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MARKOVIAN HETEROGENEOUS ARRIVAL RATES AND SINGLE SERVER WORKING VACATION QUEUE WITH IMPATIENT CUSTOMERS

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Abstract. In this paper, we analyse an $M/M/1$ queueing system with impatient customer and working vacation. The server begins the working vacation when the server finds no customer in the system and the server serves in a vacation period with slow service rates. If the server come back from a working vacation and find the queue empty, it starts the another working vacation. Otherwise, it switches to a normal busy period. If the server is busy or working vacation When a customer comes to the system, it activates an impatience timer. If the customer's service has not finished before the customer's impatience timer expires and never comes back, the customer leaves the system. The balance equations of the model are all derived using state-transition diagram. Then, we obtain various performance measures such as the mean system sizes. Some numerical results and graphical representation are also represented.

1. Introduction

Queueing system with customer's impatience are occur in our everyday life. Many authors treated the impatience phenomenon under various assumptions.When the system is empty during the working vacation period, the server takes a vacation to attend secondary jobs at different rates.

The models considered in this paper have applications in practical systems. For example, consider a leather product-inventory system with impatience timer. The job of the facility is to produce leather bags to fulfill customer's orders. The manufacturing plant may produce leather bags in a make-to-stock manner in order to meet demand. The production manager, however, does not want to maintain a higher inventory level becauseit will increase the holding expenses. Therefore, the manager may halt bag production if the final order is fulfilled and no order occurs. The manager may decide to wait for the orders if none arrive at that moment. Depending on whether a production facility is open when an order arrives, it is either temporarily out of stock or filled from the inventory. Customers whose orders are temporarily out of stock may become impatient and decide to cancel their orders if waiting time exceeds a customer's level of patience (the customer may have different impatient times). Such a system can be modeled by our models

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developed in this paper.

Altman and Yechiali [1], analyse of $M/M/1$ and $M/G/1$ queues, for multiple and the single-vacation cases, and obtain various types of results. Oliver and Isijola [8], consider $M/M/1$ queueing system with multiple vacation. Goswami [6], analyze $M/M/1$ queue with impatince customers, working vacations and Bernoulli schedule vacation interruption. Sudhesh and Azhagappan [18], studies an $M/M/1$ model with server's vacation and annoyed customers in which the server activates the timer. Laxmi et al. [13], analyse an infinite size of a server Markovian queue with a working vacation and annoyed customers.

Swathi and Kumar [19], describes for both vacation models during variant states like as busy, repair and breakdown are presented. Manoharan and Ashok [14], In the stationary state, obtained the distributions of the additional queue length and sojourn time of a customer. Gupta and Kumar [7], considers single retrial model with a waiting server subject to repair and breakdown under working vacation, vacation interruption. Chakravarthy [4], deals with the classical queueing models, the server providing services is assumed to be available at all times even when there is no customer in the system. Choudhury [5], analyses a $M/G/1$ queue with unreliable server and two phases of heterogeneous service. Amina and Latifa [2], deals with a $M/M/1$ feedback queueing system under balked customers and two differentiated multiple vacations. Dequan et al. [20], consider an $M/M/1$ queueing system with impatient customers and vacations. Perel and Yechiali [15], analyse two-phase multi-server queueing system with customer's impatience. Parimala [17], analyse a $M/M(a,b)/(2,1)$ queueing system of two heterogeneous servers and bulk service with various service rates. Bounkhel et al. [3], analyse $M/M/1$ queueing system with server adaption by using a strategy where the service provided can be either single or bulk depending on some threshold level c.

Kumar and Shinde [12], deals with bulk arrival and service under vacation interruption and also analyse the steady-state behavior. Selvaraju and Goswami [16], consider an single server Markovian queue with impatient customers and working vacations. Kumar and Sharma [11], analyse the transient solution of a Markovian queuing system with two heterogeneous servers and retention of reneging customers. Jain et al. [10], study the operating characteristics of an $M^X/H_k/1$ queueing system under multiple vacation policy. Jain [9], examine the characteristics to $M/M/1$ queue with infinite capacity and functioning vacation.

2. The Model and Analysis

We consider the multiple working vacation in $M/M/1$ queueing system with customer's impatience timer.

- Customers arrival according to a Poisson process with arrival rates λ_1 and λ_0 for busy period and working vacation respectively. Then the service rate μ_b for busy period is exponentially distributed.
- The server begins a working vacation when the queue becomes empty, and vacation time follows an exponential distribution with rate ϕ. During a working vacation an arriving customer is served at the rate of μ_v which is exponentially distributed. If the server returns from a working vacation

to find the queue empty, it begins another working vacation. If the queue is not empty, it switches to a normal busy period.

- Whenever a customer comes and finds the server is busy or working vacation, then the server activates a timer, which is exponentially distributed with parameter ϵ_1 or ϵ_0 for busy period or working vacation.

Let $N(t)$ be the number of customers in the system at the time t.

Let $J(t) = \begin{cases} 0, \text{system is in working vacation at time } t \\ 1, \text{system is in busy period at time } t \end{cases}$

The process $\{N(t), J(t); t \geq 0\}$ is defined as a continuous-time markov process with the state space $\Omega = \{(0,0) \bigcup (n,j), j = 0,1; n \geq 1\}$

$$P_{n,0} = \lim_{t\to\infty} P[N(t) = n, J(t) = 0], n \geq 0$$

$$P_{n,1} = \lim_{t\to\infty} P[N(t) = n, J(t) = 1], n \geq 1$$

The balance equations are given below:

$$\lambda_0 P_{0,0} = (\mu_v + \epsilon_0)P_{1,0} + (\mu_b + \epsilon_1)P_{1,1} \tag{2.1}$$

$$(\lambda_0 + \mu_v + n\epsilon_0 + \phi)P_{n,0} = \lambda_0 P_{n-1,0} + (\mu_v + (n+1)\epsilon_0)P_{n+1,0}, n \geq 1 \tag{2.2}$$

$$(\lambda_1 + \mu_b + \epsilon_1)P_{1,1} = \phi P_{1,0} + (\mu_b + 2\epsilon_1)P_{2,1} \tag{2.3}$$

$$(\lambda_1 + \mu_b + n\epsilon_1)P_{n,1} = \lambda_1 P_{n-1,1} + \phi P_{n,0} + (\mu_b + (n+1)\epsilon_1)P_{n+1,1}, n \geq 2 \tag{2.4}$$

The normalizing condition as follows,

$$\sum_{n=0}^{\infty} P_{n,0} + \sum_{n=1}^{\infty} P_{n,1} = 1. \tag{2.5}$$

We define the Probability Generating Functions as follows,
$G_0(z) = \sum\limits_{n=0}^{\infty} P_{n,0}z^n$ and $G_1(z) = \sum\limits_{n=1}^{\infty} P_{n,1}z^n$
Multiplying equation (2.2) by z^n, using equation (2.1) and summing all possible values of n, we get

$$\epsilon_0 z(1-z)G_0'(z) - [(\lambda_0 z - \mu_v)(1-z) + \phi z]\, G_0(z) = -[\phi z - (1-z)\mu_v]\, P_{0,0} - (\mu_b + \epsilon_1)zP_{1,1} \tag{2.6}$$

In a similar way, we get from equations (2.3) and (2.4)

$$\epsilon_1 z(1-z)G_1'(z) - (\lambda_1 z - \mu_b)(1-z)G_1(z) = -\phi z G_0(z) + [\phi P_{0,0} + (\mu_b + \epsilon_1)P_{1,1}]z \tag{2.7}$$

3. The solutions of differential equations

For $z \neq 0$ and $z \neq 1$, equation (2.6) can be written as follows

$$G_0'(z) - \left[\frac{\lambda_0}{\epsilon_0} - \frac{\mu_v}{z\epsilon_0} + \frac{\phi}{\epsilon_0(1-z)}\right] G_0(z) = \frac{1}{\epsilon_0}\left[\frac{A}{z} - \frac{B}{1-z}\right] \tag{3.1}$$

where $A = \mu_v P_{0,0}$ and $B = \phi P_{0,0} + (\mu_b + \epsilon_1)P_{1,1}$
To solve the first order linear differential equation (3.1), we get an integrating

factor (IF) as $e^{\frac{-\lambda_0 z}{\epsilon_0}} z^{\frac{\mu_v}{\epsilon_0}} (1-z)^{\frac{\phi}{\epsilon_0}}$
Multiplying both sides of (3.1) by IF, we get

$$\frac{d}{dz}\left[e^{\frac{-\lambda_0 z}{\epsilon_0}} z^{\frac{\mu_v}{\epsilon_0}} (1-z)^{\frac{\phi}{\epsilon_0}} G_0(z)\right] = \frac{1}{\epsilon_0}\left[\frac{A}{z} - \frac{B}{1-z}\right] e^{\frac{-\lambda_0 z}{\epsilon_0}} z^{\frac{\mu_v}{\epsilon_0}} (1-z)^{\frac{\phi}{\epsilon_0}}$$

Integrating both sides of above from 0 to z, we get

$$G_0(z) = e^{\frac{\lambda_0 z}{\epsilon_0}} z^{\frac{-\mu_v}{\epsilon_0}} (1-z)^{\frac{-\phi}{\epsilon_0}} \left[\frac{AK_0(z) - BK_1(z)}{\epsilon_0}\right] \tag{3.2}$$

where $K_0(z) = \int_0^z (1-s)^{\frac{\phi}{\epsilon_0}} e^{\frac{-\lambda_0 s}{\epsilon_0}} s^{\frac{\mu_v}{\epsilon_0}-1} ds$ and $K_1(z) = \int_0^z (1-s)^{\frac{\phi}{\epsilon_0}-1} e^{\frac{-\lambda_0 s}{\epsilon_0}} s^{\frac{\mu_v}{\epsilon_0}} ds$
From equation (3.2), as $0 \leq G_0(1) = \sum_{n=0}^{\infty} P_{n,0} \leq 1$ and $\lim_{z\to 1}(1-z)^{\frac{-\phi}{\epsilon_0}} \to \infty$, so we have

$$e^{\frac{\lambda_0}{\epsilon_0}}\left[\frac{A}{\epsilon_0}K_0(1) - \frac{B}{\epsilon_0}K_1(1)\right] = 0$$

After substituting the values of A and B, we get

$$P_{0,0} = \frac{(\mu_b + \epsilon_1)K_1(1)P_{1,1}}{\mu_v K_0(1) - \phi K_1(1)} \tag{3.3}$$

Equation (3.2) can be written as

$$G_0(z) = \frac{e^{\frac{\lambda_0 z}{\epsilon_0}} z^{\frac{-\mu_v}{\epsilon_0}} (1-z)^{\frac{-\phi}{\epsilon_0}}}{\epsilon_0}\left[\mu_v K_0(z)P_{0,0} - (\mu_b + \epsilon_1)K_1(z)P_{1,1} - \phi K_1(z)P_{0,0}\right] \tag{3.4}$$

Substituting equation (3.3) into equation (3.4), we obtain

$$G_0(z) = \frac{e^{\frac{\lambda_0 z}{\epsilon_0}} z^{\frac{-\mu_v}{\epsilon_0}} (1-z)^{\frac{-\phi}{\epsilon_0}} \mu_v}{\epsilon_0}\left[K_0(z) - \frac{K_0(1)K_1(z)}{K_1(1)}\right] P_{0,0} \tag{3.5}$$

For $z \neq 1$ and $z \neq 0$, equation (2.7) can be rewritten as follows:

$$G_1'(z) - \left[\frac{\lambda_1}{\epsilon_1} - \frac{\mu_b}{z\epsilon_1}\right] G_1(z) = \frac{B - \phi G_0(z)}{\epsilon_1(1-z)} \tag{3.6}$$

To solve the first order linear differential equation (3.6), we get an integrating factor (IF) as $e^{\frac{-\lambda_1 z}{\epsilon_1}} z^{\frac{\mu_b}{\epsilon_1}}$
Multiplying both sides of (3.6) by IF, we get

$$\frac{d}{dz}\left[e^{\frac{-\lambda_1 z}{\epsilon_1}} z^{\frac{\mu_b}{\epsilon_1}} G_1(z)\right] = \frac{[B - \phi G_0(z)]}{\epsilon_1(1-z)} e^{\frac{-\lambda_1 z}{\epsilon_1}} z^{\frac{\mu_b}{\epsilon_1}}$$

Integrating both sides of above from 0 to z, we get

$$G_1(z) = e^{\frac{\lambda_1 z}{\epsilon_1}} z^{-\frac{\mu_b}{\epsilon_1}} \left[\frac{B}{\epsilon_1}K_2(z) - \frac{\phi}{\epsilon_1}\int_0^z G_0(s) e^{-\frac{\lambda_1 s}{\epsilon_1}} s^{\frac{\mu_b}{\epsilon_1}} (1-s)^{-1} ds\right] \tag{3.7}$$

where $K_2(z) = \int\limits_0^z e^{-\frac{\lambda_1 s}{\epsilon_1}} s^{\frac{\mu_b}{\epsilon_1}} (1-s)^{-1} ds$

Using equation (3.3) and substituting equation (3.5) into equation (3.7), we get

$$G_1(z) = \frac{e^{\frac{\lambda_1 z}{\epsilon_1}} z^{-\frac{\mu_b}{\epsilon_1}} \mu_v}{\epsilon_1} \left[\frac{K_2(z)K_0(1)}{K_1(1)} - \frac{\phi}{\epsilon_0 K_1(1)} (K_1(1)K_3(z) - K_0(1)K_4(z)) \right] P_{0,0} \tag{3.8}$$

where $K_3(z) = \int\limits_0^z e^{\left(\frac{\lambda_0}{\epsilon_0} - \frac{\lambda_1}{\epsilon_1}\right)s} (1-s)^{\frac{-\phi}{\epsilon_0}-1} s^{-\frac{\mu_v}{\epsilon_0}+\frac{\mu_b}{\epsilon_1}} K_0(s) ds$

$$K_4(z) = \int\limits_0^z e^{\left(\frac{\lambda_0}{\epsilon_0} - \frac{\lambda_1}{\epsilon_1}\right)s} (1-s)^{\frac{-\phi}{\epsilon_0}-1} s^{-\frac{\mu_v}{\epsilon_0}+\frac{\mu_b}{\epsilon_1}} K_1(s) ds$$

Define $P_{.0} = G_0(1) = \sum\limits_{n=0}^{\infty} P_{n,0}$ and $P_{.1} = G_1(1) = \sum\limits_{n=1}^{\infty} P_{n,1}$

Put $z = 1$ in equations (2.6) and (3.3), we get

$$P_{.0} = G_0(1) = \frac{\mu_v K_0(1) P_{0,0}}{\phi K_1(1)} \tag{3.9}$$

Put $z = 1$ in equation (3.8), we get

$$P_{.1} = G_1(1) = \frac{e^{\frac{\lambda_1}{\epsilon_1}} \mu_v P_{0,0}}{\epsilon_1} \left[\frac{K_2(1)K_0(1)}{K_1(1)} - \frac{\phi}{\epsilon_0 K_1(1)} (K_1(1)K_3(1) - K_0(1)K_4(1)) \right] \tag{3.10}$$

Noting that $P_{.0} + P_{.1} = 1$, we get from Equations (3.9) and (3.10)

$$P_{0,0} = \left\{ \frac{e^{\frac{\lambda_1}{\epsilon_1}} \mu_v P_{0,0}}{\epsilon_1} \left[\frac{K_2(1)K_0(1)}{K_1(1)} - \frac{\phi}{\epsilon_0 K_1(1)} (K_1(1)K_3(1) - K_0(1)K_4(1)) \right] + \frac{\mu_v K_0(1)}{\phi K_1(1)} \right\}^{-1} \tag{3.11}$$

4. Performance Measures

Let $E[L_{sb}]$ be the the average size of the system when the server is in busy state, let $E[L_{sv}]$ be the average size of the system when the server is in working vacation state.

From equation (3.1), using L'Hopital rule, we have

$$\begin{aligned} E[L_{sv}] &= \lim_{z\to 1} G_0'(z) = \lim_{z\to 1} \frac{[((\lambda_0 - \mu_v)(1-z) + \phi)\, zG_0(z) + A(1-z) - Bz]}{\epsilon_0 z(1-z)} \\ &= \frac{\mu_v}{(\phi+\epsilon_0)\phi K_1(1)} \left[(\lambda_0 - \mu_v)K_0(1) - \phi K_1(1)\right] P_{0,0} \end{aligned}$$

From equation (3.6), using L'Hopital rule, we have

$$\begin{aligned} E[L_{sb}] &= \lim_{z\to 1} G_1'(z) = \lim_{z\to 1} \left[\frac{\lambda_1}{\epsilon_1} - \frac{\mu_b}{z\epsilon_1} \right] G_1(z) + \frac{B - \phi G_0(z)}{\epsilon_1(1-z)} \\ &= \frac{\mu_b e^{\lambda_1 \epsilon_1}(\lambda_1 - \mu_b)}{\epsilon_1^2} \left[\frac{K_2(1)K_0(1)}{K_1(1)} - \frac{\phi}{\epsilon_0 K_1(1)} (K_1(1)K_3(1) - K_0(1)K_4(1)) \right] P_{0,0} \\ &+ \frac{\mu_v}{\epsilon_1(\phi+\epsilon_0)K_1(1)} \left[(\lambda_0 - \mu_v)K_0(1) - \phi K_1(1)\right] P_{0,0} \end{aligned}$$

where $P_{0,0}$ is given by equation (3.11).
Average size of the system and throughput are obtained as,

$$E[N] = E[L_{sv}] + E[L_{sb}]$$
$$TP = \mu_v \sum_{n=0}^{\infty} P_{n,0} + \mu_b \sum_{n=1}^{\infty} P_{n,1}$$

Mean waiting time,

$$E[W] = \frac{E[N]}{\lambda_{eff}}$$

where $\lambda_{eff} = \lambda_0 P_{0,0} + \sum_{n=1}^{\infty} (\lambda_0 P_{n,0} + \lambda_1 P_{n,1})$

Mean delay time,

$$E[D] = \frac{E[N]}{TP}$$

5. Numerical Results

We find performance measures numerically in this section. We consider the parameters as $\lambda_0 = 0.3, \lambda_1 = 0.6, \phi = 0.3, \epsilon_0 = 2,\ \mu_b = 2.0, \mu_v = 1, \epsilon_1 = 3$ for all the figures and table. In figures, we varied λ_0=0.1 to 0.5 and λ_1=0.5 to 1.4. In all the figures, shows that the average size of the system by varying λ_0 and λ_1 for different parameters $\mu_b, \mu_v, \phi, \epsilon_0, \epsilon_1$. The figures shows that upward trend lines and the values are increases for both λ_0 and λ_1.

In figure 1, displays that the average size of the system by varying λ_0 and λ_1 for different parameters ϵ_0 and ϵ_1. In figure 2, explains that the average size of the system by varying λ_0 and λ_1 for different parameters μ_b, μ_v and ϕ. As the graphs for $E[N]$ in figure 1, reveals that the increasing trend with respect to λ_1 and depict that $E[N]$ increases for lowering the values ϵ_0 then for the increasing values of ϵ_1. As the graphs for $E[N]$ in figure 2, reveals that the increasing trend with respect to λ_0 and depict that $E[N]$ increases for lowering the values μ_v and μ_b, then for the increasing values of ϕ.

In table 1, we varied $\lambda_1 = 0.5$ to 1.4 and the probability $P_{.,0}$ is decreases and $P_{.,1}$ is increases. From table 2, we can see that the arrival rate (λ_1) increase then $E[L_{sb}]$ also increase. Also the percent variation indicates the increasing trend for $E[L_{sb}]$ and decrease for $P_{0,0}$ and $E[L_{sv}]$. From table 3, shows that the effect of arrival rate for vacation (λ_0) on $E[L_{sv}]$ and $E[L_{sb}]$. As λ_0 increases, $E[L_{sv}]$ also increases and hence $P_{0,0}$ and $E[L_{sb}]$ decrease. Also percent variation increases for $E[L_{sv}]$ and decrease for $P_{0,0}$ and $E[L_{sb}]$. From table 4, depicts that the effect of service rate for busy period (μ_b) on $E[L_{sv}]$ and $E[L_{sb}]$. As μ_b increases, $P_{0,0}$ and $E[L_{sv}]$ increases and hence $E[L_{sb}]$ decreases. Also the percent variation increases for $P_{0,0}$ and $E[L_{sv}]$ and decreases for $E[L_{sb}]$. From table 5, shows the effect of service rate for working vacation (μ_v) on $E[L_{sv}]$ and $E[L_{sb}]$. As μ_v increases, $P_{0,0}$ and $E[L_{sv}]$ decreases and hence $E[L_{sb}]$ increases. Also the percent variation increase for $P_{0,0}$, $E[L_{sb}]$ and decreases for $E[L_{sv}]$. From table 6 and 8, indicates the effect of ϕ and ϵ_0 on $E[L_{sv}]$ and $E[L_{sb}]$. As ϵ_0 and ϕ increases,$P_{0,0}$ and $E[L_{sb}]$ increase and hence $E[L_{sv}]$ decreases. Also the percent variation decrease for $P_{0,0}$ and $E[L_{sv}]$ and increases for $E[L_{sb}]$. From table 7, reflects the effect of vacation rate (ϵ_1) on the performance measures. As ϵ_1 increases, $P_{0,0}, E[L_{sv}], E[L_{sb}]$ increase. Also the

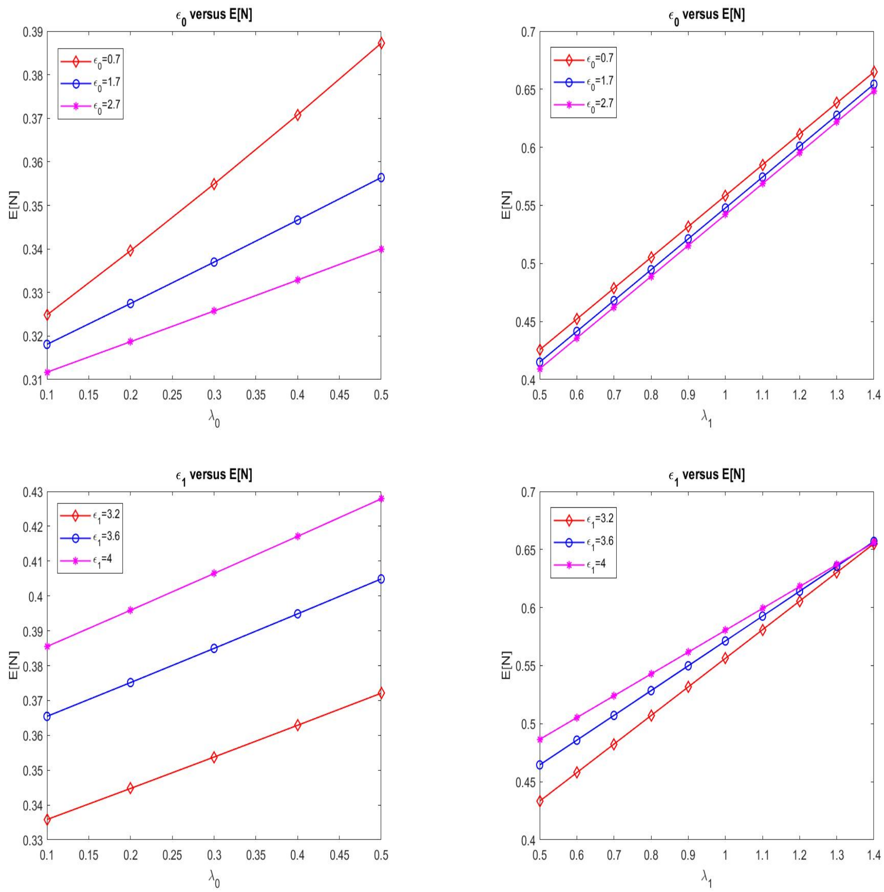

FIGURE 1. Average size of the system by varying λ_0 and λ_1 for different parameters ϵ_0, ϵ_1

percent variation increase for $P_{0,0}$, $E[L_{sv}]$ and $E[L_{sb}]$.

λ_1	$P_{.,0}$	$P_{.,1}$	λ_1	$P_{.,0}$	$P_{.,1}$
0.5	0.2079307	0.7920693	1.0	0.2066783	0.7933217
0.6	0.2076893	0.7923107	1.1	0.2064137	0.7935863
0.7	0.2074435	0.7925565	1.2	0.2061442	0.7938558
0.8	0.2071931	0.7928069	1.3	0.2058696	0.7941304
0.9	0.2069381	0.7930619	1.4	0.2055899	0.7944101

TABLE 1. Effect of λ_1 on probabilities

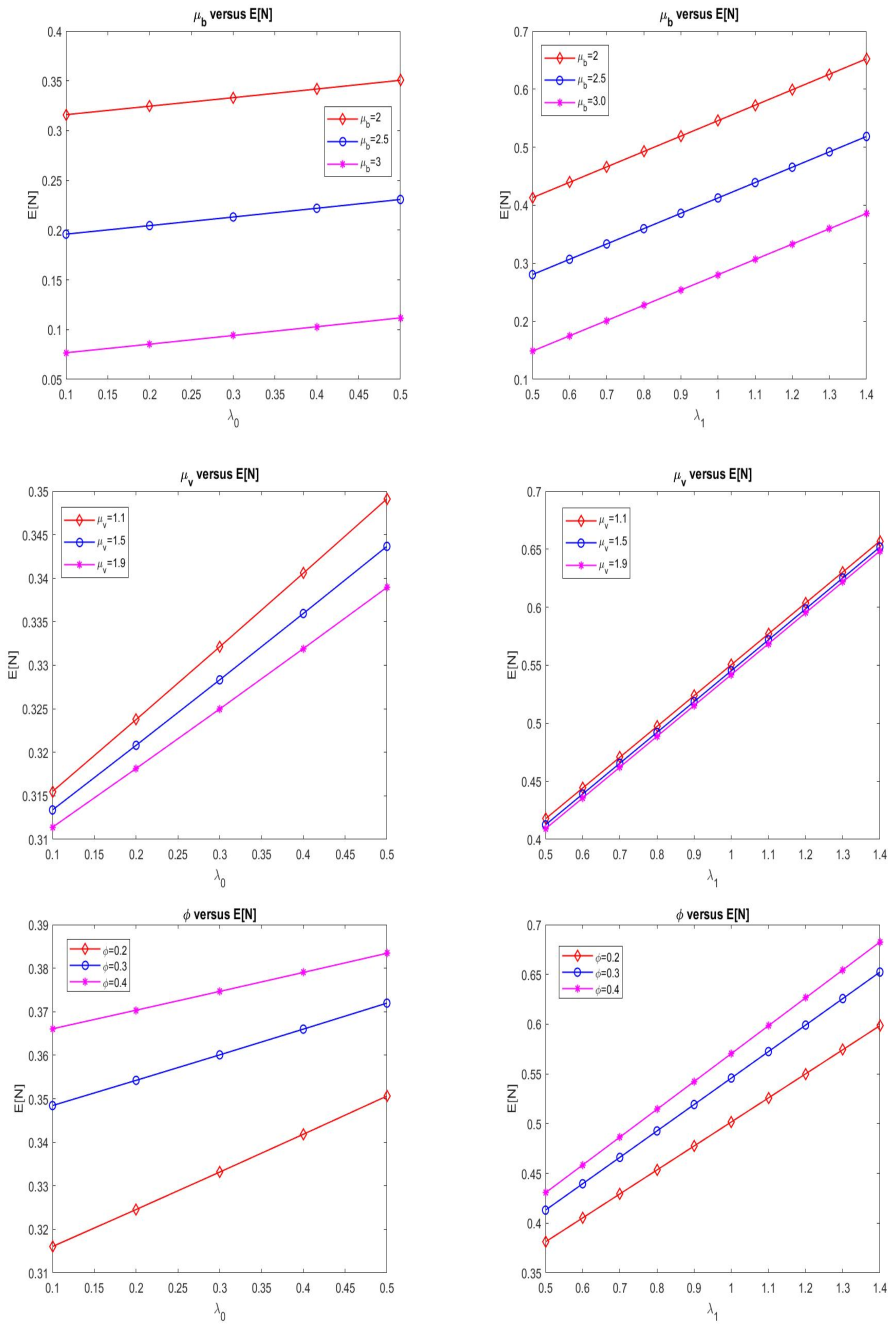

FIGURE 2. Average size of the system by varying λ_0 and λ_1 for different parameters μ_b, μ_v, ϕ

λ_1	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
0.5	0.1890460	0.0189107	0.3941436
0.7	0.1886030	0.0188664	0.4472287
0.9	0.1881435	0.0188204	0.5003905
1.1	0.1876668	0.0187727	0.5536331
1.3	0.1871721	0.0187232	0.6069610
Percentage variation	-0.18739	-0.01875	21.28174

TABLE 2. Effect of λ_1 on performance measures

λ_0	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
0.1	0.2007112	0.0059960	0.4219661
0.2	0.1946892	0.0124077	0.4213249
0.3	0.1888266	0.0188887	0.4206768
0.4	0.1831204	0.0254378	0.4200219
0.5	0.1775681	0.0320537	0.4193603
Percentage variation	-2.31431	2.60577	-0.26058

TABLE 3. Effect of λ_0 on performance measures

μ_b	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
2.1	0.1892738	0.0189334	0.3940161
2.2	0.1896891	0.0189750	0.3674048
2.3	0.1900763	0.0190137	0.3408376
2.4	0.1904387	0.0190500	0.3143100
2.5	0.1907788	0.0190840	0.2878182
Percentage variation	0.1505	0.01506	-10.61979

TABLE 4. Effect of μ_b on performance measures

μ_v	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
1.1	0.1893211	0.0183051	0.4207352
1.2	0.1897893	0.0177508	0.4207906
1.3	0.1902332	0.0172234	0.4208434
1.4	0.1906547	0.0167210	0.4208936
1.5	0.1910554	0.0162417	0.4209415
Percentage variation	0.17343	-0.20634	0.02063

TABLE 5. Effect of μ_v on performance measures

ϕ	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
0.1	0.3357471	0.0131419	0.2981150
0.2	0.2504307	0.0240329	0.3812105
0.3	0.1888266	0.0188887	0.4206768
0.4	0.1502763	0.0146950	0.4437486
0.5	0.1246913	0.0118234	0.4588807
Percentage variation	-21.10558	-0.13185	16.07657

TABLE 6. Effect of ϕ on performance measures

ϵ_1	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
3.1	0.1935108	0.0193573	0.4297943
3.2	0.1981217	0.0198185	0.4380660
3.3	0.2026609	0.0202726	0.4455750
3.4	0.2071303	0.0207197	0.4523945
3.5	0.2115314	0.0211599	0.4585890
Percentage variation	1.80206	0.18026	2.87947

TABLE 7. Effect of ϵ_1 on performance measures

ϵ_0	$P_{0,0}$	$E[L_{sv}]$	$E[L_{sb}]$
2.1	0.1891207	0.0182243	0.4207433
2.2	0.1893471	0.0175858	0.4208071
2.3	0.1895068	0.0169709	0.4208686
2.4	0.1896013	0.0163773	0.4209280
2.5	0.1896324	0.0158035	0.4209853
Percentage variation	0.05117	-0.24208	0.0242

TABLE 8. Effect of ϵ_0 on performance measures

6. Conclusion

In this paper, we have analyzed in an $M/M/1$ queueing system under the server in working vacation and busy period with impatient timer. In real life situations, this model can be applied for industrial overcrowding problems like computer communication networks, telecommunications and manufacturing system. Also we discussed a example about leather product-inventory system. The steady state equations, various performance measures, some numerical analysis and graphs are presented in this paper.

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