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AN INTRODUCTION OF GRAPH THEORY IN APPLIED MATHEMATICS

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Abstract: Graph theory is an important area of Applied Mathematics with a broad spectrum of applications in many fields. Graph theoretical concepts are widely used to study and model various applications, in different areas. Further, with connections to other branches of mathematics, many various tools are being employed to considerable effect from algebra, analysis, geometry, number theory, probability, and topology. The main objective of this paper is to introduce the main concepts of Graph theory. Graph theory is a branch of mathematics which has wide application in the area of mathematics as well as in other branches of science. This paper aims to emphasize the applications of graph theory in Applied Mathematics, in Computer science, Operation Research, Chemistry etc

Keywords: Graph theory, Graph theoretical concepts, graph algorithms, Graph theory and its Applications

Introduction

Graph theory is becoming interestingly significant as it is being actively applied in biochemistry, nanotechnology, electrical engineering, computer science, and operations research. The powerful combinatorial method found in graph theory has also been used to prove the results of pure mathematics.

Graph theoretical ideas are extremely utilized by computer discipline applications. particularly in research areas of computer discipline such data mining, image segmentation, clustering, image capturing, networking etc., For example a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly modelling of network topologies can be done using graph concepts. In the same way the most important concept of graph colouring is utilized in resource allotment, setting up. Also, paths, walks and circuits in graph theory are used in wonderful applications say travelling salesman problem, database design concepts, resource networking. This leads to the development of new algorithms and new theorems that can be used in tremendous applications.

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges. A graph is a pair (V,E), where V is a finite set and E is a binary relation on V. V is called a vertex set whose elements are called vertices.

E is a collection of edges, where an edge is a pair (u,v) with u,v in V. Graphs are one of the prime objects of study in discrete mathematics. Certain discrete problems can be profitably analyzed using graph theoretic methods.

Origin of graph theory

1. The origin of graph theory started with the problem of Koinsber bridge, in 1735. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called **Eulerian graph.**
2. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems.
3. The concept of tree, which is **a connected graph without cycles** was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits.
4. In 1852, Thomas Gutherie found the famous four color problem.
5. Iin 1856, Thomas. P. Kirkman and William R.Hamilton studied cycles on polyhydra and invented the concept called **Hamiltonian graph** by studying trips that visited certain sites exactly once.
6. In 1913, H. Dudeney mentioned a puzzle problem.
7. Even though the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken.
8. Caley studied particular analytical forms from differential calculus to study the trees. This had many implications in theoretical chemistry. This lead to the invention of enumerative graph theory.
9. The term "Graph" was introduced by Sylvester in 1878 where he drew an analogy between "Quantic invariants" and co-variants of algebra and molecular diagrams.
10. In 1941, Ramsey worked on colorations which lead to the identification of another branch of graph theory called **extremel graph theory**.
11. In 1969, the four color problem was solved using computers by Heinrich. The study of asymptotic graph connectivity gave rise to random graph theory.

Review of Literature

Sahar Abbasi and Sadoullah Ebrahimnejad (2011) in this paper they considered the dynamic shortest path problem, motivated by its applications in dynamic minimum cost flows in transformation problem. They showed that this problem is equivalent to a classical shortest path problem in a so-called time-expanded network.

Lili Cao, Xiaohan Zhao, Haitao Zheng, and Ben Y. Zhao conclude that search for shortest paths is an essential primitive for a variety of graph-based applications, particularly those on online social networks.

S.G.Shirinivas, S.Vetrivel and Dr. N.M.Elango (2010) presented the importance of graph theoretical ideas in various areas of computer applications like Shortest path algorithm in a network, Finding a minimum spanning tree, Finding graph planarity, Algorithms to find adjacency matrices, Algorithms to find the connectedness, Algorithms to find the cycles in a graph, Algorithms for searching an element in a data structure (DFS, BFS)

A. J. Baddeley and H. J. A. M. Heijmans describe the use of mathematical morphology in image analysis. They explain mathematical morphology to examine the geometrical structure of an image by matching it with small patterns at various locations in the image. By varying the size and the shape of the matching patterns one can extract useful information about the shape of the different parts of the image and their interrelations.

Objectives of the Study

This paper aims to emphasize the applications of graph theory in Applied Mathematics, in Computer science, Operation Research, Chemistry etc.

Graph theory and its Applications :

Graph theory has a very wide range of applications in engineering, in physical, social, and biological sciences, in linguistics, and in numerous other areas. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them.

They are

1. study of molecules,
2. construction of bonds in chemistry and the study of atoms.
3. graph theory is used in sociology
4. It is used in biology
5. Graph theoretical concepts are widely used in Operations Research.
6. It is also used in modeling transport networks, activity networks and games.
7. It is used in computational biochemistry.

In briefly, graph theory has its unique impact in various fields and is growing large now a days. The subsequent section analyses the applications of graph theory especially in computer science.

Algorithms and graph theory:

The most important role of graph theory in computer applications is the growth of graph algorithms. several algorithms are used to solve problems that are modeled in the form of graphs. These algorithms are used to solve the graph theoretical concepts which intern used to solve the corresponding computer science application problems.

Some algorithms are as follows:

1. Shortest path algorithm in a network
2. Finding a minimum spanning tree
3. Finding graph planarity
4. Algorithms to find adjacency matrices.
5. Algorithms to find the connectedness
6. Algorithms to find the cycles in a graph
7. Algorithms for searching an element in a data structure (DFS, BFS) and so on.

Use of graph enumeration techniques:

Graph enumeration technique is used to identify the computerized chemical identification. The list of all distinct chemical structures will be generated based on the given chemical formula and the valence rules for any new substance. To identify the chemical compounds automatically, a computer language called DENDRAL has been developed.

Graph Theory in OR:

Graph theory is a very natural and powerful tool in combinatorial operations research. Some important OR problems that can be solved using graphs are given here. A network called transport network where a graph is used to model the transportation of commodity from one place to another. The objective is to maximize the flow or minimize the cost within the prescribed flow. The graph theoretic approach is found more efficient for these types of problems though they have more constraints .

Graph Coloring:

Graph coloring is one of the most vital concepts in graph theory and is used in many real time applications in computer science. Various coloring methods are available and can be used on necessity basis. The proper coloring of a

graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the **chromatic number** and the graph is called properly colored graph.

Precoloring extension:

In certain scheduling problems, the assignments of jobs are already decided. In such cases precoloring technique can be adopted. Here some vertices of the graph will have preassigned color and the precoloring problem has to be solved by extending the coloring of the vertices for the whole graph using minimum number of colors.

List coloring:

In list coloring problem, each vertex v has a list of available colors and we have to find a coloring where the color of each vertex is taken from the list of available colors. This list coloring can be used to model situations where a job can be processed only in certain time slots or can be processed only by certain machines.

Minimum sum coloring:

In minimum sum coloring, the sum of the colors assigned to the vertices is minimal in the graph. The minimum sum coloring technique can be applied to the scheduling theory of minimizing the sum of completion times of the jobs.

The multicolor version of the problem can be used to model jobs with arbitrary lengths. Here, the finish time of a vertex is the largest color assigned to it and the sum of coloring is the sum of the finish time of the vertices. That is the sum of the finish times in a multicoloring is equal to the sum of completion times in the corresponding schedule.

Use of Graph Coloring

In many real-time applications of computer science, graph coloring technique is very important. There are many coloring methods available depending upon the requirement of the real-time applications. In graphs, proper coloring is used for the vertices and edges where no two vertices/ edges should have the same color. This type of graph is called a colored graph, and the minimum number of colors used in a graph is known as the chromatic number. The coloring is done in a way that two adjacent vertices/edges get distinct colors for the clarity. The example of edges and vertex coloring is shown in **Figure 1** a,b, respectively.

(a) Edge coloring graph having chromatic number 4. (b) Vertices coloring graph having chromatic number 4.

Figure 1. Overview of the colored graphs (vertex and edges).

Uses of Graph Theory in Algorithms

In the field of CS, algorithms have very important roles for developing and upgrading applications. Generally, software developers make the complete design of their applications prior to development, and then they follow this design to develop applications, so that applications are flawless. GT plays a very important role in designing algorithms. There are many algorithms developed with the help of graphs to solve different real-world problems. Some of them are listed as: (1) Depth First Search (DFS) and Breath First Search (BFS) algorithms used in the data structure for searching a node in a directed or undirected graph, (2) MST

(Minimum Spanning Tree) algorithm, (3) algorithms for finding the shortest path in a network, (4) Graph Planarity algorithm, and (5) depicting the data transfer in complex applications.

APPLICATIONS OF GRAPH THEORY

Applications in Computer Science: There is a major role of graph theory in computer science. Various applications that deal with computers are using graph theory concepts.

Some Applications are as follows:

1. Map coloring and GSM mobile phone networks
2. Graph algorithm in computer network security
3. Graph theory relevant to ad-hoc networks
4. A graph model for fault tolerant computing systems
5. The optimal k-FT single loop system
6. Automatic channel allocation for small wireless local areanetworks using graph coloring algorithm approach
7. Clustering of web documents using graph model
8. Modeling sensor networks as graph.

Applications in Google map:

Now a days, Google map is a very useful tool for travelling anywhere in the world. Using Google map we can find all routes from one place to any other place and also can find the shortest route. In case of Google map, we can consider the places as vertices of graph and the routes as the edges. Then the software of Google map, when we find the routes between two places, it will find all edges between these two places or vertices and also gives the shortest edge as the shortest path.

Applications in Internet:

Internet is a very useful invention of modern science. In the working technique of internet the concepts of graph theory are used. In case of connectivity of internet, all the users are considered as vertices and the connection between them are edges. Similarly, in case of social networking sites one friend is connected to other friends and his friends are also connected to others. If we consider the friends as vertices of graph and define an edge in between them, then it will form a graph.

Graphs in Chemistry:

Graph theory is used in chemistry for mathematical modeling of chemical phenomena. We can make natural model of a molecule where vertices represent atoms and edges represent bond. There is a branch of mathematical chemistry called Chemical graph theory (CGT) which deals with the non trivial applications of graph theory to solve molecular problems.

Applications in Operation Research:

Graph theory is a very useful tool in operation research. There are some Operation research problems that can be solved using graphs. In transportation problem, when we need to minimize the transportation cost or maximize the profit, then the graph theoretical approach is very useful. It is also used in different assignment problems such as assigning different peoples to different jobs, manage time table for school, college etc.

Conclusion and Scope of Research Work:

Graph Theory has been realized as one of the most flourishing branches of modern Mathematics finding widest applications in all most all branches of

Sciences, Social Sciences, Engineering, Computer Science, etc. Number Theory in one of the oldest branches of Mathematics, which inherited rich contributions from almost all greatest mathematicians, ancient and modern. The main aim of this paper is to present the importance of graph theoretical ideas in various areas of compute applications for researches that they can use graph theoretical concepts for the research. This paper is valuable for students and researchers to get the overview of graph theory and its application in diverse fields like everyday life, computer science, Operation Research, Chemistry.

References:

[1] Sahar Abbasi and Sadoullah Ebrahimnejad, International Journal of Business and Social Science: 2011, Finding the Shortest Path in Dynamic Network using Labeling Algorithm.

[2] Atlas: Approximating Shortest Paths in Social Graphs by Lili Cao, Xiaohan Zhao, Haitao Zheng, and Ben Y. Zhao, Computer Science Department, U. C. Santa Barbara.

[3] S.G.Shirinivas, S.Vetrivel and Dr. N.M.Elango, International Journal of Engineering Science and Technology: 2010, Application of Graph theory in Computer Science an overview.

[4] H. J. A. M. Heijmans. Mathematical morphology: a modern approach in image processing based on algebra and geometry. SIAM Review, Vol. 37, pp.1-36, 1995.

[5] A. J. Baddeley and H. J. A. M. Heijmans. Incidence and lattice calculus with applications to stochastic geometry and image analysis. Applicable Algebra in Engineering, Communication and Computing, Vol. 6, pp. 129-146, 1995.

[6] Albert R Meyer, "Mathematics for Computer Science", chapter 13:Communication Networks, pp:253-272, May 9, 2010.

[7] Adam Schenker, Mark Last, horst Banke,Abraham andel, "Clustering of Web documents using a graph model", Springer werlog, September 2007.

[8] Anindya J.Pal, Samar S.Sarma, Biman Ray, "CCTP, Graph Coloring algorithms" Soft computing Solutions IEEE, 2007

[9] J.A. Bondy and U.S.R. Murty, "Graph Theory with Applica-tions", Elsevier Science Publishing Company Inc. 1976.

[10] E. Burman and P. Horak, "Some applications of graph theory to other Parts of Mathematics", *The Mathematical Intelligencer*, Springer-Velag, New York, 1996, 6-11.

[11] S. Pirzada and A. Dharwadkar, "Graph Theory", Orient Longman and Universities Press of India, 2007

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