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A FUZZY INVENTORY MODEL BY CONTROLLING CARBON EMISSIONS WITH GREEN ENVIRONMENT

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Abstract: The aim of this model is to suggest a fuzzy inventory model that is environmentally friendly by controlling carbon emissions. The issue is explored in both a crisp and a fuzzy sense. Defuzzification is accomplished using the graded mean integration representation process. Additionally, hexagonal fuzzy numbers are used to represent fuzzy parameters such as keeping cost, setup cost, ordering cost, variable cost, social cost, and carbon emission price. The paper comes to a close with a numerical illustration.

Introduction

The release of carbon into the atmosphere is referred to as carbon pollution. Talking regarding carbon emissions is essentially the same as talking about greenhouse gas emissions. When addressing global warming or the greenhouse effect, greenhouse gas emissions are often addressed to as carbon emissions because they are often measured as carbon di oxide equivalents. The consumption of fossil fuels has risen since the industrial revolution, and is directly related to the rise in carbon dioxide levels in our atmosphere and, as a result, the dramatic increase in global warming.

To cut carbon emissions costs, Elhedhli and Merrick (2012) devised a green supply chain network concept. According to Den elzen (2013), emissions from manufacturing, biomass, and transportation activities in the city are influenced by citizen conduct. As a result, the consumption-based approach is linked to higher overall emissions in cities than the production-based approach. Sarkar et al. (2015) expanded on Sarkar's (2013) definition by considering the impact of carbon emissions during the transport of items from vendor to consumer. Sana et al. (2014) looked at a three-layer supply chain of various vendors, distributors, and retailers for a variety of products. When demand is based on sales teams' initiatives, Cardenas Barron and Sana (2014) established a production inventory model for a two-tier supply chain. Yang et al. (2015) presented a two-stage optimization approach for a multi-objective apply chain network design issue with unknown transportation costs and unknown customer demands. When the quantity of demand is unknown, Chang (1999) explored how to determine the economic output quantity. To address the inventory dilemma, Chen et al (2000) developed a fuzzy economic development model with all parameters and variables being fuzzy numbers. Heish (2002), Lee et al (1998), and Lin et al (2000) are among the authors of papers on fuzzy development models. In certain cases, uncertainty is caused by fuzziness, which is mainly introduced by Zadeh. Zadeh et al. suggested several decision-making methods in a blurry environment in 1070. Jain's research focused on making decisions in the presence of ambiguous variables. Kcpryzk et al used fuzzy decision-making models to address certain long-term inventory policy-making. We will suggest an inventory model to manage carbon emissions using fuzzy principles in this paper. The solution is explored in both a clear and a hazy light. To discuss the same model in a fuzzy world, we use hexagonal fuzzy numbers. A numerical illustration is also given to demonstrate the proposed model, and the mathematical model is finally concluded.

Definitions and Methodologies:

Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$.Here $\mu_{\tilde{A}}:X\to[0,1]$ is a mapping called the membership value of $x\in X$ in a fuzzy set $\tilde{A}$.

Hexagonal Fuzzy Number: A fuzzy number $\tilde{A}$ is a hexagonal fuzzy number denoted by

$\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6)$ Where $(a_1\le a_2\le a_3\le a_4\le a_5\le a_6)$ are real numbers satisfying $(a_2-a_1\le a_3-a_2)$ and $(a_5-a_4\le a_6-a_5)$ and its membership function $\mu_{\tilde{A}}(x)$ is given as

$$\mu_{\tilde{A}}(x)=\begin{cases}0, x<a_1\\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), a_1\le x\le a_2\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), a_2\le x\le a_3\\ 1, a_3\le x\le a_4\\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), a_4\le x\le a_5\\ 0, x>a_6\end{cases}$$

Graded Mean Integration Representation Method: If $\tilde{A}=(a,b,c,d,e,f)$ is a hexagonal fuzzy number then the graded mean representation (GMIR) method of $\tilde{A}$ is defined as

$$P\left(\tilde{A}\right)=\frac{1}{12}\left[a+3b+2c+2d+3e+f\right]$$

Arithmetic Operations under Function Principle: The arithmetic operations between hexagonal fuzzy numbers proposed are given below.

Let us consider $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6)$ and $\tilde{B}=(b_1,b_2,b_3,b_4,b_5,b_6)$ be two hexagonal fuzzy numbers. Then

- The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\oplus\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5,a_6+b_6)$
- The subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\Theta\tilde{B}=(a_1-b_6,a_2-b_5,a_3-b_4,a_4-b_3,a_5-b_2,a_6-b_1)$
- The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\otimes\tilde{B}=(a_1b_1,a_2b_2,a_3b_3,a_4b_4,a_5b_5,a_6b_6)$
- The division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\o\tilde{B}=\left(\frac{a_1}{b_6},\frac{a_2}{b_5},\frac{a_3}{b_4},\frac{a_4}{b_3},\frac{a_5}{b_2},\frac{a_6}{b_1}\right)$
- If α is a scalar, $\alpha\tilde{A}$ is defined as $\alpha\tilde{A}=\begin{cases}(\alpha a_1,\alpha a_2,\alpha a_3,\alpha a_4,\alpha a_5,\alpha a_6),\alpha\geq 0\\(\alpha a_6,\alpha a_5,\alpha a_4,\alpha a_3,\alpha a_2,\alpha a_1),\alpha<0\end{cases}$

Notations:

A_d – Annual demand

P_r – Production rate

Q – Order quantity per cycle

n – Number of times the order is produced

d – Distance travelled

O_c – Order cost per cycle

C_e - Carbon emission quantity from order per cycle

S_c – Setup cost

C_s - Carbon emission from setup cost

H_C – Inventory holding cost per unit time

C_h - Carbon emission quantity from inventory holding

y – Variable cost per unit transported

k – Social cost

C_p – Carbon emission price

$\tilde{O}_c$ - Fuzzy Order cost per cycle

$\tilde{S}_c$ - Fuzzy Setup cost

$\tilde{H}_c$ - Fuzzy Inventory holding cost per unit time

$\tilde{y}$ - Fuzzy Variable cost per unit transported

$\tilde{k}$ - Fuzzy Social cost

$\tilde{C}_p$ - Fuzzy Carbon emission price

Assumption:

- The carbon emission amounts are consistent with the EOQ assumptions.
- The total demand is assumed to be constant.
- A single product is evaluated over a set amount of time.
- Time of plan is constant.

Mathematical model in crisp sense:

In the first place, we consider an inventory model using the above notations and assumptions. The cumulative cost is calculated using the equation.

$$T_C = \frac{A_d}{Q}\left[\left(O_C + C_P C_e\right) + \left(\frac{S_C + C_P C_S}{n}\right) + 2k + \frac{Q}{2}\left[\left(H_C + C_P C_h\right) + \frac{nC_P A_d}{P_r} + \frac{yd}{A_d}\right]\right]$$

In order to find the optimal order quantity the above equation is differentiated with respect to Q and equated to zero.

The optimal order quantity is derived as Q^*

$$Q^* = \sqrt{\frac{2A_d\left[\left(O_C + C_P C_e\right) + \left(\frac{S_C + C_P C_S}{n}\right) + 2k\right]}{\left[\left(H_C + C_P C_h\right) + \frac{nC_P A_d}{P_r} + \frac{yd}{A_d}\right]}}$$

Mathematical Model in Fuzzy sense:

The ordering cost, setup cost, variable cost, social cost, carbon emission price, and inventory holding cost are all fuzzy in nature, so we consider the model in a fuzzy setting. We use hexagonal fuzzy numbers to represent them. The annual demand as well as the order quantity per cycle are treated as constants.

Now we fuzzify total cost,The fuzzy total cost is given by

$$\tilde{T}_C = \frac{\dfrac{A_d}{Q}\left[\begin{array}{l}\left(O_{C_1}+C_eC_{P_1}+3O_{C_2}+3C_eC_{P_2}+2O_{C_3}+2C_eC_{P_3}+2O_{C_4}+2C_eC_{P_4}+3O_{C_5}+3C_eC_{P_5}+O_{C_6}+C_eC_{P_6}\right)+\\ \dfrac{1}{n}\left(S_{C_1}+C_SC_{P_1}+3S_{C_2}+3C_SC_{P_2}+2S_{C_3}+2C_SC_{P_3}+2S_{C_4}+2C_SC_{P_4}+3S_{C_5}+3C_SC_{P_5}+S_{C_6}+C_SC_{P_6}\right)+\\ 2\left(k_1+3k_2+2k_3+2k_4+3k_5+k_6\right)\end{array}\right]}{\dfrac{Q}{24}\left[\begin{array}{l}\left(H_{C_1}+C_hC_{P_1}+3H_{C_2}+3C_hC_{P_2}+2H_{C_3}+2C_hC_{P_3}+2H_{C_4}+2C_hC_{P_4}+3H_{C_5}+3C_hC_{P_5}+H_{C_6}+C_hC_{P_6}\right)+\\ \dfrac{nA_d}{P_r}\left(C_{P_1}+3C_{P_2}+2C_{P_3}+2C_{P_4}+3C_{P_5}+C_{P_6}\right)+\dfrac{d}{A_d}\left(y_1+3y_2+2y_3+2y_4+3y_5+y_6\right)\end{array}\right]}$$

And, the fuzzy optimum order quantity,

$$Q^* = \sqrt{\frac{2A_d\left[\begin{array}{l}\left(O_{C_1}+C_eC_{P_1}+3O_{C_2}+3C_eC_{P_2}+2O_{C_3}+2C_eC_{P_3}+2O_{C_4}+2C_eC_{P_4}+3O_{C_5}+3C_eC_{P_5}+O_{C_6}+C_eC_{P_6}\right)+\\ \dfrac{1}{n}\left(S_{C_1}+C_SC_{P_1}+3S_{C_2}+3C_SC_{P_2}+2S_{C_3}+2C_SC_{P_3}+2S_{C_4}+2C_SC_{P_4}+3S_{C_5}+3C_SC_{P_5}+S_{C_6}+C_SC_{P_6}\right)+\\ 2\left(k_1+3k_2+2k_3+2k_4+3k_5+k_6\right)\end{array}\right]}{\left[\begin{array}{l}\left(H_{C_1}+C_hC_{P_1}+3H_{C_2}+3C_hC_{P_2}+2H_{C_3}+2C_hC_{P_3}+2H_{C_4}+2C_hC_{P_4}+3H_{C_5}+3C_hC_{P_5}+H_{C_6}+C_hC_{P_6}\right)+\\ \dfrac{nA_d}{P_r}\left(C_{P_1}+3C_{P_2}+2C_{P_3}+2C_{P_4}+3C_{P_5}+C_{P_6}\right)+\dfrac{d}{A_d}\left(y_1+3y_2+2y_3+2y_4+3y_5+y_6\right)\end{array}\right]}}$$

Numerical Example:

In Crisp Sense

Let,

P_r = 70000 units per year,

A_d = 50000 units per year,

O_c = 150 per cycle,

C_p = Rs.0.2/-,

C_e = 350 Order per cycle,

S_c = Rs.700/-,

C_s = 350,

H_c = 0.6 Per unit time,

C_h = 2,

n=1,

k=Rs. 0.5/-,

y=5 per unit transported,

d=250 Km

Then, the optimal order quantity

$$Q^* = \sqrt{\frac{2A_d\left[\left(O_C + C_P C_e\right) + \left(\frac{S_C + C_P C_S}{n}\right) + 2k\right]}{\left[\left(H_C + C_P C_h\right) + \frac{nC_P A_d}{P_r} + \frac{yd}{A_d}\right]}}$$

Q^* =9211.74627

And, the total cost

$$T_C = \frac{A_d}{Q}\left[\left(O_C + C_P C_e\right) + \left(\frac{S_C + C_P C_S}{n}\right) + 2k + \frac{Q}{2}\left[\left(H_C + C_P C_h\right) + \frac{nC_P A_d}{P_r} + \frac{yd}{A_d}\right]\right]$$

TC^* = Rs.12804.08/-

Conclusion:

We present a fuzzy inventory model with a green environment by regulating carbon emissions in this research. The issue is explored in both a crisp and a fuzzy light. Defuzzification is accomplished using the graded mean integration representation approach. Additionally, hexagonal fuzzy integers are used to represent fuzzy characteristics such as holding cost, setup cost, ordering cost, variable cost, social cost, and carbon emission price. Numerical examples are used to verify the calculated solutions. This work can be further upon in the future for additional research projects.

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