentricity (g-eccentricity)  $e_g(u)$  of a node  $u$  in a connected FG  $G$  is given by  $e_g(u) =$

$\max\{d_g(u, v), v \in V\}$ . The  $g$ -eccentric set of a vertex  $u$  is defined and denoted by  $E_g(u) = \{v: v \in d_g(u, v) = e_g(u)\}$ . The minimum  $g$ -eccentricity among the vertices of  $G$  is called  $g$ -radius and denoted by  $r_g(G) = \min\{e_g(u), u \in V\}$ . The maximum  $g$ -eccentricity among the vertices of  $G$  is called  $g$ -diameter and denoted by  $d_g(G) = \max\{e_g(u), u \in V\}$ . A vertex  $v$  is said to be a  $g$ -central node if  $e_g(v) = r_g(G)$ . Also, a vertex  $v$  in  $G$  is said to be a  $g$ -peripheral node if  $e_g(v) = d_g(G)$ . A FG  $G$  is said to be self centered if  $r_g(G) = d_g(G)$ .

**Definition 2.4.**[7, 11] A subset  $D$  of  $V$  is called a dominating set of a FG  $G$  if for every  $v \in V - D$  there exists  $u \in D$  such that  $u$  dominates  $v$ . A dominating set  $D$  of  $V$  of a FG  $G$  is called a perfect dominating set if for every  $v \in V - D$ , there exists exactly one vertex  $u \in D$  such that  $u$  dominates  $v$ .

**Definition 2.5.**[4] A subset  $S \subseteq V$  in a FG  $G$  is said to be a  $g$ -eccentric point set ( $g$ EP-set) if for every  $v \in V - S$ , there exists at least one  $g$ -eccentric point  $u$  of  $v$  in  $S$ .

**Definition 2.6.**[4] A dominating set  $D$  of  $V$  of a FG  $G$  is called a  $g$ -eccentric dominating set ( $g$ ED-set) of  $G$  if for every  $v \in V - D$  there exists at least a  $g$ -eccentric vertex  $u \in D$  of  $v$ .

### 3. Perfect $g$ -Eccentric Point set in Fuzzy Graphs

In this section, the perfect  $g$ -eccentric point set, perfect  $g$ -eccentric point number and some observations are given in a FG with applicable examples.

**Definition 3.1.** A set  $S \subseteq V$  in a FGG on  $G^*(V, E)$  is said to be a perfect  $g$ -eccentric point set ( $Pg$ EP-set) if for every  $x \in V - S$ , there exists exactly one  $g$ -eccentric point  $y$  of  $x$  in  $S$ . The perfect  $g$ -eccentric point set  $S$  of  $G$  is a minimal perfect  $g$ -eccentric point set if there is no proper subset  $S'$  of  $S$  is a perfect  $g$ -eccentric point set of  $G$ . The minimum cardinality of a minimal perfect  $g$  eccentric point set of  $G$  is called the perfect  $g$ -eccentric number and is denoted by  $e_{pg}(G)$  and simply denoted by  $e_{pg}$ . The maximum cardinality of a minimal perfect  $g$ -eccentric point set is called upper perfect  $g$ -eccentric number and is denoted by  $E_{pg}(G)$  and simply denoted by  $E_{pg}$ .

Note 3.1. The minimum PgEP-set is denoted by  $e_{pg}$  - set.

Example 3.1.

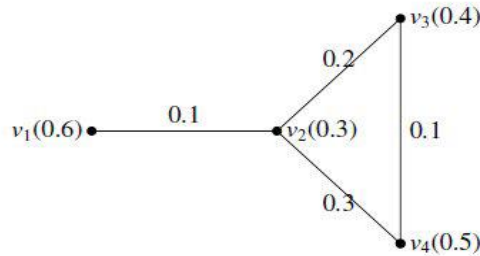


Figure 1: Fuzzy Graph  $G(\sigma, \mu)$

In the FG  $G$  given in figure 1, the  $g$ -eccentricity of  $v_1, v_2, v_3$  and  $v_4$  are  $e_g(v_1) = 2, e_g(v_2) = 1$  and  $e_g(v_3) = 2, e_g(v_4) = 2$  respectively and the  $g$ -eccentric point set of  $v_1, v_2, v_3$  and  $v_4$  are  $E_g(v_1) = \{v_3, v_4\}, E_g(v_2) = \{v_1, v_3, v_4\}, E_g(v_3) = \{v_1, v_4\}$  and  $E_g(v_4) = \{v_1, v_3\}$  respectively. The  $e_{pg}$ -sets are  $S_1 = \{v_3\}$  and  $S_2 = \{v_1\}$ . Hence,  $e_{pg}(G) = 0.4$  and  $E_{pg}(G) = 0.6$ .

Observation 3.1.

- 1 If  $S$  is a PgEP-set, then  $S' \supset S$  is also a PgEP-set.
- 2 If  $S$  is a minimal PgEP- set, then  $S' \subset S$  is not a PgEP-set.
- 3 In a fuzzy tree, every PgEP-set contains at least one pendent vertex.
- 4 For any FG  $G, e_{pg}(G) \leq E_{pg}(G)$ .
- 5 For any FGG,  $e_g(G) \leq e_{pg}(G)$ .
- 6 The complement of a  $e_{pg}$ - set need not be a PgEP-set.

Example 3.2.

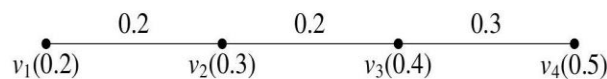


Figure 2: Path Fuzzy Graph  $P_\sigma, |\sigma^*| = 4$

The path  $FG$  given in figure 2, the  $e_{pg}$ -set is  $S = \{v_1, v_4\}$ . The complement of  $SV - S = \{v_2, v_3\}$  is not a PgEP-set.

**Proposition 3.1.** Let  $K_\sigma$  be any complete  $FG$ . Then  $e_{pg}(K_\sigma) = \sigma_0, |\sigma^*| = n$ , where  $\sigma_0 = \min\{\sigma(u), u \in V\}$ .

**Proof.** Let  $K_\sigma$  be any complete  $FG$ . Then  $\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V$ . Therefore, each vertex has exactly  $(n - 1)$   $g$ -eccentric points. Hence, every singleton set is a minimum  $pg$ -set. Therefore,  $e_{pg}(K_\sigma) = \min\{\sigma(u), u \in V\} = \sigma_0$ .

**Proposition 3.2.** Let  $K_{\sigma_1, \sigma_2}$  be a complete bipartite  $FG$ , where  $|\sigma_1^*| = m$  and  $|\sigma_2^*| = n$ . Then  $e_{pg}(K_{\sigma_1, \sigma_2}) = \sigma_{10} + \sigma_{20}$ , where  $\sigma_{10} = \min_{u \in V_1} \sigma(u)$  and  $\sigma_{20} = \min_{v \in V_2} \sigma(v)$ .

**Proof.** Let  $K_{\sigma_1, \sigma_2}$  be a complete bipartite  $FG$ , where  $|\sigma_1^*| = m$  and  $|\sigma_2^*| = n$ . For  $\sigma_1$ , the  $g$ -eccentric set of each vertices are  $E_g(u_1) = \{u_2, u_3, \dots, u_m\}, E_g(u_2) = \{u_1, u_3, \dots, u_m\}, \dots, E_g(u_n) = \{u_2, u_3, \dots, u_{(m-1)}\}$  and similarly, for the vertex set  $\sigma_2$  the  $g$ -eccentric set of each vertices are  $E_g(v_1) = \{v_2, v_3, \dots, v_n\}, E_g(v_2) = \{v_1, v_3, \dots, v_n\}, \dots, E_g(v_n) = \{v_2, v_3, \dots, v_{(n-1)}\}$ . Therefore, the  $e_{pg}$ -set is  $(u, v)$ , let  $\sigma_{10} = \min\{\sigma(u), u \in V_1\}$  and  $\sigma_{20} = \min\{\sigma(v), v \in V_2\}$ . Hence,  $e_{pg}(K_{\sigma_1, \sigma_2}) = \sigma_{10} + \sigma_{20}$

**Proposition 3.3.** Let  $S_\sigma$  be a star  $FG$ . Then  $e_{pg}(S_\sigma) \leq 1, |\sigma^*| = n, n \geq 3$ .

**Proof.** Let  $S_\sigma$  be any star  $FG$ . Let  $c \in V$  be  $g$ -central vertex of a star  $FG$ . Then, all vertices in  $V - \{c\}$  are the  $g$ -peripheral vertices. Now, all the  $g$ -peripheral vertices is the  $g$ -eccentric point of  $g$ -central vertex and a vertex in  $V - \{c\}$  has a  $g$ -eccentric points as all other  $g$ -peripheral vertices except  $g$ -central vertex  $c$ . Therefore, the PgEP-set is  $= \min\{\sigma(u), u \in V - (c)\}$ . Hence,  $e_{pg}S_\sigma \leq 1$ .

**Proposition 3.4.** Let  $P_\sigma$  be any path  $FG$ . Then  $e_{pg}(P_\sigma) = \sigma_0, |\sigma^*| = 2, 3$ .

**Proof.** Let  $P_\sigma, |\sigma^*| = n$  be any path  $FG$ .

**Case(i) If  $n = 2$**

One of the vertex in a path  $FG$  is a perfect  $g$ -eccentric point of other and Therefore,

$$\begin{aligned} e_{pg}(P_\sigma) &= \min_{u \in V} \sigma(u) \\ &= \sigma_0 \end{aligned}$$

**Case(ii) If  $n = 3$**

Every  $g$ -peripheral vertex is a  $g$ -eccentric point of other  $g$ -peripheral vertex and a  $g$ -central vertex has a  $g$ -eccentric point of every  $g$ -peripheral vertex. Therefore, every singleton minimum  $g$ -peripheral vertex is a perfect  $g$ -eccentric point set of  $P_\sigma$ . Therefore,  $e_{pg}(P_\sigma) = \sigma_0$ .

**Proposition 3.5.** Let  $P_\sigma$  be any path  $FG$ .

$$e_{pg}(P_\sigma) \leq \begin{cases} 2, & \text{if } |\sigma^*| = n, n \geq 4, n \text{ is even} \\ 3, & \text{if } |\sigma^*| = n, n \geq 5, n \text{ is odd} \end{cases}$$

**Proof.** Let  $P_\sigma, |\sigma^*| = n$  be any path fuzzy graph

**Case(i) If  $n$  is even,  $n \geq 4$  :**

Let the  $g$ -eccentric set of vertices are  $E_g(u_1) = E_g(u_2) = \dots = E_g(u_{\frac{n}{2}}) = \{u_n\}$  and  $E_g(u_{\frac{n}{2}+1}) = E_g(u_{\frac{n}{2}+2}) = \dots = E_g(u_n) = \{u_1\}$ . Then the PgEP-set is  $S = \{u_1, u_n\}$ . Since, every vertex of  $V - S$  has exactly one  $g$ -eccentric point in  $S$ . Hence,  $e_{pg}(P_\sigma) \leq 2$

**Case(ii): If  $n$  is odd,  $n \geq 5$  :**

Now,  $E_g(u_1) = E_g(u_2) = \dots = E_g(u_{\frac{n+1}{2}-1}) = \{u_n\}, E_g(u_{\frac{n+1}{2}}) = \{u_1, u_n\}$  and  $E_g(u_{\frac{n+1}{2}+1}) = \dots = E_g(u_n) = \{u_1\}$ . Then, the PgEP-set is  $S = \{u_1, u_{\frac{n+1}{2}}, u_n\}$ . Since, every vertex of  $V - S$  has exactly one  $g$ -eccentric point in  $S$ . Hence,  $e_{pg}(P_\sigma) \leq 3$

#### 4 Perfect $g$ -Eccentric Domination in Fuzzy Graphs

In this section, the perfect  $g$ -eccentric dominating set and its numbers are defined in a FG. The relations between domination numbers,  $g$ -eccentric number,  $g$ -eccentric domination number and perfect  $g$ -eccentric domination number are obtained. The perfect  $g$ -eccentric domination numbers of some well known  $FG$  are found and theorems related to perfect  $g$ -eccentric domination numbers are given and demonstrated.

**Definition 4.1.** A dominating set  $D \subseteq V(G)$  is said to be a perfect  $g$ -eccentric dominating set (PgED-set) in a FG  $G$  if for every  $v \in V - D$ , there exists exactly one  $g$ -

eccentric point  $u \in D$  of  $v$ . A perfect  $g$ -eccentric dominating set is a minimal perfect  $g$ -eccentric dominating set if no proper sub set  $D' \subset D$  is a perfect  $g$ -eccentric dominating set. The minimum cardinality taken over all minimal perfect  $g$ -eccentric dominating set is called perfect  $g$ -eccentric domination number and is denoted by  $\gamma_{pged}(G)$ . The maximum cardinality taken over all minimal perfect  $g$ -eccentric dominating set is called upper perfect  $g$ -eccentric domination number and is denoted by  $\Gamma_{pged}(G)$ .

**Note 4.1.** The minimumPgED-set of a  $FG$  is denoted by  $\gamma_{pged}$ -set .

**Example 4.1.**

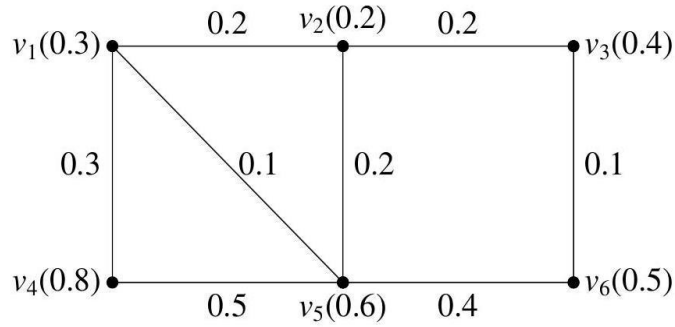


Figure 3: Fuzzy Graph  $G(\sigma, \mu)$

The FG given in figure 3, we observe that,

- 1 The set  $D_1 = \{v_2, v_5\}$  is  $\gamma$ -set and  $\gamma(G) = 0.8$ .
- 2 The set  $D_2 = \{v_1, v_3, v_6\}$  is  $\gamma_{ged}$ -set and  $\gamma_{ged}(G) = 1.2$ .
- 3 The set  $D_3 = \{v_1, v_3, v_5, v_6\}$  and  $D_4 = \{v_2, v_3, v_4, v_6\}$  are minimal PgED-sets. Hence,  $\gamma_{pged}(G) = 1.8$  and  $\Gamma_{pged}(G) = 1.9$ .

**Observation 4.1.**

- 1 If  $D$  is a PgED-set, then  $D' \supset D$  is also PgED-set.
- 2 If  $D$  is a minimal PgED-set, then  $D' \subset D$  is not a PgED-set.
- 3 For any FG  $G$ ,  $\gamma(G) \leq \gamma_{ged}(G) \leq \gamma_{pged}(G) \leq \Gamma_{pged}(G)$ .

- 4 If a connected FG  $G$  has more than one pendent vertex then the PgED-set contains at least two pendent vertex.
- 5 Every PgED-set is a gED-set but the converse need not be true.
- 6 The complement of a  $\gamma_{pged}$ -set need not be PgED-set.

**Example 4.2.**

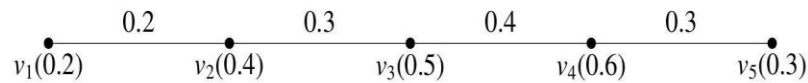


Figure 4: Path Fuzzy Graph  $P_\sigma, |\sigma^*| = 5$

Let us consider the figure 4 given in example 4.2, We observe that

- 1 The set  $D = \{v_1, v_3, v_5\}$  is gED-set and PgED-set. Also, the set  $D_1 = \{v_1, v_2, v_5\}$  is a gED-set but not a PgED-set.
- 2 Let the set  $D = \{v_1, v_3, v_5\}$  is a PgED-set but the complement of  $D$  is  $V - D = \{v_2, v_4\}$  not a PgED-set.

**Observation 4.2.** Let  $D$  be dominating set in a  $FGG$  and  $S$  be a  $e_{pg}$ -set of  $G$ . Then clearly  $D \cup S$  is  $\gamma_{pged}$ -set of  $G$ .

**Observation 4.3.** For any connected  $FGG$ .

- (i)  $\gamma(G) \leq \gamma_{ged}(G) \leq \gamma_{pged}(G)$
- (ii)  $\gamma_{pged}(G) \leq \Gamma_{pged}(G)$ .
- (iii) If  $\text{diam}_g(G) = \text{rad}_g(G)$ , then  $\gamma_{pged}(G) = \gamma(G)$ .

**Theorem 4.1.** For any connected  $FGG, \gamma_{pged}(G) \leq \gamma(G) + e_{pg}(G)$ .

**Proof.** By the observation 4.2, every  $\gamma_{pged}$ -set is a union of dominating set and perfect g-eccentric point set. Hence,  $\gamma_{pged}(G) \leq \gamma(G) + e_{pg}(G)$ .

**Result 4.1.** If  $G(\sigma, \mu)$  is a disconnected  $FG$ , then  $\gamma_{pged}(G) = \gamma(G)$ . Since, vertices from different components are  $g$ -eccentric to each other.



**Note 4.2.**  $\gamma_{pged}(G) = p$  if and only if  $G(\sigma, \mu) = \bar{K}_\sigma$ .

**Theorem 4.2.** For any complete  $FGK_\sigma$ ,  $\gamma_{pged}(K_\sigma) = \sigma_0$ , where  $\sigma_0 = \min_{u \in V} \sigma(u)$ .

**Proof.** If  $G = K_\sigma$  be a complete FG, then  $\text{rad}_g(G) = \text{diam}_g(G) = \sigma_0$ . Hence any vertex  $u \in V(G)$  dominates all other vertices and also perfect g-eccentric point of other vertices. Hence,  $\gamma_{pged}(K_n) = \sigma_0$ .

**Observation 4.4.** For any  $FGG$ ,  $\sigma_0 \leq \gamma_{pged}(G) \leq p$ , where  $\sigma_0 = \min_{u \in V} \sigma(u)$  and  $p$  is the order of  $FGG$ .

**Theorem 4.3.** For a complete bipartite FG,  $\gamma_{pged}(K_{\sigma_1, \sigma_2}) = \sigma_{10} + \sigma_{20}$ , where  $\sigma_{10} = \min_{u \in V_1} \sigma(u)$  and  $\sigma_{20} = \min_{v \in V_2} \sigma(v)$ .

**Proof.** If  $(K_{\sigma_1, \sigma_2})$ ,  $\sigma = \sigma_1 \cup \sigma_2$ , be any complete bipartite FG, where  $|\sigma_1^*| = m$  and  $|\sigma_2^*| = n$  then each point  $u$  of  $V_1$  is adjacent to every point  $v$  of  $V_2$  and vice versa. By theorem 4.1  $D = \{u, v\}$ ,  $u = \min_{u \in V_1} \sigma(u)$  and  $v = \min_{v \in V_2} \sigma(v)$  is a PgEP-set and also  $D$  is a minimum dominating set of  $K_{\sigma_1, \sigma_2}$ . Hence,  $\gamma_{pged}(K_{\sigma_1, \sigma_2}) = \sigma_{10} + \sigma_{20}$ .

**Corollary 4.3.1.** For a star  $FG$ ,  $\gamma_{pged}(K_{\sigma_1, \sigma_2}) = \sigma_{10} + \sigma_{20}$ ,  $|\sigma_1^*| = 1$  and  $|\sigma_2^*| \geq 1$ .

**Theorem 4.4.** Let  $D_1$  and  $D_2$  be two disjoint PgED-sets of a  $FGG$ . Then  $|D_1| = |D_2|$

**Proof.** Let  $G$  be a FG. Let  $D_1$  and  $D_2$  be any two disjoint PgED-sets. For every vertex  $x$  in  $D_1$  there is unique vertex  $v(x)$  in  $D_2$  which is adjacent to  $x$  and exactly a g-eccentric vertex to  $x$ . Also, for every vertex  $y$  in  $D_2$  there is unique vertex  $u(y)$  in  $D_1$  which is adjacent to  $y$  and exactly a g-eccentric vertex to  $y$ .  $\therefore |D_1| = |D_2|$ .

**Corollary 4.4.1.** Let  $G$  be any  $FG$  and if  $D_1$  and  $D_2$  be any two PgED-sets such that  $|D_1| = |D_2|$  then  $D_1 \cap D_2 = \emptyset$ .

**Corollary 4.4.2.** Let  $G$  be a  $FG$  with  $n$  vertices. If there is a PgED-set  $D$  with  $|D| < n/2$  or  $|D| \geq n/2$  then  $V - D$  is not a PgED-set.

**Theorem 4.5.** Let  $D$  be a (minimal)  $\gamma_{pged}$ -set of connected  $FGG$ . Then  $V - D$  is a gED-set of  $G$

**Proof.** Let  $D$  be a minimal PgED-set of connected FG  $G$ . Suppose  $V - D$  is not a gED-set. Then there exists a vertex  $v \in D$  such that  $v$  (is not dominated by any vertex) has

no  $g$ -eccentric vertex in  $V - D$ . Since  $G$  is connected,  $v$  is strong neighbor of atleast one vertex in  $D - \{v\}$ . Then  $D - \{v\}$  is a  $gED$ -set, which is contradicts to the minimality of  $D$ . Thus every vertex in  $D$  is strong neighbor of atleast one vertex in  $V - D$ . Hence  $V - D$  is a  $gED$ -set.

**Result 4.2.** Every  $\gamma_{pged}$ -set of a FG  $G$  is a minimal  $PgED$ -set but the converse need not be true.

**Theorem 4.6** Let  $G$  be a FG and let  $D$  be a  $PgED$ -set. Then  $D$  is a minimal  $PgED$ -set if and only if for each vertex  $u \in D$ , satisfies one of the following condition:

1.  $N_s(u) \cap D = \phi$  or  $E_g(u) \cap D = \phi$ .
2. There exists some  $v \in V - D$  such that (i)  $N_s(v) \cap D = \{u\}$  or (ii)  $E_g(v) \cap D = \{u\}$  or (iii)  $E_g(v) \cap D \neq \phi$ .

**Proof:** Suppose that  $D$  is a minimal  $PgED$ -set of a FGG. Then for every  $u \in D$ ,  $D - \{u\}$  is not a  $PgED$ -set. Then (i) there exists some vertex  $v \in V - D \cup \{u\}$  which is not dominated by any vertex in  $D - \{u\}$  or (ii) there exists  $v \in V - D \cup \{u\}$  such that  $v$  has no  $g$ -eccentric point in  $D - \{u\}$  or (iii) there exists  $v \in V - D \cup \{u\}$  such that  $v$  has at least two  $g$ -eccentric point in  $D - \{u\}$ .

**Case (1)** If  $u = v$ , then (i)  $u$  has no strong neighbor in  $D - \{u\}$ . Hence  $N_s(u) \cap D = \phi$  or (ii)  $u$  has no  $g$ -eccentric point in  $D - \{u\}$ . Hence  $E_g(u) \cap D = \phi$  or (iii)  $u$  has at least two  $g$ -eccentric point in  $D - \{u\}$ . Hence  $\mathbb{N}_g(u) \cap D \neq \emptyset$ .

**Case (2)**  $u \neq v$ , then (i) If  $v$  is not dominated by  $D - \{u\}$  but is dominated by  $u$ , then  $v$  is adjacent to only  $u$  in  $D$ , that is  $\mathbb{N}_s(v) \cap D = \{u\}$ . (ii) Suppose  $v$  has no  $g$ -eccentric point in  $D - \{u\}$  but  $v$  has a  $g$ -eccentric point in  $D$  that is  $\mathbb{N}_g(v) \cap D = \{u\}$ . (iii) Suppose  $v$  has at least two  $g$ -eccentric point in  $D - \{u\}$  but  $v$  has at least two  $g$ -eccentric point in  $D$  that is  $\mathbb{N}_g(v) \cap D \neq \emptyset$ .

Conversely, suppose that  $D$  is a  $PgED$ -set and for each  $u \in D$  one of the conditions holds.

Assume that  $D$  is not a minimal  $PgED$ -set, then there exists a vertex  $u \in D$  such that  $D - \{u\}$  is a  $PgED$ -set. Therefore,  $u$  is strong neighbor to at least one vertex  $v$  in  $D - \{u\}$  or  $u$  has a  $g$ -eccentric point in  $D - \{u\}$ . Hence, condition (1) does not hold.

Also, if  $D - \{u\}$  is a  $PgED$ -set, then every element  $v$  in  $D - \{u\}$  is strong neighbor to exactly one vertex in  $D - \{u\}$  or  $v$  has exactly one  $g$ -eccentric point in  $D - \{u\}$  or  $\mathbb{N}_g(v) \cap D \neq \emptyset$ .

Therefore, condition (2) does not hold. This is a contradiction to our assumption that for each  $\emptyset \in \mathbb{I}$ , satisfies one of the condition. Hence,  $\mathbb{I}$  is a minimal PgED-set.

## 6. Conclusion

The perfect  $g$ -eccentric point set, perfect  $g$ -eccentric dominating set, its number and bounds for this numbers in fuzzy graphs are discussed in this paper.  $g$ -Eccentric perfect dominating set and perfect  $g$ -eccentric perfect dominating set in fuzzy graphs may be discuss in future.

## References

- [1] Kiran R Bhutani and Azriel Rosenfeld. Geodesies in fuzzy graphs. *Electronic Notes in Discrete Mathematics*, 15:49-52, 2003.
- [2] Ernest J Cockayne and Stephen T Hedetniemi. Towards a theory of domination in graphs. *Networks*, 7(3):247-261, 1977.
- [3] F. Harary. *Graph Theory*. Addition - Wesley Publishing Company Reading, Mass, 1992.
- [4] A. Mohamed Ismayil and S. Muthupandiyar. Complementary nil  $g$ -eccentric domination in fuzzy graphs. *Advances in Mathematics: Scientific Journal*, 9(4):1719-1728, 2020.
- [5] TN Janakiraman, M Bhanumathi, and S Muthammai. Eccentric domination in graphs. *International Journal of Engineering Science, Advanced Computing and Bio-Technology*, 1(2):1-16, 2010.
- [6] JP Linda and MS Sunitha. On  $g$ -eccentric nodes  $g$ -boundary nodes and  $g$ -interior nodes of a fuzzy graph. *Int Jr Math SciAppl*, 2:697-707, 2012.
- [7] SV Rashmi, Subramaniyan Arumugam, Kiran R. Bhutani and Peter Gartland, Perfect Secure domination in Graphs, *Categories and General Algebraic Structures with Applications*, 7(Special Issue on the occasion of Banaschewski's 90<sup>th</sup> Birthday (II)):125-140, 2017.
- [8] Azriel Rosenfeld. Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes*, pages 77-95. Elsevier, 1975.

- [9] A Somasundaram and S Somasundaram. Domination in fuzzy graphs-i. Pattern Recognition Letters, 19(9):787-791, 1998.
- [10] MS Sunitha and Sunil Mathew. Fuzzy graph theory: a survey. Annals of Pure and Applied mathematics, 4(1):92-110, 2013.
- [11] Paul M. Weichsel, Dominating sets in n-cubes, *Journal of Graph Theory*, 18(5): 479-488, 1994.
- [12] L.A. Zadeh. Fuzzy sets. Information and Control, 8(3):338-353, 1965.

<sup>1st</sup> S. MUTHUPANDIYAN AND <sup>2nd</sup> A. MOHAMED ISMAYIL

<sup>1</sup> Research Scholar, PG & Research Department of Mathematics, Jamal Mohamed College(Affiliated to Bharathidasan University), Tiruchirappalli-620020, Tamilnadu, INDIA

<sup>2</sup> Associate Professor, PG & Research Department of Mathematics, Jamal Mohamed College(Affiliated to Bharathidasan University), Tiruchirappalli-620020, Tamilnadu, INDIA.

"Corresponding Author E-Mail Address:muthupandianmaths@gmail.com