

## A STUDY OF THE PROBLEMS OF HEAT TRANSFER IN THE FLOW OF NON – NEWTONIAN SECOND ORDER FLUID

POOJA GOEL, & DR. MANOJ SRIVASTAVA \*\*

**Abstract:** The goal of this research is to use a regular perturbation approach to investigate heat transfer in the flow of a second-order fluid through a channel with porous walls in a transverse magnetic field. For varying values of the Hartman and Reynolds numbers, the second-order effects on the temperature profile are shown. The Newtonian fluid's findings are likewise obtained by setting the second-order parameter to e zero.

### Introduction

Because it involves both iological and non-iological disciplines, the study of non-Newtonian fluids (fluids that do not obey the Newtonian rule of viscosity) is of great interest and importance. A non-Newtonian fluid is one whose viscosity is affected by the force applied to it (and sometimes time and temperature as well). Fluids like water and gasoline obey Newton's model and are referred to as Newtonian fluids; however, ketchup, blood, yoghurt, gravy, pie fillings, mud, and cornstarch paste do not. Because doubling the speed at which the layers slide past each other does not double the resisting force, they are non-Newtonian fluids. It might be less than double (like ketchup) or more than double (like ketchup) (as in the case of quicksand and gravy). Stirring gravy thickens it, and battling in quicksand makes it much more difficult to get out. We can push some fluids (such as dirt or snow) and receive no flow until we press hard enough and the substance begins to flow like a normal liquid. Mudslides and avalanches are the result of this. The movement of fluids and the deformation of solids under stress and strain are referred to as rheology. Rheometers are instruments that are used to test the rheological qualities of a substance. Hook's law, which asserts that deformation is proportionate to applied force, is perhaps the first recognised law. Newton analysed the behaviour of an imaginary fluid filling all space, in which resistance to motion was proportional to what is known as the Newton-Cauchy-Poisson law, which has variously been dubbed rate of strain, rate of deformation, velocity strain, or flow tensor dij. Accordingly,

$$2d_{ij} + \delta_{ij} = p_{ij} + 2d_{ij} + \delta_{ij}$$

where

$$(u_{i,j} + u_{j,i})/2$$

p denotes pressure, and  $\delta_{ij}$  denotes material constants, commonly known as viscosity coefficients, and  $\delta_{ij}$  denotes Kronecker's delta tensor. Newtonian fluids are fluids that satisfy the relation (1.1), such as honey, glycerin, and some thick oils. The relation (1.1) applies to incompressible fluids.

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Although this classical theory successfully explains certain phenomena such as skin friction, form drag, separation, secondary flows, and so on, it has proven insufficient to explain the rheological properties of certain materials such as paints, slurries, ceramics, melts poly-iso-utylene solution in mineral oils or in tetralin, poly-methylmethacrylate solutions in dimethyl-pthalate, ruer-toluene solutions, and so on. Certain phenomena in these fluids, such as anomalous viscosity\*, the Weissenberg effect\*\*, Merrington effect\*\*, and spinnaility effect\*\*\*\*, could not be explained by the solutions of Navier-Stokes equations, necessitating a systematic study into the foundations of fluid dynamics.

The motion of electrically conducting fluids in the presence of electric and magnetic fields is studied through MHD. When an electro-magnetic field is applied to a conducting fluid, it behaves differently than when the field is not applied. The Lorentz force, which is a cross product of electric and magnetic fields (Sir Flemming's right hand law), is primarily responsible for this. The existence of a strong magnetic field alters the flow pattern even when there is no external electric field.

Electric current is generated by a magnetic field and the velocity of conducting fluid particles. With a chain reaction, the current and magnetic fields interact and affect the flow motion. All three fields (velocity, magnetic, and electric) are interrelated and display extremely distinctive properties.

Heat transfer is a branch of research that aims to anticipate energy transfer between material bodies as a result of temperature differences. The discipline of heat transfer, in its most basic form, is concerned with only two things: temperature and heat movement. The quantity of thermal energy available is represented by temperature, whereas heat flow is the transfer of thermal energy from one location to another. Thermal energy is proportional to the kinetic energy of molecules on a tiny scale. The thermal agitation of a material's component molecules increases as its temperature rises (as evidenced in both linear and vibrational modes). It's only natural for areas with more molecular kinetic energy to transfer that energy to regions with less.

#### FLUIDS OF THE SECOND ORDER:

Green et. Al., Coleman and Noll proposed a theory of a more broad sort of incompressible fluid. The theory is based on the idea that stress is a function of the deformation gradient, and that the stress at a given material location is solely determined by the deformation gradient's prior history. Simple materials by Noll are the materials that are used in this theory. If an incompressible simple fluid has the feature that all local states with the same mass density are inherently identical in response, it is an incompressible simple material. A retarded history  $g_c(s)$  can be defined as follows for a given history  $g(s)$ :  $g_c(s) = g(s)$ ,  $0 \leq s \leq c$ ,  $g_c(s) = g(s) = g(s) = g(s) = g(s)$  (1.3) where  $c$  is the retardation factor, and  $0 < c < 1$  is the retardation factor. Coleman and Noll established that the theory of simple fluids gives the theory of perfect fluids by taking into account this concept of delayed history and assuming that stress is more responsive to recent deformation than to deformations that happened in the distant past ( in which deviatoric stress is independent of strain-rate) The theory of simple fluids produces the theory of perfect fluids (in which



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Non-Newtonian flows over an oscillating plate with variable suction have been studied by Hayat.

The flow past of a torsionally oscillating plane has been studied by Chawla. Riley and Wyrow looked at the flow caused by an elliptic cylinder's torsionally oscillations. A study of two-dimensional flow past an oscillating cylinder has been considered by Bluckurn.

In the presence of a transverse magnetic field, Sadhna Kahre investigated the constant flow between a revolving and porous stationary disc.

In the case of Newtonian fluid, Sharma and Agarwal addressed heat transmission from an enclosed spinning disc. Following it, K. R. Singh and H.G. Sharma discussed heat transport. The heat transmission from an enclosed spinning disc in the case of Newtonian fluid has been explored by K. R. Singh and H.G. Sharma.

Following that, a second-order fluid flows between two enclosed rotating discs. Rosenlat has addressed the torsional oscillations of Newtonian fluids. He's also spoken about the situation where the Newtonian fluid is trapped between two infinite torsionally rotating discs. Sharma and Gupta studied the flow of a second-order fluid between two infinite torsionally oscillating discs in a generic example. The problem of heat transfer in the flow of non-Newtonian second-order fluid between torsionally oscillating planes was then solved by Sharma & K. R. Singh. Riley and Wyrow studied the flow caused by an elliptic cylinder's torsional oscillations. In the presence of a transverse magnetic field, Sadhna kahre investigated the constant flow between a revolving and porous stationary disc.

Terrill and Shrestha studied the effects of a magnetic field on steady laminar flow of an incompressible viscous fluid in a two-dimensional channel when the walls are of different permeability's and discussed the problem of steady laminar flow of an incompressible viscous fluid in a two-dimensional channel when the walls are of different permeability's. Agrawal has studied the problem of flow of a second-order fluid with heat transfer in a tube with porous walls. Sharma and Singh investigated the numerical solution of a second-order fluid flow via a porous channel in a transverse magnetic field.

In the case of Newtonian fluid, Sharma and Agarwal addressed heat transmission from an enclosed spinning disc. Following it, K. R. Singh and H.G. Sharma discussed heat transport. The heat transmission from an enclosed spinning disc in the case of Newtonian fluid has been explored by K. R. Singh and H.G. Sharma. Following that, a second-order fluid flows between two enclosed rotating discs. Rosenlat has addressed the torsional oscillations of Newtonian fluids. He's also spoken about the situation where the Newtonian fluid is trapped between two infinite torsionally rotating discs. Sharma and Gupta studied the flow of a second-order fluid between two infinite torsionally oscillating discs in a generic example. The problem of heat transfer in the flow of non-Newtonian second-order fluid between torsionally oscillating planes was then solved by Sharma & K. R. Singh. Riley and Wyrow studied the flow caused by an elliptic cylinder's torsional oscillations. In the presence of a transverse magnetic field, Sadhna kahre investigated the constant flow between a revolving and porous stationary disc.

### The Study's Objectives

We explore the flow pattern of an incompressible second-order fluid between two parallel infinite discs in the presence of a transverse magnetic field while one is spinning (named rotor) and the other is at rest in our current problem (called stator). The stator receives a consistent infusion, producing the paper's subject content. The stator is aligned with the plane  $z = d$ , whereas the rotor is aligned with the plane  $z = 0$ . The effects of elastic-viscosity and cross-viscosity are regulated by the dimensionless parameters  $1(2/pd^2)$  and  $2(2/pd^2)$ , respectively, whereas the effects of injection are guided by a non-dimensional parameter  $k (=w_0/2d)$ , where  $w_0$  is the uniform suction velocity (negative for injection).

### Methodology of Study

The following are the governing equations that will be utilised in the problems:

#### Continuity Equation:

According to the rule of conservation of mass, fluid mass cannot be generated or destroyed. The goal of the equation of continuity is to explain the rule of mass conservation in a mathematical manner.

The equation of continuity explains the fact that the increase in the mass of fluid within any closed surface drawn in the fluid at any time must equal the excess of the mass that flows in over the mass that flows out in continuous motion.  $\rho \frac{d}{dt} \int_V dV + \int_V \nabla \cdot (\rho \mathbf{u}) dV = 0$

Where  $\mathbf{u}_i$  and  $\rho$  are the fluid's velocity vector and density, respectively. This equation is reduced to for incompressible fluids.  $\nabla \cdot \mathbf{u} = 0$  (1.7)

#### 2. The Equation of Momentum:

These equations are based on Newton's law of motion, which remains the focal point of all continuum mechanics except relativistic mechanics.

$$\rho \frac{d\mathbf{u}}{dt} = \nabla \cdot \mathbf{T} + \mathbf{f} \quad (1.8)$$

The impressed force per unit mass of fluid is  $\mathbf{F}$ , and the stress tensor is  $\mathbf{T}$ . For no additional force, the momentum equation is simple.

$$\rho \frac{d\mathbf{u}}{dt} = \nabla \cdot \mathbf{T} \quad (1.9)$$

#### 3. Energy Equation:

This equation is based on Thermodynamics' first law. The energy balance of an incompressible fluid is dictated by the internal energy, heat conduction, heat convection with the stream, and heat creation through friction. When the volume of a compressible fluid is altered, there is an extra term owing to the work of expansion (or compression). Radiation may be present in all scenarios, but its



$$J_2 / + kgigT, IJ + cv(T/t + umT, m) = J_2 / + kgigT, IJ + (1.17)$$

As a result, the equation of motion will change.

$$(u_i/t + um_{ui,m}) = J x + m_i, m (u_i/t + um_{ui,m}) (u_i/t + um_{ui,m}) (u_i/t + um_{ui,m})$$

$$(u_i/t + um_{ui,m}) (1.18)$$

## CONCLUSION AND RESULTS

The fluctuation of radial velocity for different elastic-viscous parameters  $\lambda = 1.3, -2, -2.6$ ; when cross-viscous parameter  $\lambda = 10$ , injection parameter  $k = 5$  Reynolds number  $R = 0.05$ , magnetic field  $m = 5$  reveals that the radial velocity w.r.t. curve is ell shaped with maximum at about  $\approx 0.5$ .

It is also clear that as  $\lambda$  grows from  $\approx 0.0-0.28$ , the radial velocity reduces, then begins to increase as  $\lambda$  increases up to  $\approx 0.72$ , and finally declines as  $\lambda$  increases from  $\approx 0.8-0.95$ . For all values of  $\lambda$ , the radial velocity is about equal at  $\approx 0.28$  and  $\approx 0.72$ . For all values of the elastic-viscous parameter  $\lambda$ , the point of maximum lies in the centre of the gap length.

No one has attempted to tackle the most practical problems of enclosed torsionally oscillating discs so far due to the complexity of the differential equations and the time consuming computations of the solutions. The authors looked at the current problem of flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating disc in the presence of a magnetic field and computed the steady and unsteady parts of both flow functions satisfactorily. The flow functions are multiplied by the powers of the amplitude (which is assumed to be minimal) of the disc oscillations. The non-Newtonian effects are exhibited by two dimensionless factors  $\lambda_1 (= \eta_2 / \eta_1)$  and  $\lambda_2 (= \eta_3 / \eta_1)$ , where  $\lambda_1, \lambda_2, \lambda_3$  are Newtonian viscosity coefficients, elastic viscosity coefficients, and cross viscosity coefficients, respectively, and  $\omega$  is the oscillation's uniform frequency.

The variation of the radial velocity with  $\lambda = 2, \lambda = 5, R = 5, R_m = 0.05, R_L = 0.049, R_z = 2, m = 2$  for different values of elastic-viscous parameter  $\lambda = 0, -0.3$  and phase difference  $\phi = \pi/3, 2\pi/3$  shows that at  $\phi = \pi/3$ , the radial velocity increases with an increase in  $\lambda$  near the lower disc, reaches its maximum value at  $\lambda = 0.2$ , then begins to decrease, reaches its minimum value at  $\lambda = 0.8$ , and then increases near It is evident that the radial velocity rises with an increase in  $\lambda$  near the lower disc, then begins to decrease with an increase in  $\lambda$  towards the top disc following the point of junction.

For  $\phi = 2\pi/3$ , the radial velocity increases with an increase in  $\lambda$  and then starts dropping at  $\lambda = 0$ , but for  $\lambda = -0.3$ , it declines first, reaches its minimum value at  $\lambda = 0.1$ , then starts growing, reaches its greatest value at  $\lambda = 0.7$ , and finally decreases up to the top disc's surface. The radial velocity similarly increases with an increase in  $\lambda$  up to the middle of the gap-length and then reduces with an increase in  $\lambda$  up to the top disc's surface.

The authors looked at the current problem of heat transfer in the flow of a non-Newtonian second-order fluid over enclosed torsionally oscillating discs with

uniform suction and injection in the presence of a magnetic field and calculated the steady and unsteady parts of the flow and energy functions successfully. In the powers of the amplitude (assumed to be modest) of the disc oscillations, the flow and energy functions are extended. The non-Newtonian effects are exhibited by two dimensionless parameters  $1 (=n^2/1)$  and  $2 (=n^3/1)$ , where 1, 2, 3 are Newtonian viscosity coefficients, elastic viscosity coefficients, and cross viscosity coefficients, respectively, and  $n$  is the oscillation's uniform frequency. The variation of temperature distribution with elastic-viscous parameter 1, cross-viscous parameter 2 (based on the relation  $1 = a 2$ , where  $a = -0.2$  as for 5.46 percent poly-iso-utylenes type solution in cetane at 300C (Markowiz38), Reynolds number  $R_1$  magnetic field  $m$ , and suction parameter  $k$  at different phase differences is graphically shown.

Figures 1 and 2 demonstrate the fluctuation of the temperature distribution with  $R = 7$ ,  $P = 6$ ,  $\omega = 5$ , and  $\omega = 0.02$ ,  $k = 15$ ,  $m = 10$ ,  $E = 5$  for different values of  $1 = 1, 1.2, 3$  when  $\omega = \sqrt{3}$  and  $2/\sqrt{3}$  respectively. The temperature change is paraolic from the vertex downwards, as shown by the results. It's also evident that the temperature is lowest in the centre of the gap and remains negative throughout the gap, with the exception of at the bottom disc's surface. Temperature rises with an increase in the elastic-viscous parameter in the first half of the gap-length, before being overlapped in the second half. It is expected that when 1 increases in the midst of the gap-length, the temperature lowers and is then overlapped.

The temperature fluctuation is paraolic with vertex downwards at  $1 = 5$ ,  $P = 6$ ,  $\omega = 5$ ,  $\omega = 0.02$ ,  $k = 15$ ,  $m = 10$ ,  $E = 5$  for different values of  $R = 1, 1.5, 2$  when  $\omega = \sqrt{3}$  and  $2/\sqrt{3}$ . It's also clear that the temperature is lowest in the centre of the gap length and remains negative throughout, with the exception of at the bottom disc's surface. It is also obvious from these that as the Reynolds number  $R$  increases, the temperature lowers across the gap-length.

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**POOJA GOEL:** PH.D. RESEARCH SCHOLAR DEPT. OF MATHEMATICS, MAHARISHI SCHOOL OF SCIENCE, MUIT UNIVERSITY, LUCKNOW, U.P.

**DR. MANOJ SRIVASTAVA:** PROFESSOR, RESEARCH GUIDE, DEPT. OF MATHEMATICS, MAHARISHI SCHOOL OF SCIENCE, MUIT UNIVERSITY, LUCKNOW, U.P.